VOLUME III

16 March 1959

To Teachers Using the PSSC COURSE:

The Teacher's Guide material for the laboratory experiments associated with Volume 3 is enclosed.

The green sheets (For Home, Desk, and Lab) of the Teacher's Guide for each chapter of Vol 3 have included a table listing the problems and experiments related to each section. You should note that, on pages 3 and 4 of the enclosed yellow sheets, there is a detailed recommendation as to the optimum time for each experiment--before, during, or after the development of the section in class. In the detailed discussion of each experiment, the reasons for the recommended time-placement are given.

Teacher's Guide

LABORATORY EXPERIMENTS

PHYSICS

Volume 3, Mechanics

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These materials were prepared by the group in Cambridge that developed the experiments, apparatus and Laboratory Guide for Volume 3 of the text.

PREFACE

This Teacher's Laboratory Guide deals only with experiments in Volume III of the Text but, since this is the first Teacher's Laboratory Guide, it may not be out of place to include some suggestions in the preface on laboratory work in general.

1. The Place of the Laboratory in this Course

Our knowledge of physics is the result of years of experimentation. Of course we cannot have every high school student experience all the discoveries himself but we wish him to come as close as possible to this goal for at least the most fundamental discoveries. Whenever possible, we would like the student to learn the new material in the laboratory, i.e., to make discoveries for himself. To enable him to do this, the laboratory guide provides procedural instruction and directs his attention to the important points by raising questions. Some students may feel a little insecure at the beginning and may ask it they have the "right" answer. No result is ever wrong. If a student has not measured what he has set out to measure, a discussion is in order rather than a "yes" or "no" answer. A good student however, enjoys doing experiments whose results he does not know in advance.

In this course, the laboratory not only replaces teacher demonstrations to a large extent but also reduces the time spent in classroom explanations. The laboratory is closely correlated with the text and is of equal importance.

2. Doing an Experiment

When a physicist does an experiment, he has a purpose for doing it. We suggest that you take time in the class period before the laboratory to discuss the purpose of the experiment with the students. The purpose of an experiment may be very general. For example, in the first ripple tank experiment, the main purpose is to observe waves and to learn about their general features. Curiosity usually leads the student to make explorations of his own and prepares the ground for later quantitative measurements. In Volume III, the purpose of the experiment on slow collisions is to provide a study, on a convenient time scale, of what happens during a collision.

Other experiments are quantitative in nature. The purpose of Experiment #460, for example, is to discover how a constant force changes the velocity of a cart. Its objective is not to verify that a constant froce produces a constant acceleration.

When discussing any experiment prior to the laboratory period, be sure not to give away the expected results.

Once the apparatus is set up and is in working order, we are ready to make measurements and take data. Finally we analyze the data, discuss the accuracy of the measurements and draw our conclusions. Together all of this makes an experiment, just taking data does not.

Often data taking requires equipment beyond the means of a high school. In such cases we divide the experiment in the following way: the Laboratory Guide provides the data and leaves the analysis to the student (Experiment 490 and the alternative method in 510). This is not a poor compromise. It is common practice among research workers who need specialized equipment such as high energy accelerators to have their photographic plates exposed in an accelerator in a different part of the country and then analyze the tracks in their own laboratories.

Whenever time permits, have the student repeat a measurement several times, take an average and judge its accuracy from the fluctuations. (For example, Experiment 540). When this is not possible, at least have the students try to estimate the expected error and express this as a fraction or percent. The Guide gives the maximum accuracy to expect in each experiment.

Very often experimental results are best summarized in graphs. Make sure your students learn how to draw and read them (See Volume 1, Chapter 4). The slide rule is a very useful, time saving device. We recommend its use.

The analysis of the data in many experiments in Volume III is quite time consuming and should be completed at home. This raises the questions of laboratory reports. More often than not, these are a chore for the student who has to write them and for the teacher who has to read them. We recommend that each student keep a neat record of what he does at the time he does it. Only an occasional experiment should be written up as a formal report.

3. Planning

The value of each experiment is greatly increased when done at the proper time. The table at the end of the preface and the first paragraph in the discussion of each experiment in this guide will help you in setting up your timetable.

The experiments are presented in a flexible way to accomodate a variety of class conditions such as the length of period and the ability of students. A class period usually allows enough time to take the basic measurements and to do some of the paper work. The rest should be done at home. Not all students should do everything suggested in the Laboratory Guide. Usually the last few questions are for the ambitious student only and may involve additional experimental work in the nature of a special project. Such open-ended parts are often indicated in the Guide.

4. Best Time and Relative Priority of Volume III Experiments

Number	Experiment	Time	Priority
450	A Variation on Galileo's Experiment	Before or just after Chapter 1, section 2.	*
460	Changes in Velocity with a Constant Force	Before Chapter 1, section 3.	***
470	The Dependence of Acceleration on Force and Mass	Before Chapter 1, section 4.	***
480	Inertial and Gravitational Mass	After Chapter 1, sections 5 and 6	**
490	Forces on a Ball in Flight	After Chapter 2, section 4.	**
500	Centripetal Force	Before Chapter 2, section 5.	***
510	Law of Equal Areas	During middle of Chapter 3.	*

Number	Experiment	Time	Priority
520	Momentum Changes in an Explosion	During Chapter 4, section 2.	***
530	The Cart and the Brick	During Chapter 4, section 4.	**
540	A Collision in Two Dimensions	During Chapter 4, section 4.	***
550	Slow Collisions	During Chapter 5, section 5.	***
560	Changes in Potential Energy	After Chapter 6, section 3.	***
570	The Energy of a Simple Pendulum	After Chapter 6, section 3.	**
580	A Head-on Collision	During Chapter 7.	*

*** essential
** desirable
* optional

A VARIATION ON GALILEO'S EXPERIMENT

This modification of Galileo's "thought" experiment, described in Volume 3, Chapter 1, Section 2 and shown in Figure 4 can be done after it has been studied in the text or you may also try to start the study of mechanics with this experiment by raising the question [illegible] whether objects can keep moving without being pushed. If time is at a premium, this experiment may be done as a demonstration or omitted in favor of the more important experiments (#460 and #470) on Newton's Law.

Galileo reasoned that, since a ball launched down a ramp tended to rise to the same height on an opposing ramp, a ball launched onto a horizontal surface should roll forever if there were no friction. The purpose of this experiment is to help the student to reach the same conclusion.

The arc the pendulum traces as it moves from its release position to its lowest point is analogous to Galileo's launching ramp. The arc it swings through from the lowest point after leaving the barrier corresponds to the plane of gradual slope in Figure 4 in the text. Raising the clamp is analogous to diminishing the slope which the ball climbs in Galileo's experiment. It is best to use as long a pendulum as possible but reasonable results can be obtained with a pendulum about one meter long.

Testing the pendulum first without a barrier (Fig. 1) shows the student that the distance to either side of the center point and the heights reached are approximately equal under these conditions. When asked to conjecture about the distance and height after release when the barrier is used (Fig. 2), he is faced with a dilemma, resolved by experimentation, which makes him realize that the height of rise is the critical factor in determining the horizontal distance.

At this point, the student, in looking for ways to improve his estimation of the bob's height of rise, may want to put a horizontal string at the level of release. This will also help him to duplicate the release level. He will probably find he does not let the bob go the same way each time. To obtain a more constant release, he may want to use a thread to hold the bob to one side, releasing the bob by burning the thread. However, the essential results can be obtained without these improvements in technique. They should be optional and not required.

Make sure the student observes that the bob nearly reaches its release height regardless of the length of the second arc. This will lead him to conclude that the bob should continue to attain its release height as the arc increases. An infinitely long pendulum would swing in an "arc" which is a straight line. The bob on such a pendulum would never reach its original height and consequently would swing out forever.

Here we assume a flat, infinitely large earth. Note that on the surface of the earth a horizontal surface is only flat for distances small in comparison to the radius of the earth. For larger distances a horizontal surface follows the curve of the earth.

* * *

An air puck is an excellent illustration of uniform motion in the absence of forces. Have several on hand when the students do the pendulum experiment. Their construction is shown in Figures (a) and (b). The operation of the puck is similar to that of the dry ice discs described in the text. In both cases the disc rides on a nearly frictionless air bearing between the disc and the table. The air flow is limited by a small hole. If it is too small, air will not escape fast enough to support the disc. If it is too large, the balloon will deflate too rapidly.

Let the students experiment with the pucks qualitatively, noting that on a smooth table at very low velocities, the puck will slide the length of a table, apparently without slowing down. If the table is not level, the pucks will accelerate from rest. Experimenting with these pucks makes Galileo's law more real.

Square pucks are much simpler to make than round pucks. You may use any flat wood available but single grain wood is either warped when you get it or warps later and is not very satisfactory. Quarter-inch plywood will work but not as well as three-eights or one-half inch plywood. Masonite can also be used. Make the pucks about ten centimeters square. Larger sizes can be made but it is more difficult to make them flat.

It is not worthwhile to apply known forces to the air puck and to measure its accelerations quantitatively since very small irregularities in the table will cause large fluctuations in velocity at low speed. The thick aluminimum table used in the photographs of the motion of dry ice discs in the text was very carefully made to be as level as possible. Since it is not easy to duplicate such a surface, carts with roller skate wheels, run at high velocities on ordinary tables, are

used in the following experiments in the laboratory manual. Although they encounter more friction, they give more satisfactory results on an ordinary table than do the air bearing pucks.

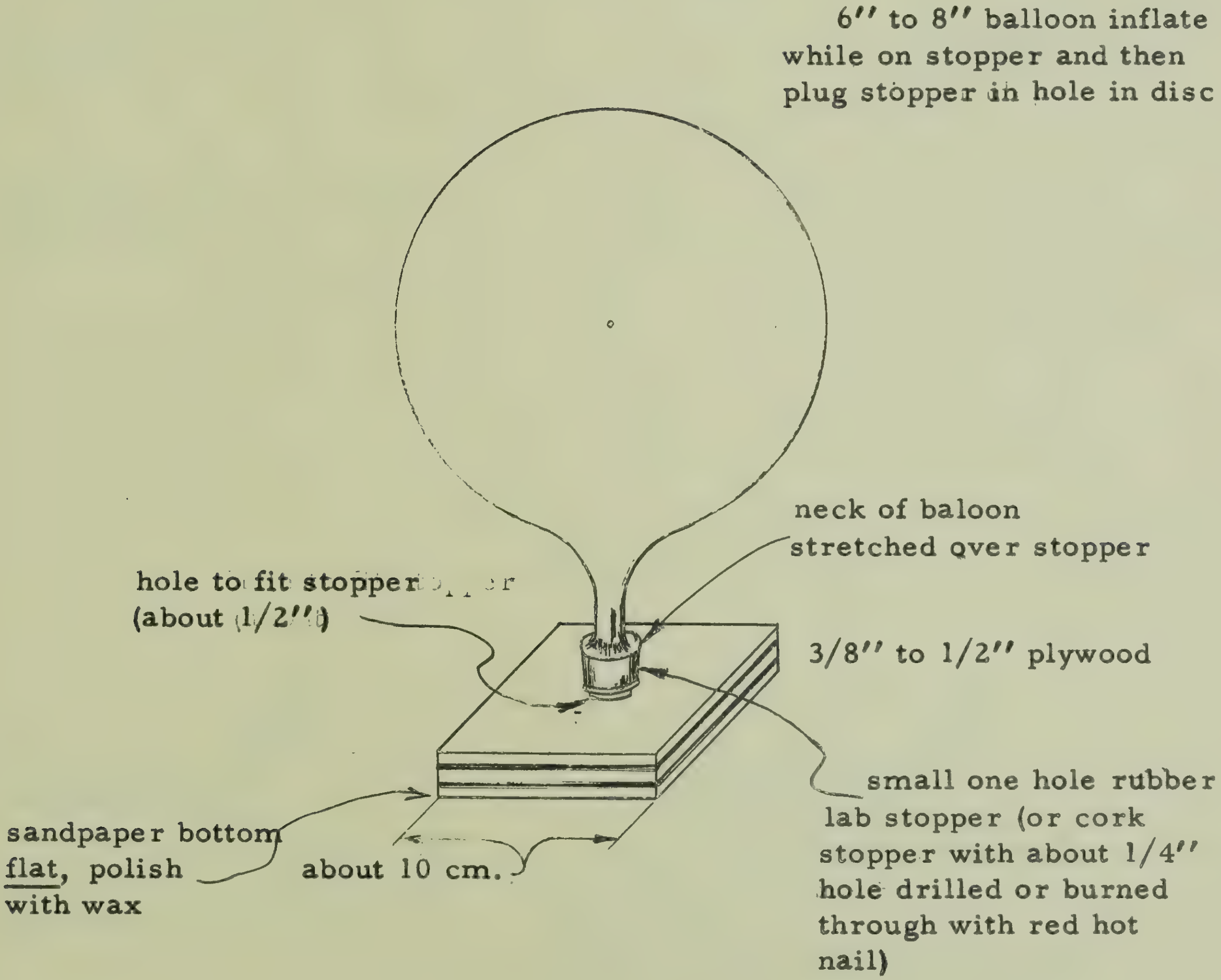
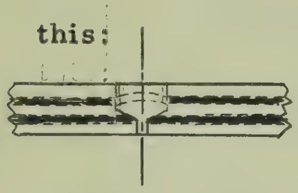

6" to 8" balloon inflate while on stopper and then plug stopper in hole in disc
neck of baloon stretched over stopper
hole to fit stopper (about 1/2")
3/8" to 1/2" plywood
small one hole rubber lab stopper (or cork stopper with about 1/4" hole drilled or burned through with red hot nail)
sandpaper bottom flat, polish with wax
about 10 cm.

FIG. (a)

SEVERAL WAYS TO MAKE THE AIR VENT:

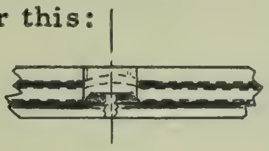
this:



Drill with twist drill to take stopper.

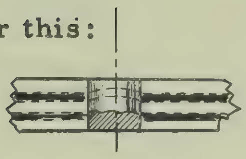
Drill or punch with nail a hole about 1 mm. in dia. and sand flat.

or this:



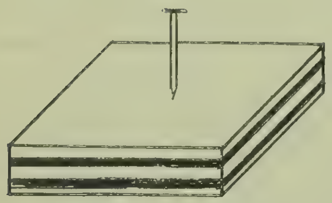
With auger bit, bore part way through--let lead screw of bit make small hole in bottom. Sand flat.

or this:

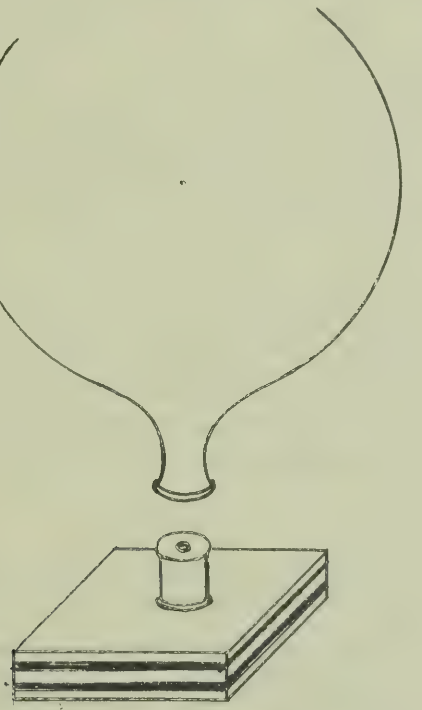


Bore all the way through. Drip hot paraffin or wax to fill bottom part of hole. Make small hole in wax with pin. Sand flat.

AN EASY PUCK TO MAKE USING A SPOOL:



Drive a small nail all the way through, and pull it out. Turn the block over and---



Cement a small spool over the nail hole. Sand the bottom flat and polish with wax.
Inflate balloon---hold neck closed between second and third fingers leaving index finger free to stretch neck over spool.

FIG. (b)

CHANGES IN VELOCITY WITH A CONSTANT FORCE

This experiment and the following one (#470 - The Dependence of Acceleration on Force and Mass) are very important and should not be omitted. Since they lead the student to discover Newton's law, they should be done before it is discussed in class.

As a result of doing this first Newton's law experiment, the student will conclude that, within his experimental error, applying a constant force to an object changes its velocity at a constant rate, i.e. its acceleration is constant. Stress that the units used in measuring forces and velocities are not important in discovering or expressing this fundamental law. The unit of force in this experiment is given by a rubber loop stretched a constant amount and velocities are expressed in meters per tick of the timer.

The experiment cannot be run properly on the floor since it is too difficult to apply a constant force to the cart as it accelerates. A smooth, level table about two meters long is desirable. If the table cannot be leveled this will show up when the tapes made by a coasting cart are examined.

The timing tape should pass smoothly through a timer clamped to the table with a "C" clamp and should offer minimum friction to the motion of the cart. The total frictional drag on the cart is about 0.3 newtons, a small force compared to the force applied through the rubber strand. The tapes should be labeled for future identification as soon as they are made.

We do not use gravity as a pulling force on the cart because it raises too many questions that cannot be answered at this stage of the course. The student has no real way of knowing that gravity supplies a steady force to a moving object and, in the next experiment where masses must be measured, it is hard for him to see that the mass of the falling weight must be included as part of the total mass being accelerated.

Practice in using the apparatus is necessary before making any quantitative runs. It is not easy to keep the rubber strands extended the length of the meter stick at the start and while the cart is accelerating at high velocities near the end of the run. In starting the cart, synchronization is important between the one who tows the cart and his partner who releases it.

The need for practice in operating the apparatus will mean that, if the laboratory period is short, the quantitative analysis of the tapes when the cart is pulled with a constant force will have to be done as homework.

A final word of warning. The rubber loops are ideal for shooting paper clips and miscellaneous missiles about the lab!

Answers to Questions

The fractional changes in velocity of a coasting cart are less at high velocity than at low velocity. The distance covered between two successive ticks of the timer at the beginning and end of the tape can be compared by holding the two ends of the tape side by side. There is no need for accurate quantitative measurements here. It is sufficient to see that a rapidly coasting cart moves at a reasonable constant velocity on a horizontal surface.

The reason for the nearly constant speed at high velocity can be explained as follows: a gentle depression in the middle of the table, only 1 mm below the surface and many centimeters long, undetectable to the eye and scarcely detectable with a good spirit level, will change the velocity of a cart starting from rest by 14 cm/sec!

change the velocity of a cart starting from rest by 14 cm/sec

($v = \sqrt{2\ gd} = \sqrt{2 \times 9.8 \times 10^{-3}} = 0.14$ m/sec). If the cart has an initial velocity v_o,

$$v^2 - v_o^2 = 2\ gd$$

$$(v + v_o)(v - v_o) = 2\ gd$$

$$\Delta v = v - v_o = \frac{2\ gd}{v + v_o}$$

if $v_o >> 2\ gd$

then $v \approx v_o$

$$\Delta v = \frac{gd}{v_o}$$

and the fraction change in velocity will be

$$\frac{\Delta v}{v_o} = \frac{gd}{v_o^2}$$

which decreases rapidly as v_o increases.

For this reason all experiments with carts are best done at high velocities; low velocity runs are very erratic. Be sure, however, that the student does not analyze the end of the tapes representing the highest velocities, where he was probably unable to keep the applied force constant.

A constant force produces a constant acceleration. The points on the graph are likely to be randomly distributed close to a straight line. These deviations from a straight line result primarily from the student's inability to keep the rubber bands stretched a constant amount. The student should offer this explanation on the basis of his own observations.

The force exerted by the rubber strands is not the only force exerted on the cart. The drag of the timing tape, friction in the wheel bearings, and between the wheels and the table exert forces opposite to the motion. These forces produce negligible velocity changes compared to those produced by the applied force, as we can conclude from the coasting run.

It is easy to see from inspection of the graphs that with the same force and different loads on the cart, the acceleration of the larger mass is less than that of the smaller mass. Quantitative measurements of the effect of mass on acceleration will be done in the next experiment.

SUPPLEMENTARY NOTES ON CONSTRUCTION AND USE OF CARTS AND TIMERS

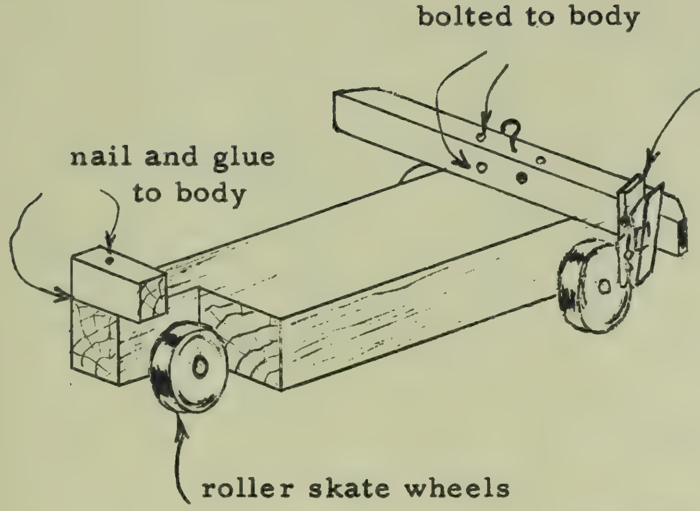
The Carts

The construction of the carts used in this experiment and in Experiments 470, 520, 530, 550 and 580 is shown in Figure (a). The body of the cart is a piece of 2"x 4" and the wheels are ball-bearing roller skate replacement wheels. The wheels should be oiled only with the thinnest oil or kerosene since heavy oil and grease increase the friction. The guide holes for the screws should be accurately drilled so the wheels will run true. The spring clothespin that holds the timing tape is fastened to the crossbar by a long narrow screw running through the center of the spring. If the long crossbar is made of hardwood it will be less likely to break if the cart is dropped.

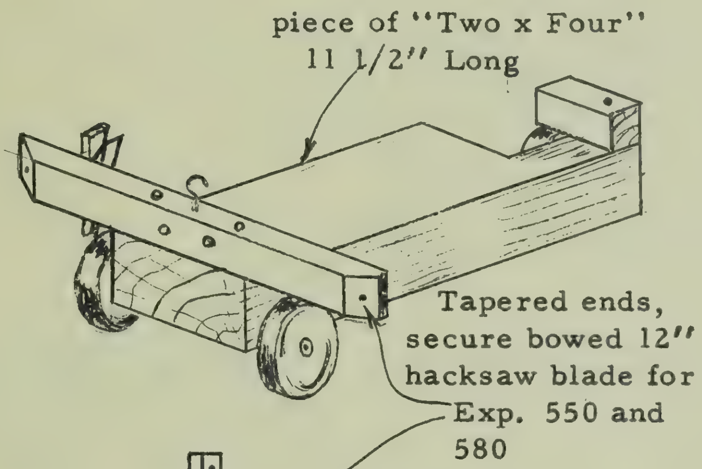
The Timer

The timing device, made from an electric door bell, is shown in Figure (b). Ideally, the frequency of the vibrating clapper should be dependent only upon the inertia of the clapper and the stiffness of the spring it is mounted on. It should not depend on the driving voltage. This will be nearly true if the timer is supplied with one to three volts and current flows through the bell only in short pulses for a fraction of a cycle. It then operates like a pendulum clock which receives energy only in short bursts, once a cycle, from the escapement. Most bells operate this way, but if the electric make-and-break contact is not far enough from the magnet, the clapper will vibrate at different frequencies when electrically driven than it will when plucked with the finger. Bending the contact away from the clapper a small amount will remedy this. It is best to use a bell whose clapper has a large inertia--one with a solid, half-ball clapper, not a hollow, pressed steel, hemispherical clapper.

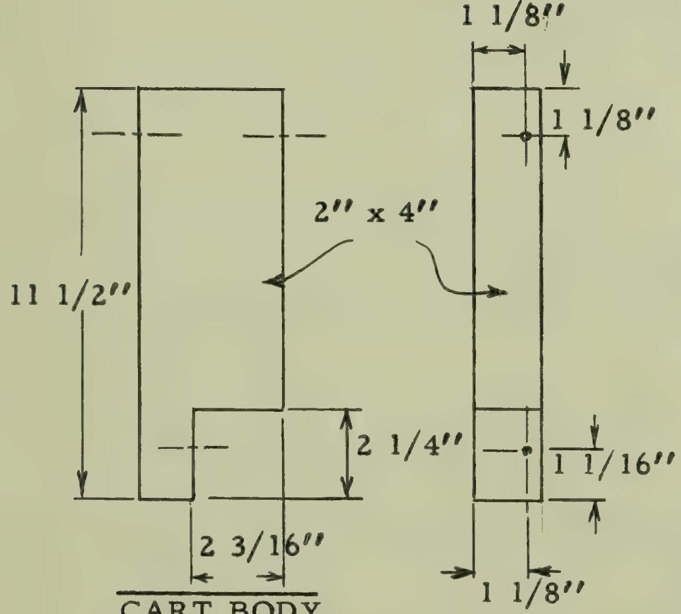
In addition to its use in the experiment described in the Laboratory Guide, the timer can be used for other experiments such as measuring the acceleration of gravity. The frequency of most bells is near fifty cycles per second but this can be reduced by at least a factor of two by increasing the inertia of the clapper. Wire can be wound tightly around it or pinch clamps or battery clips can be fastened to it to reduce the frequency. For very low frequencies when it is heavily loaded, the timer should be mounted so the clapper vibrates in



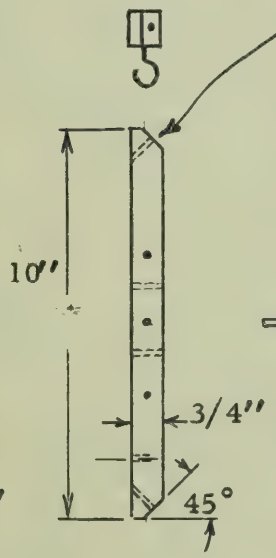
REAR VIEW


piece of "Two x Four" 11 1/2" Long
Tapered ends, secure bowed 12" hacksaw blade for Exp. 550 and 580

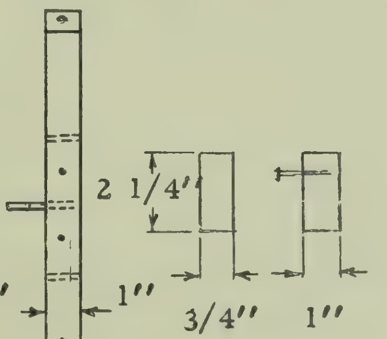
FRONT VIEW



CART BODY

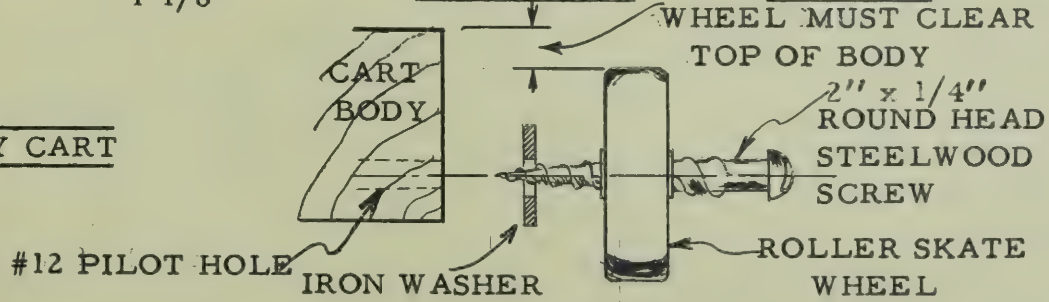


FRONT BAR



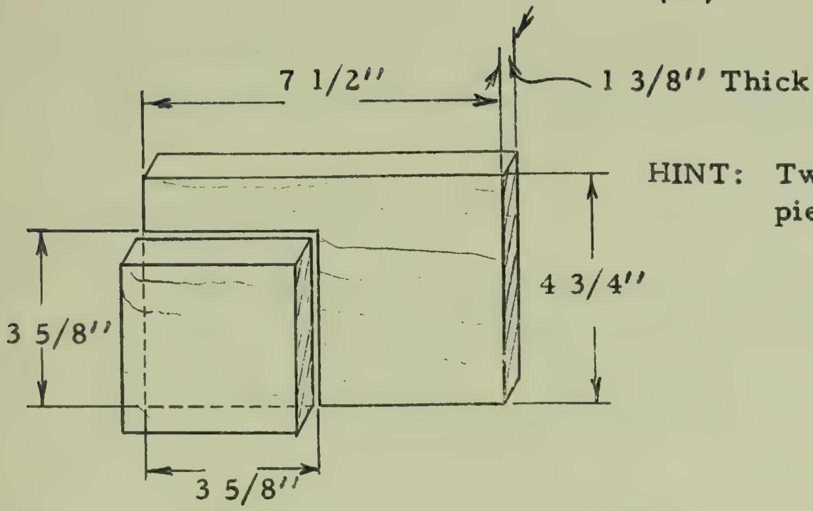
REAR BAR

LABORATORY CART

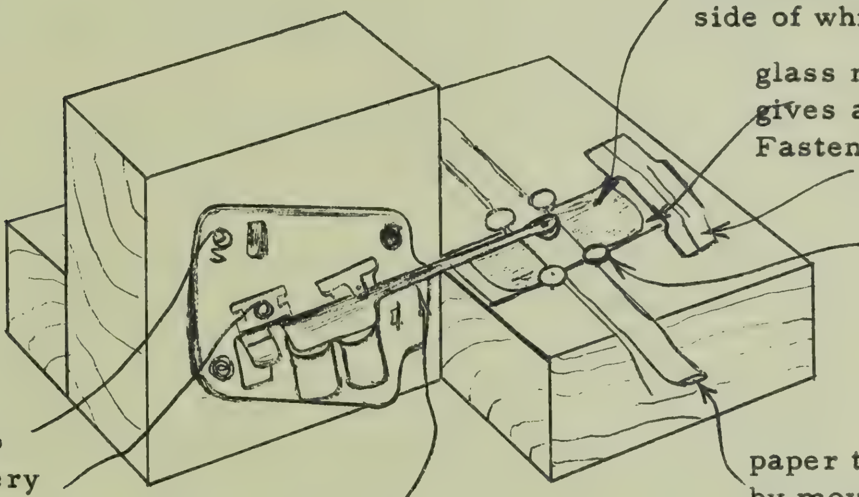

WHEEL MUST CLEAR TOP OF BODY
CART BODY
2" x 1/4" ROUND HEAD STEELWOOD SCREW
#12 PILOT HOLE
IRON WASHER
ROLLER SKATE WHEEL

WHEEL DETAIL

FIG. (a)



HINT: Two saw cuts gives you both pieces


strip of carbon paper, black side up. Timing marks are made on under side of white tape

glass miscroscope slide gives a hard striking surface Fasten down with scotchtape

thumbtack guides for paper tape

paper tape pulled thru timer by moving body

To Battery

- In some bells this armature is slanted. In this case, skew the bell frame around until armature is horizontal. Use bell with straight armature

- Adjust bell so clapper just barely strikes paper at end of its stroke

- Attach bell frame with screws smaller than the holes

FIG. (b)

a horizontal plane. Otherwise the clapper will be so heavy that either the spring cannot pull it back to the contact or the magnet cannot lift it.

Calibrating the Timer

For most of the experiments in the Laboratory Guide, the timer need not be calibrated; time measurements are made in units of ticks (ten ticks, etc.) It is only in Experiment 570, "The Energy of a Simple Pendulum", that the timer must be calibrated in seconds in order to compare kinetic and potential energy changes.

Calibration can be done by pulling a tape through the operating timer for a measured time interval and then counting the number of ticks. The tape does not have to be pulled at constant speed, only fast enough so the marks on the paper are resolved.

Pulling the tape through a 50 cycle/sec timer for ten seconds may give only five percent accuracy in timing and yet there will be 500 ticks to count.

This can be simplified by pulling the tape through a timer for one minute to give about one percent accuracy and then cutting the tape into as many pieces as there are students. Each student counts a small number of ticks and the results are pooled. The timer is so calibrated becomes a "standard" and all other timers can easily be calibrated from the standard by pulling two tapes simultaneously through the standard and the uncalibrated timer, mounted side by side. The tapes are pulled only long enough (at a roughly constant speed) to give about 40 or 50 ticks. The uncalibrated tape is then calibrated from the "standard" tape.

A timer can also be calibrated by free fall. A heavy "C" clamp is clamped to the end of a piece of tape as heavy as the tape will support and is allowed to fall, pulling the tape through the timer for about a meter. Assuming the acceleration is 9.8 m/sec^2, the time between ticks can then be found. Friction should be kept to a minimum and the timer should be mounted in the position in which it is to be used since its frequency is slightly different when the clapper vibrates horizontally instead of vertically.

The timer could be calibrated with a stroboscope, although even with well arranged lighting it is hard to keep the clapper "stopped" for the minute or so needed for an accurate calibration.

THE DEPENCENCE OF ACCELERATION ON FORCE AND MASS

Having learned in Experiment 460 that a constant force causes a constant acceleration, the student proceeds to investigate the effect of different forces on the acceleration of a fixed mass and the effect of applying the same force to different masses. He will discover by experimentation the relationship between force, mass, and acceleration and the independence of this fundamental law of any selection of units for the different quantities.

The technique is the same as in the previous experiment and the same precautions should be followed. Care is needed in the second part where a considerable force is exerted by several rubber strands extended the length of the meter stick. The meter stick should not be used for archery practice since it easily covers the length of the classroom and curves erractically in its flight! Because the screwhook in the end of the meter stick may possibly work loose and pull out, check it after each run involving large forces. A safer arrangement is to fasten a screw bolt to the meter stick with washers and a nut. Be sure to slow the cart gently at the end of the run to prevent damage to it, and to the toes from falling bricks.

In analyzing the tape, the student may assume, after doing Experiment 460, that the acceleration is constant throughout the run. This saves time since he need only measure the distance traveled in a given number of ticks to find the acceleration from $a = \frac{2d}{t^2}$. Since the marks may not be very clear at the start of the tape, counting may begin at the first clear tick. The velocity is so small near the start of the run that this procedure will introduce only a negligible error. Keeping the force constant near the end of a fast run is difficult and the student should take this into consideration in choosing the number of ticks to use for his constant time interval. It is advisable to complete all the runs first and then to decide upon the desired number of ticks.

The mass of the bricks and cart must be found on a balance to plot the graph of acceleration as a function of mass. Ideally, we should not use the balance, which measures gravitational mass, but

should accelerate different numbers of objects of the same size and material, for example, a number of identical carts. This is, of course, clumsy and impractical. The different concepts of inertial and gravitational mass will be clarified in the next experiment.

Answers to Questions

The graph of acceleration as a function of mass is a hyperbola, but since it is difficult to recognize it, the graph will not lead to any quantitative conclusions about the relationship between mass and acceleration. One can only conclude that as the mass increases, the acceleration decreases. If the student recalls plotting the reciprocal of the volume of a gas as a function of the pressure in Experiment 220 (Boyle's Law) he may, without being told, plot the mass as a function of the reciprocal of the acceleration.

A conclusion drawn from the graph of the reciprocal of acceleration as a function of mass is unreliable if the graph contains only three data points. If students work in groups of four and five, each one can analyze a different tape and the results can be pooled for drawing the graph. Ideally, the straight line should not pass exactly through the origin since the wheels on the cart are given rotational motion and their rotational inertia may show up on the graph as equivalent in the apparatus to an unaccounted mass of about 0.2 kg. Therefore, one would expect the straight line to cross the mass axis at about - 0.2 kg. The experimental data will not, however, be accurate enough to show this.

The graph of the acceleration of a given mass as a function of force comes close enough to a straight line to convince the student that force and acceleration are proportional. It would pass through the origin if there were no friction. With friction, the graph should show a positive value of force at zero acceleration. At this point the net force is zero; the applied force is equal and opposite to the friction force. In this experiment the friction is so small compared to the pull of the rubber strands that failure to maintain a constant pull may obscure the force of friction. The graph may cross the force axis slightly above or below the origin and cannot be depended upon to measure the force of friction.

The two experiments lead to the conclusion that the equation relating force, mass and acceleration is $F = kma$. The value of $\underline{k}$ depends upon the choice of units. The answer $F = ma$ is correct only if the student uses correctly related units like kilograms for mass, meters/$(n \text{ ticks})^2$ for acceleration, and consequently kilogram-meters/$(n \text{ ticks})^2$ for the force. (n is the number of ticks taken for a fixed time interval in making the analyses and drawing the graphs).

The timer must be calibrated in seconds per tick to express the accelerations in m/sec^2. Suppose there are 0.02 seconds per tick of the timer and 80 ticks were taken as the time interval, then an acceleration of 0.4 meters per $(80 \text{ ticks})^2$ is equal to

$$a = \frac{0.4 \text{ m}}{(80 \times .02 \text{ sec})^2} = 0.16 \text{ m/sec}^2$$

The accelerating force in newtons would be this acceleration times the accelerated mass in kilograms. To answer this question the student will have to read ahead in the text to find the definition of a newton (Volume 3, Chapter 1, sec. 7).

The mass of a chunk of lead or a heavy stone is most easily determined with the apparatus used in this experiment in the following way: pull the unknown mass on the cart with one rubber loop and measure the acceleration $\underline{a}$ in meters per $(n \text{ ticks})^2$. Then, in terms of the acceleration $\underline{a'}$ with one brick on the cart (data he already has),

$$\frac{m_x + m_{cart}}{m_{brick} + m_{cart}} = \frac{a'}{a}$$

$$m_x = \frac{a'}{a}(m_{brick} + m_{cart}) - m_{cart}$$

This experiment is long and probably all of the analysis will have to be done as homework unless the experiment is divided into two parts and done in two consecutive laboratory periods.

INERTIAL AND GRAVITATIONAL MASS

This experiment is not as essential to the course as the two experiments on Newton's law but it gives quite accurate results and will help clarify the relation between inertial and gravitational mass. It is best done after discussion of Sections 5 and 6 of Chapter 1.

The experiment is intended to show that although gravitational and inertial mass are measured by independent experiments, the ratios of inertial masses of different objects are always equal to the ratios of their gravitational masses. The apparatus used in this experiment gives results accurate to about one percent. Such a small error warrants careful consideration of not only the sources of error but also their magnitude, expressed as fractions or percents.

Two-inch "C" clamps are the most convenient units of mass because they can be easily clamped to the platform of the balance. If time allows, adjust them, as closely as you can, to weigh the same to about one percent. This is to ensure that the objects are all nearly equal, as the student assumes they are when he begins the experiment. Acceptable results can be obtained, however, from unadjusted clamps since they are reasonably uniform. To adjust their weight, place two clamps chosen at random on an equal arm platform balance. Remove the heavier one and replace it with another. As soon as one lighter than the original "light" one is found, use this to balance the additional clamps until a still lighter one is found. When you have gone through all the clamps in this manner, the final "light" clamp will be the lightest. Using this clamp as a "standard" weight on the balance, file or grind the others to match within one percent.

Other objects having similar sizes, shapes, and compositions can be used as standard masses. A piece of sandpaper glued to the wooden platform of the inertial balance prevents these masses from sliding when the balance vibrates. If they slide, the balance will come to a stop before an adequate count can be made.

It is advisable not to adjust the units of inertial mass to exactly 100 grams since one of the things for the student to learn from this experiment is that the standard unit for inertial mass can be completely independent of the gravitational unit of mass. However, if your time is at a premium and you don't have time to check and

equalize a large number of mass units, standard 100 gram masses can be used.

Any object of different material from the standard can be used as the unknown mass. But, to allow for accurate interpolation, it should have a mass that does not fall too near the ends of the range for which the balance has been calibrated. The unknown mass can be clamped to the balance with a "C" clamp of known mass.

The slot in the wooden block that is hung over the two nails at the end of the balance should be wide enough so the block will not bind on the nails. The supporting thread can be tied to a nail or screw eye inserted in the top of the block and the other end of the thread should be at least two feet above the block. The center of the hanging wooden block should be between the two nails to prevent the block from wobbling around the nails as it vibrates back and forth.

If finding the period of the rapidly vibrating, unloaded balance, it may be impossible for the student to count enough vibrations to get the period with an error no greater than about two percent. This less accurate value is still worth plotting on the graph but it should not be given too much importance in drawing the curve. The strobe cannot be easily used to measure this rate of vibration because it is not rapid enought for the persistance of vision to wipe out bad "flicker".

When the load on the balance is too large (above about 750 grams) there is a visible up and down motion as well as horizontal motion. This makes the plotted relationship between T and m diverage from $T = 2\pi\sqrt{\frac{m}{k}}$, the relationship that holds for simple harmonic motion. (Do not discuss this relation in class at this time! It will come up later in Chapter 2 of the text.) The more complicated graph (Fig. a) that one actually gets is still perfectly good as a calibration curve.

Unlike a simple pendulum, the frequency is independent of amplitude over a very wide range of amplitudes.

Answers to Questions

The student's qualitative examination of the acceleration for a given amplitude and different masses makes him aware that, even

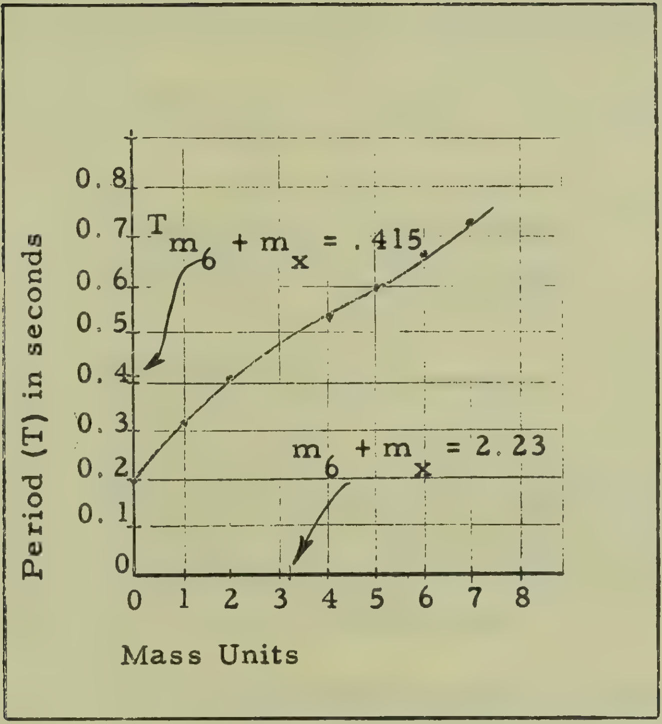

Period (T) in seconds
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$T_{m_6 + m_x} = .415$
$m_6 + m_x = 2.23$
0
1
2
3
4
5
6
7
8
Mass Units

FIG. (a)

though the acceleration is not constant for a given mass, it is greater at each point in the swing for smaller masses. That is, it is the inertial effect of the mass, the same effect that occurs in Newton's law, that changes the period of the balance.

If the vibrations of the balance are measured for 100 seconds, the error in timing, assuming the time can be determined to the nearest second, will be $\frac{1}{100} = 1\%$. If 100 whole vibrations are timed, the error in counting will also be one percent. Now, if in 100 seconds the balance completes more than 100 vibrations, the ratio of $\frac{\text{no. vib.}}{\text{100 sec.}}$ will be accurate to within 2 percent. If, on the other hand, it completes 100 vibrations in less than 100 sec, it is necessary to count 100 vibrations to get a ratio accurate to within 2 percent.

The gravitational mass of the "unknown" is found from its inertial mass m_i:

$$\frac{(m_g)\text{ unknown}}{(m_i)\text{ unknown}} = \frac{(m_g)\text{ standard}}{(m_i)\text{ standard}}$$

In our experiment (m_i) standard = 1 by choice.

Inertial and gravitational mass are proportional within the experimental error. They are not equal since different units ("C" clamps and grams) have been used. Here they are, of course, equal in most physical work since the same standard piece of matter, the standard kilogram, is used for both scales.

On the moon we would get the same results. Neither gravitational no inertial mass changes in going from earth to moon.

If the apparatus is used as shown in Figure 4, it will have a shorter period for a given mass since the force of gravity now acts as a restoring force in addition to the restoring force of the bent blades when the balance is pushed to one side. This part of the experiment is optional. Its usefulness depends on the timing of the experiment.

This apparatus can be used as an accelerometer by fastening a straw or other indicator to the moveable platform and marking a force scale on the base (Fig b). The balance can then be clamped to a table and pulled by a calibrated spring balance to give a force scale in newtons. With a known mass (mass of object plus mass of platform) in place, the device can be calibrated in acceleration units from Newton's law ($a = \frac{F}{M}$). The answer to this question as well as the actual calibration and use of the apparatus as an accelerometer should be considered only as a project for particularly interested students. It is not meant to be answered by all.

Later, in Chapter 2 this apparatus can be used as a demonstration of simple harmonic motion. With a heavy mass on the platform, its motion will be slow enough so the change in velocity at different positions can be easily seen. To show that this is simple harmonic motion, the force-deflection curve can be plotted as described above in calibrating the device as an accelerometer to show that $F = -Kx$. It is best to have the students draw the graph and calculate their results as they proceed, but if time does not allow, they can complete the analysis of their data outside the lab.

The accelerometer can also be held or mounted in an automobile so it is horizontal with the hacksaw blades at right angles to the direction of motion. Upon acceleration of the car, the accelerometer will show a deflection but there may be some oscillation back and forth if the acceleration is rapid. The mass should be damped in some way to prevent this. Be careful if you try this. The driver's attention should be focused on the road and not on the accelerometer.

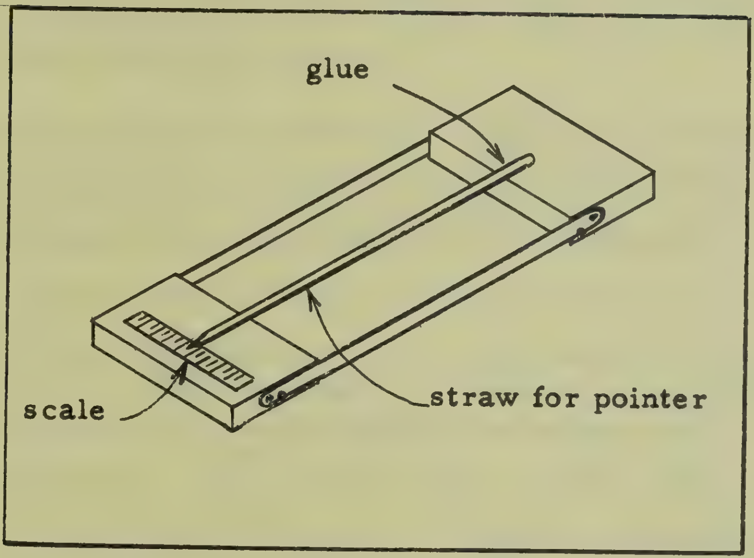


FIG. (b)

FORCES ON A BALL IN FLIGHT

So often students are instructed to "neglect" friction or "neglect air resistance" that they may think only simple, idealized motion can be analyzed. In this experiment the student can see how an application of what he has learned about simple projectile motion is the starting point which leads to an understanding of more complex motion. This is the most important thing for him to learn from this experiment. He also makes a discovery for himself; by analyzing Figure 1 he learns that air resistance acts in a direction opposite to the motion and that it increases with an increase in speed.

This experiment, though important, involves no experimental equipment and can be done as a homework assignment after projectile motion has been studied in the text.

Centimeter graph paper provides a convenient scale although any tracing paper will suffice. Figure (a) is a sample analysis of Figure 1. Students may have trouble reducing the change in velocity resulting from Δv_g to a vector they can subtract on their diagram. The first step is $\Delta v_g = g\Delta t = 9.8\ (0.1) = 0.98$ m/sec. This must then be converted to meters per tenth of a second. This will be $0.98\ (0.1) = 0.098$ m/0.1 sec. This is equivlent to converting the acceleration of gravity from meters per second squared to meters per (tenth of a second)^2, $9.8\ \text{m/sec}^2 = \frac{9.8}{100} = .098\ \text{m}/(0.1\ \text{sec})^2$. This value is then reduced to the scale of Figure 1. Since the photograph is 1/10 actual size, the length of the velocity change vector due to gravity Δv_g that must be subtracted will be $0.098/10 = 0.0098$ m. This change is , of course, in the vertical direction and is subtracted from the velocity change Δv by adding its negative as shown in Figure 2b.

If a student is interested in analyzing Figures 3 and 4, he should check the scale reduction by measuring the meter stick in the photographs.

Answers to Questions

A qualitative examination of the photograph shows that the horizontal velocity decreases from left to right. One concludes from

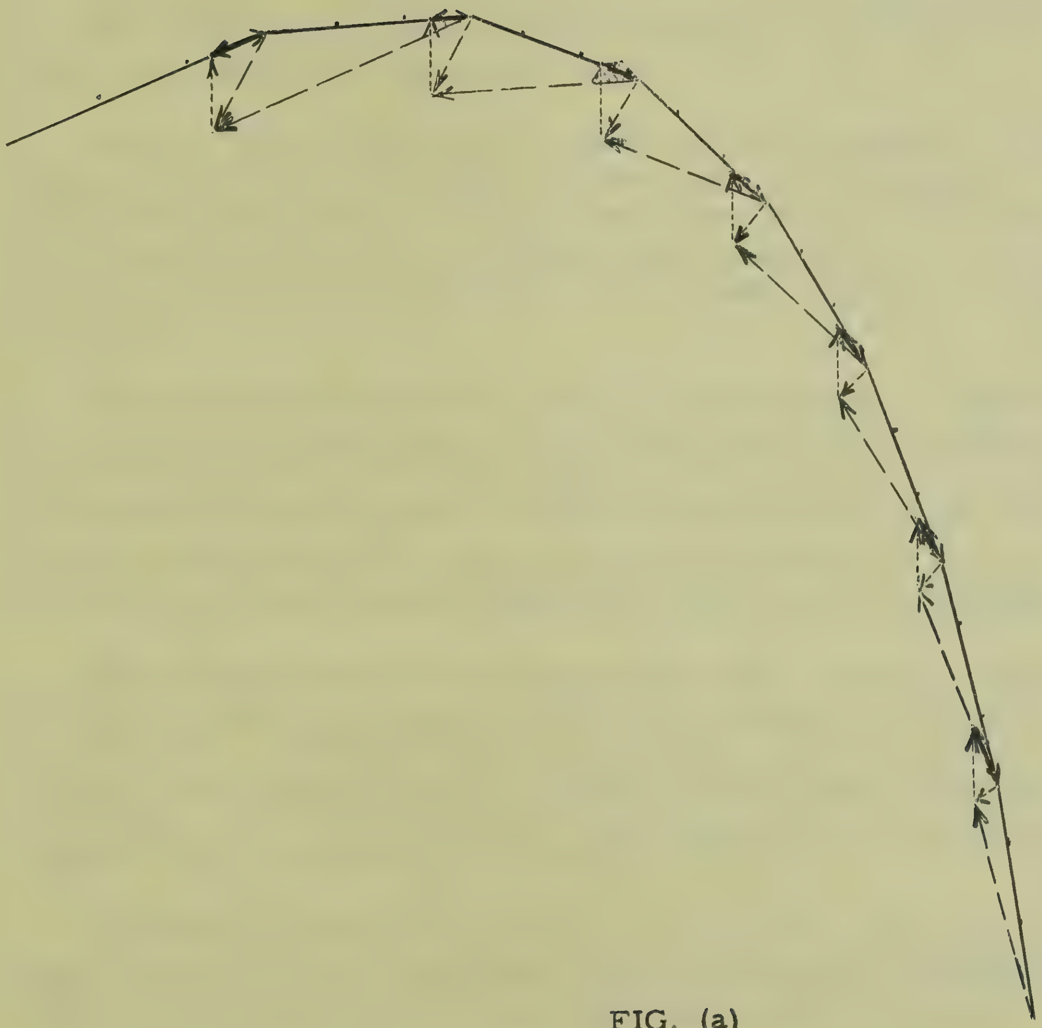


FIG. (a)

this, that in addition to the force of gravity, there is a force that has a horizontal component.

The total velocity changes found by subtracting velocity vectors on the diagram vary from interval to interval in both direction and magnitude. The magnitude of the Δv decreases during the motion and the direction slowly becomes more horizontal. Since velocity changes are in the direction of the forces which produced them, there is at least a horizontal component of the net force acting on the sphere. Without further analysis it is impossible to tell how the unknown force changes.

The change in velocity of the ball in 0.1 sec. caused by gravity is 0.98 m/sec and it acts vertically downward. In meters per tenth of a second it is 0.098 m/0.1 sec.

When the effect of gravity is subtracted, the resultant velocity changes due to the remaining force have both a vertical and a horizontal component and are opposite to the direction of motion. The larger the velocity, the larger is the opposing force. The student cannot conclude, however, that velocity and force are proportional.

The retarding force is the result of air resistance. This force is proportional to the square of the velocity under the conditions of our experiment. The total variation of the velocity, however, is not great enought to show this proportionality by a graph of the force as a function of v^2, particularly with the limited accuracy with which one can measure the force in the analysis of the data.

The ball must have a very large area to mass ratio (a very low density) to show such large effects of air resistance at relatively low velocities. If the ball had been the same size but much heavier, it would have followed a path more like the parabolas illustrated in the text. The ball used in all three photographs was a 0.05 gm sphere 1.5 cm in diameter cut from styrofoam.

If gravity has been the only force acting on the sphere, its trajectory would have looked like Figure (b) where the average velocity in the first interval was taken as the initial velocity.

The ball in Figure 1 was thrown by hand with little or no spin. Figures 3 and 4 were made with the same flash rate as Figure 1

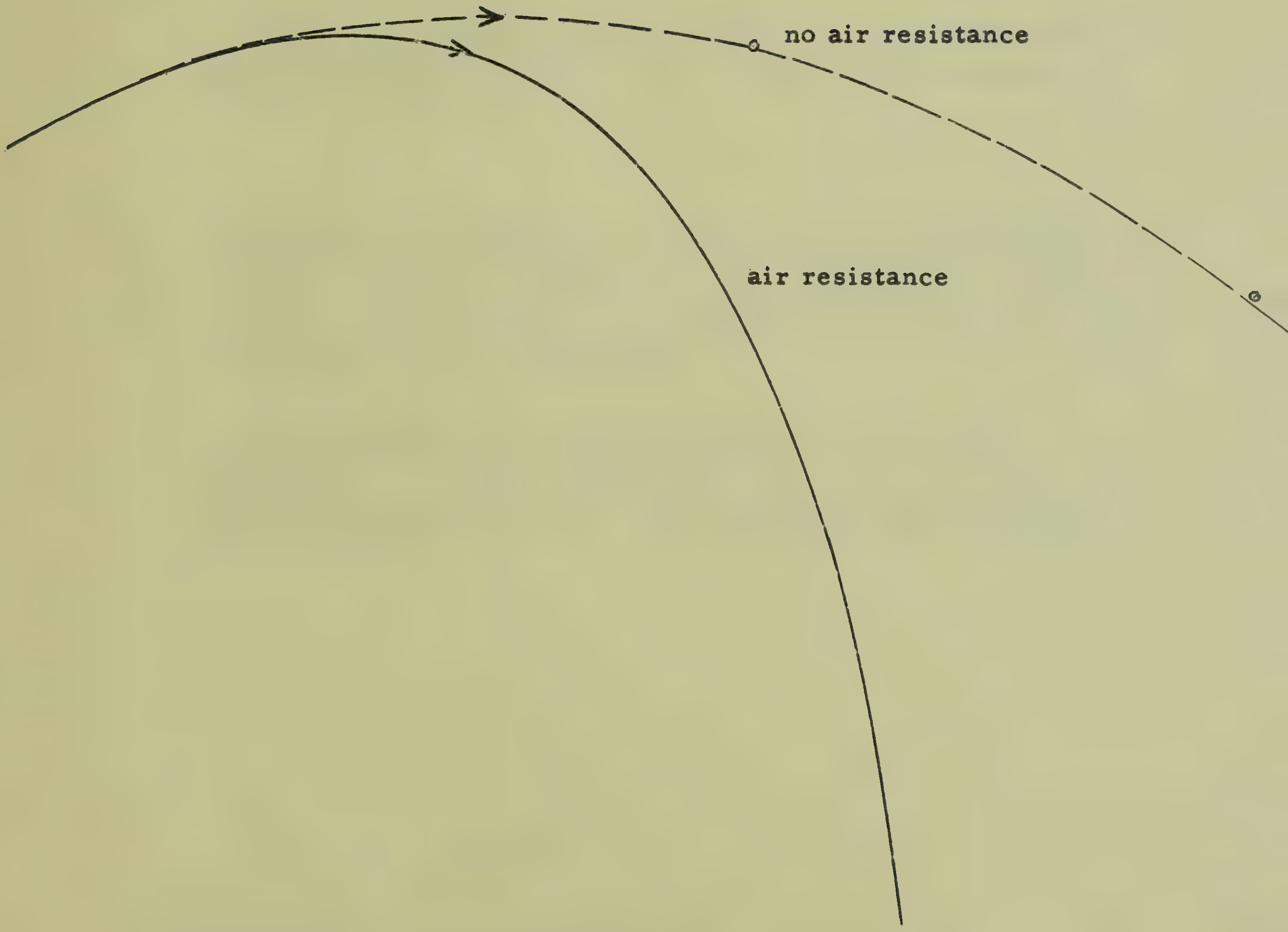
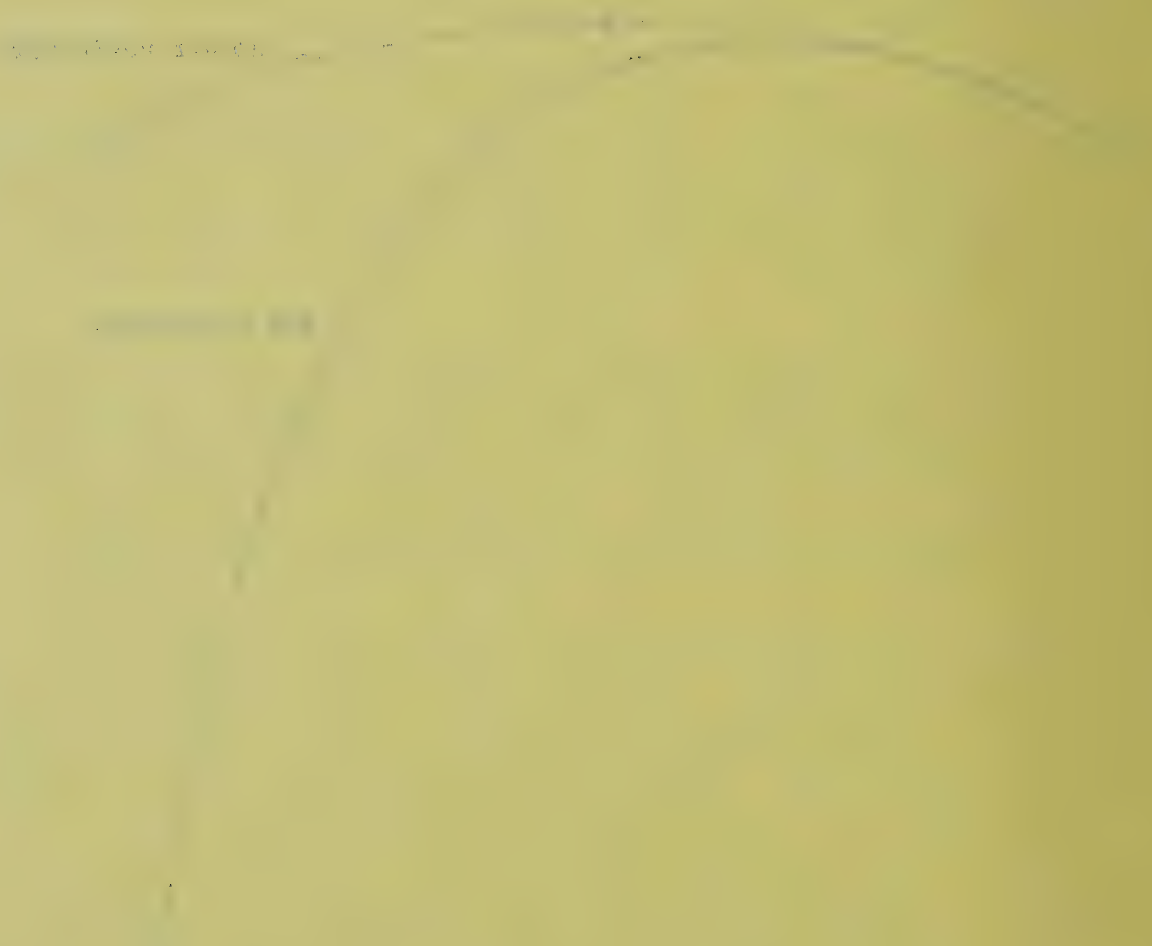


FIG. (b)



[illegible]

[illegible]

but the ball was given a spin. In Figure 3, the ball was given a counterclockwise spin about an axis perpendicular to the page. In Figure 4 the spin was clockwise. The analyses of Figures 3 and 4 are shown in Figures (c) and (d).

The analyses of the spin photographs and the plotting of the trajectory with no air resistance should be considered supplementary to the main experiment and done only by particularly interested students. In Figure (c) the force which remains after subtracting the effect of gravity is no longer opposite to the direction of motion but has a component at right angles to the path. This component becomes smaller as the rate of spin decreases during the motion. Figure (d) shows a similar component perpendicular to the path but in the opposite direction.

A complete mathematical treatment of this spin motion (or even the motion with air resistance and no spin) is a formidable task not even to be attempted.

- - - -

In all three photographs, the speed of the ball near the end of its trajectory is nearly constant. The ball is close to its terminal velocity. When the ball reaches its terminal velocity, the force of gravity on the ball equals the air resistance, the net force is zero, and it moves at constant velocity.

Students may wish to fire ping-pong balls with a spin. A cardboard mailing tube makes a good gun. Put the ball in the tube and swing it in an overhead arc or an underhand arc depending on the desire direction of spin. This method was used in making Figures 3 and 4.

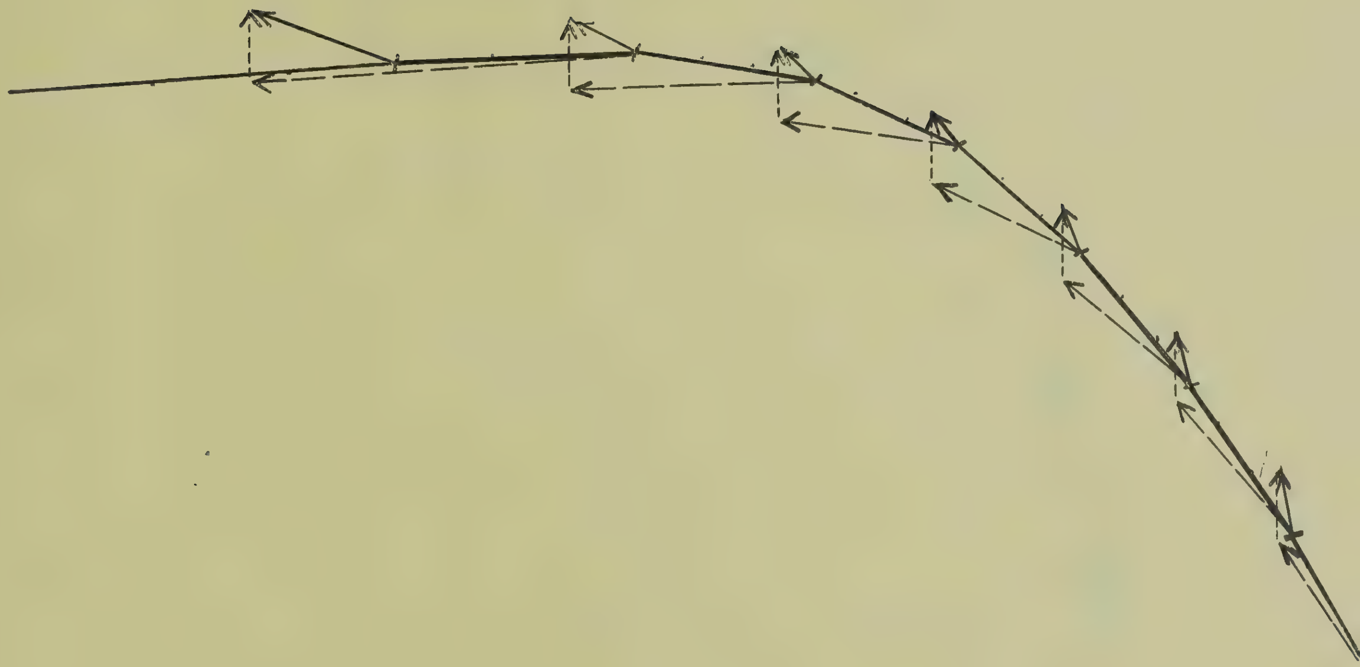


FIG. (c)

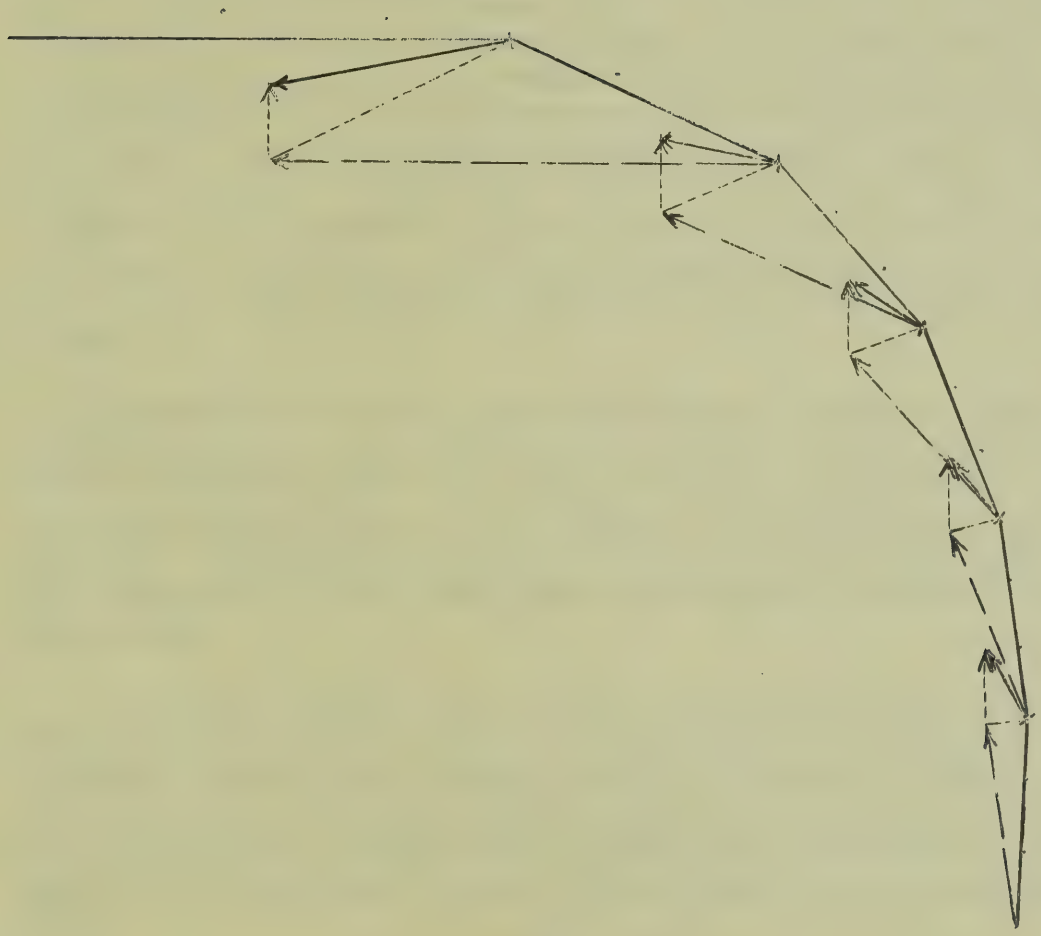


FIG. (d)

PSSC Teacher's Guide Experiment 500 Vol 3

CENTRIPETAL FORCE

The purpose of this experiment is to find the dependence of the centripetal force on the speed, mass and radius of an object moving in a circle. The experiment is best done before these relations are derived in the text (Volume 3, Chapter 2, Sec. 5), so it leads to discovery rather than mere confirmation of the equation for centripetal force.

As a result of doing the first part of the experiment, the student can conclude that the centripetal force is proportional to the square of the speed. If he has time and wishes to go further, he will conclude from experimental evidence that the centripetal force is proportional to the mass and inversely proportional to the radius of the circle. As in many of the experiments in this volume, he learns that the discovery of fundamental laws does not require the use of standard units of measure. Here he uses arbitrary units of force and mass: the unit for measuring centripetal force is the force of gravity on a steel washer; the mass unit is the mass of a # 3 rubber stopper.

The experiment can be done alone at home instead of in the laboratory, but it is best done by two or three students together--one to swing the stopper in a circle, another to count the number of revolutions and a third to do the timing. If the laboratory is too small for swinging many stoppers in a circle of one meter radius, it is advisable to do the experiment out-of-doors or in the school gymnasium.

The size of the glass tube is not critical but the bearing surface for the string at the top end must be carefully fire polished so it is smooth and free from bumps to keep friction at a minimum. The tube should be wrapped with Scotch tape so there will be no danger from jagged pieces of glass if it should break. The forces used, however, are such that the tube will not break unless it has a bad flaw.

The cord used is critical if friction is to be kept low. Fisherman's hard nylon braided casting line, 12 lb. test works well.

The stopper size, radius of the circle, and size and minimum number of washers have been carefully chosen to give good results. Large variations from the recommended values will cause the angle the cord makes with the horizontal to be so large that the student cannot use the length of the cord as a good approximation of the radius of the circle. These variations may also make the graph of force as a function of velocity look like a straight line.

To keep the stopper moving at constant speed, the top of the tube must be moved in a small circle. This provides a small component of force along the path to counteract friction. However, this circle should be less than a centimeter or two in diameter or the circle described by the stopper cannot be accurately measured by the length of the cord. The radius should be measured from the middle of the stopper near its center of mass. Since the dimensions of the stopper are small compared to the radius of the circle, measurement from this point will give a value for the radius which is close to the correct value.

The laboratory instructions specify a minimum number of washers so the cord will be horizontal enough to permit the use of its length as a reasonably accurate measure of the radius of the circle. Actually, using the "slant" radius given by the length of the cord leads to correct results no matter how large the angle between the cord and the horizontal. This can be verified by analyzing Figure (a). The force of gravity $m_w g$ acting on the washers is transmitted along the cord and acts on the stopper along $\underline{L}$. The horizontal component of this force provides the centripetal force F_c along $\underline{R}$. We see that $R = L \cos \theta$ and $F_c = m_w g \cos \theta$. Writing the equation for the centripetal force $F_c = \frac{m_s v^2}{R}$ in the form

$$F_c = \frac{4\pi^2 m_s R}{T^2}$$

by substituting $V = \frac{2\pi R}{T}$ where T is the period, we have by

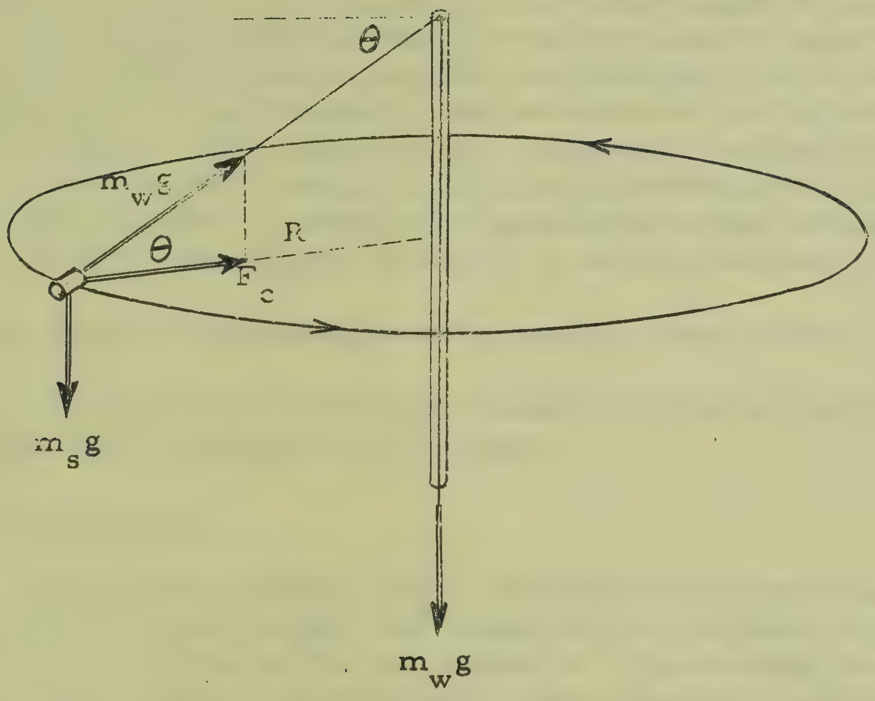


FIG. (a)

substitution

$$m_w g \cos \theta = \frac{4\pi^2 m_s L \cos \theta}{T^2}$$

$$m_w g = \frac{4\pi^2 m_s L}{T^2}$$

This means that the force exerted by the washers is inversely proportional to the period squared for a constant length of cord L. Since the student assumes that R stays very nearly equal to L, he measures the period of rotation and assumes this to be inversely proportional to the velocity v. He has no error due to the changes in the angle θ. This explanation is somewhat involved; it is best to let most students simply assume the approximation and explain the real state of affairs to those who question the effect of the angle θ. You will note from Figure (a) that θ is determined only by the weight of the stopper and the weight of the washers since $\sin \theta = \frac{m_s g}{m_w g}$. Using less than the recommended number of washers with a #3 stopper will therefore make θ larger.

Answers to Questions

The faster the stopper whirls, the harder one must pull down on the string. If one lets go, the stopper ceases to move in a circle and pulls the string up the tube behind it. By exerting the force himself, the student becomes aware that a force at right angles to the motion is necessary to make an object move in a circle.

A graph of the speed of the stopper as a function of the number of washers looks like Figure (b). It is, of course, not clear what function this graph represents although one might suspect that it is a parabola. The student may think, if his points are somewhat scattered, that the curve is a straight line which does not pass through the origin, but if he is asked what the velocity should be with zero force, he should see that it would have to be zero, that is, the stopper must be stationary. Hence the curve must pass through the origin, curving down from his assumed straight line.

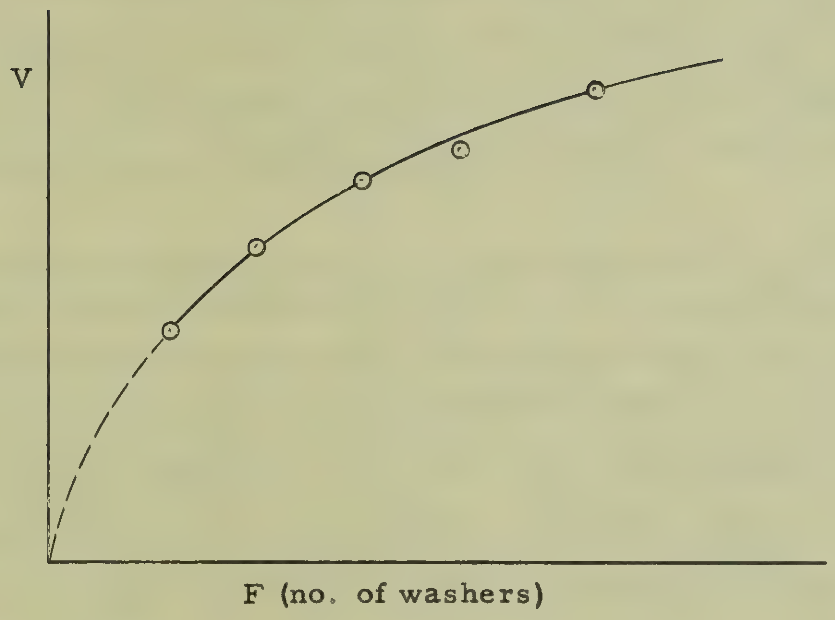


FIG. (b)

A plot of V^2 as a function of force will be a straight line passing through the origin, showing that the centripetal force varies as V^2.

The student can conclude, from Newton's law F = ma, that doubling the mass whirling in a given circle with a given centripetal acceleration should require twice the force to keep it moving at the same speed in the circle. If he has time, he can try this.

It is difficult to find out how the force depends upon the radius because it is hard to whirl the stopper at the same velocity each time the radius is changed. A better way of getting the relation between force and radius is to repeat the first part of the experiment several times, using different radii and only one value of centripetal force for each radius. Then a family of curves of V^2 as a function of $\underline{F}$ can be drawn (Fig. c). Since we know from the first part of the experiment that these curves are straight lines, the single value of the centripetal force for each radius suffices to establish the straight line through the origin.

For a constant value of V^2, represented by the line AB, the various values of the force (F_1, F_2, F_3, F_4) lie on curves corresponding to different values of $\underline{R}$. When these values of $\underline{F}$ and $\underline{R}$ are plotted, they give curves which resemble Figure (d), and Figure (e). (Note that Figures (c) through (e) do not represent actual data taken with the apparatus. They are merely approximate sketches of the results that would be obtained.) Students who are well versed in "ham" radio may recognize this method of analyzing a family of curves as similar to the method of finding the amplification factor of a vacuum tube from a plot of the plate current as a function of the plate voltage.

If there is not enough time to investigate experimentally the effect of mass and radius on the centripetal force, it may be worthwhile to do this part of the experiment as a class demonstration.

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Figure (f) shows all the forces that act on the apparatus. $\underline{F}$ is the resultant force that the operator exerts on the apparatus (neglecting the weight of the tube). Note that the gravitational component of this force ($m_w g + m_s g$) is constant in direction while the horizontal component F_c, the centripetal force, is constantly changing direction in the horizontal plane.

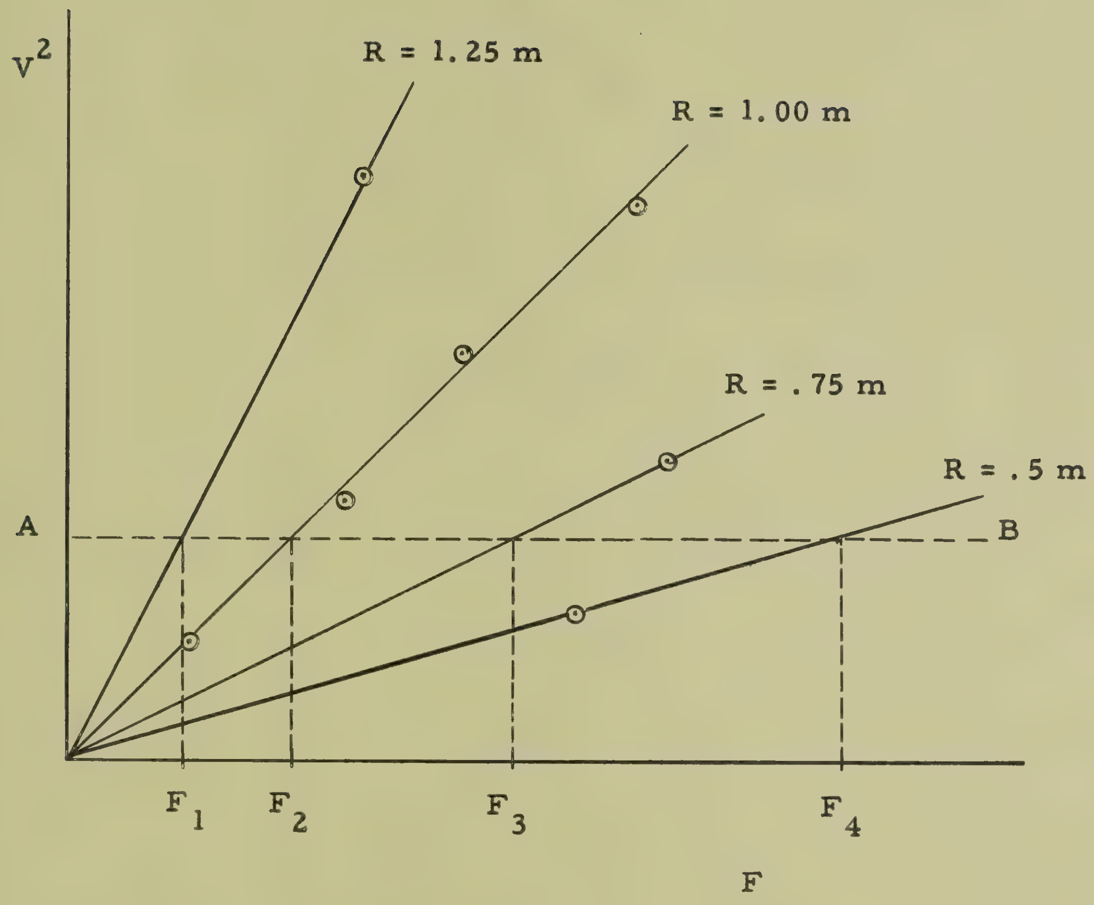

FIG. (c)

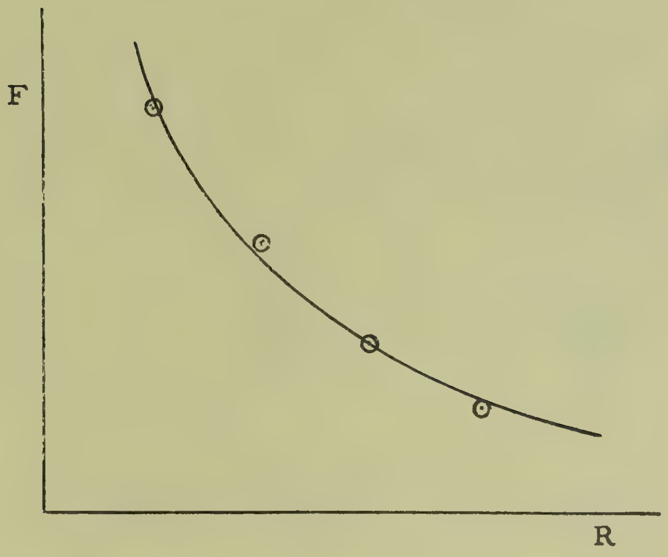

FIG. (d)

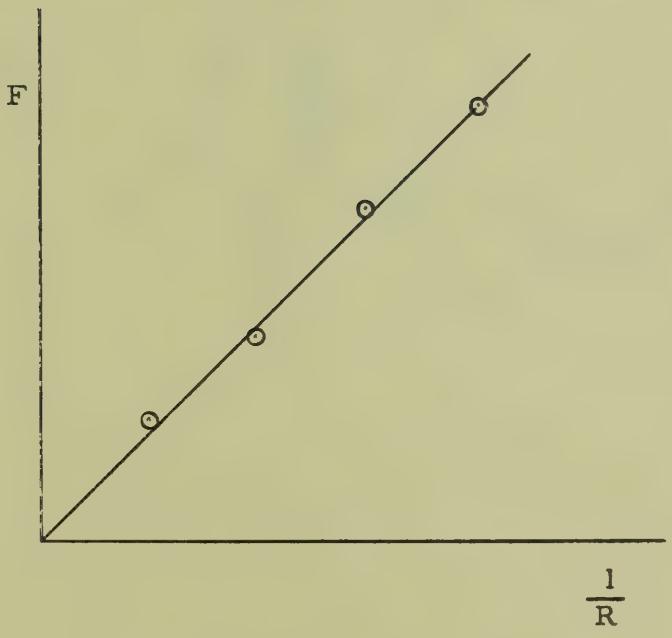

FIG. (e)

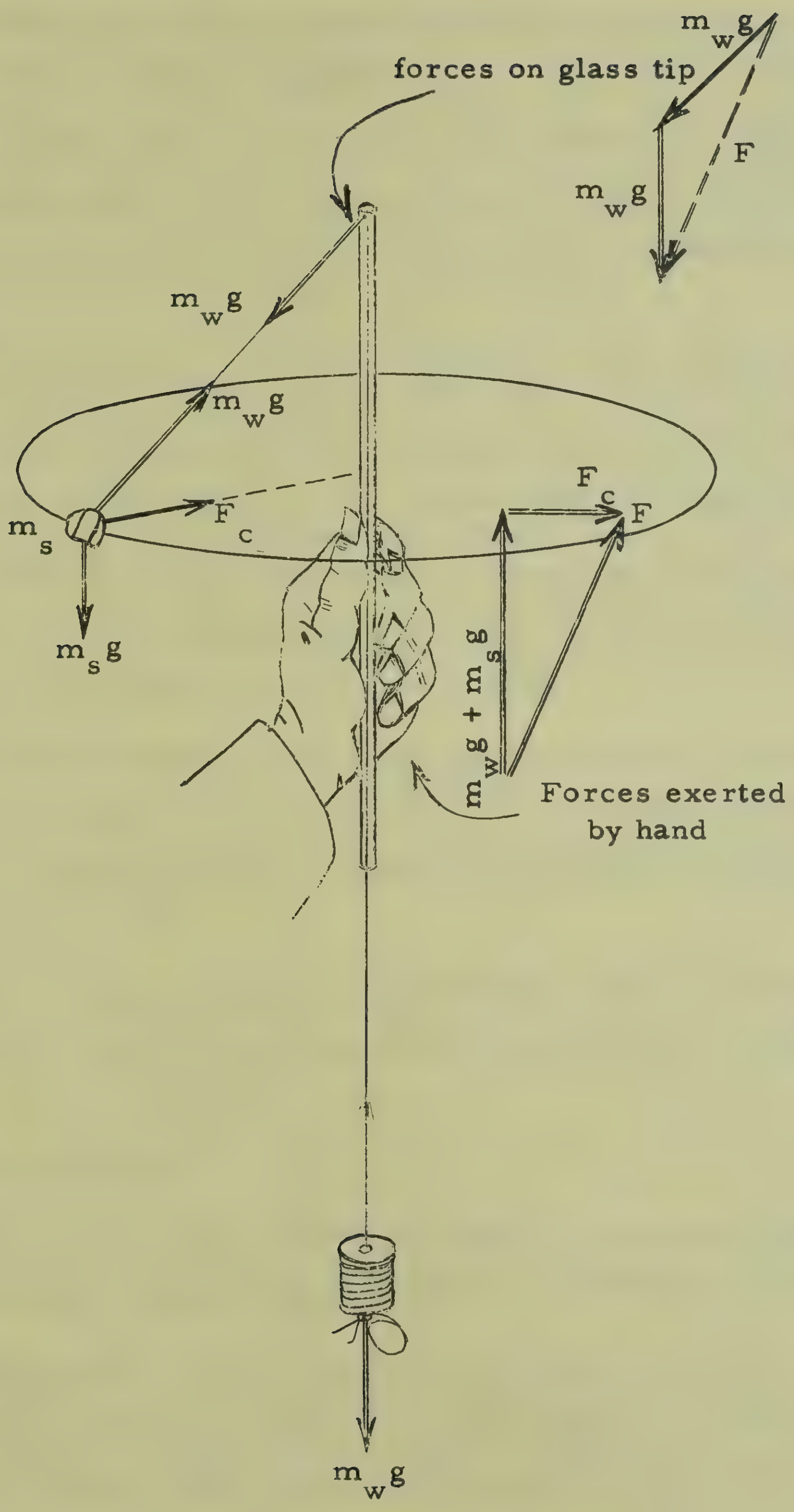
forces on glass tip
$m_w g$
F
$m_w g$
$m_w g$
$m_w g$
m_s
F_c
$m_s g$
F_c
F
$m_w g + m_s g$
Forces exerted
by hand
$m_w g$

FIG. (f)

LAW OF EQUAL AREAS

Whenever a central force operates, an object will move in orbit such that it sweeps out equal areas in equal times, regardless of how the force varies with the distance from the central point. The law of equal areas expresses the conservation of angular momentum, but since angular momentum is not covered in the course, it is probably unwise to discuss it in relation to this experiment. This experiment serves only to illustrate the law of equal areas and is not among the most important experiments.

Salt, except under humid conditons, is the best substance to use as a time indicator. It scatters less than sand but some varieties of fine sand may work as well as salt. Conical cups should be used since salt does not run out of flat bottomed cups evenly. To stop the flow of salt, the cup can be plugged by pushing the pointed end of a pencil down through the salt and into the hole. The scales for weighing the salt should have a sensitivity of at least 0.1 gm although a sensitivity of 0.01 gm would be better. More sensitive balances are not worthwhile since other sources of error will be greater than those incurred in weighing.

A rigid support for the pendulum is necessary for if the support wobbles, the pendulum will precess in its orbit, reducing the accuracy when several revolutions are used to collect enough salt to be weighed accurately. Ideally, the pendulum should be supported from the ceiling.

It takes time to collect and to analyze data in a short laboratory period so it may be best to make several runs with the pendulum, weigh the sand and then analyze the results of one or two of the best runs as homework.

Answers to Questions

Several complete revolutions permit a more accurate weighing of the accumulated sand. Too many revolutions will result in inaccuracies caused by precession.

The net force on the pendulum always acts toward a point directly beneath the point of suspension. This can be checked by pulling the pendulum to one side and releasing it without giving it any sideward motion. In this case, if salt is allowed to run out as the pendulum

starts from different points around its orbit, all the salt tracks will intersect under the point of suspension. This question suggests to the student that this is the point toward which the force is directed and around which equal areas are swept out by a conical pendulum, unlike planetary motion where the force acts toward a focus of the ellipse and the planet sweeps out equal areas in equal times around this focus.

Careful work will show that equal areas are swept out to an accuracy of about five to ten percent, even for a narrow elliptical orbit where the variation in velocity is very large.

For both the pendulum and the planets, the force is a central force acting toward a single point regardless of the position of the object. The force on the planet varies inversely as the square of the distance between it and the sun. The force on the pendulum is directly proportional to the distance of the bob from the center of the ellipse as long as the angle of the suspension string with the vertical is small.

An inverse square force causes a planet to move in an ellipse with the center of force at one focus whereas the linear force on the pendulum causes it to move in an ellipse with the center force at the center of the ellipse.

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Figure 3 was taken by the method described in the Laboratory Guide. The plane of the film was parallel to the plane of the ellipse. Figure 4 was taken the same way with the same equipment as Figure 3 but the size of the orbit was different.

There is, however, an error in Figure 2. If the experiment is done by this method, both lights should be on the floor. If one lamp is on the table as shown in the figure, it will illuminate the background so much that the resulting photographs will suffer from lack of contrast.

If Figures 3 and 4 are analyzed to get the ratio $\frac{R^3}{T^2}$, the ratios will not be equal for the two cases since Kepler's law of periods applies only to an inverse square force.

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MOMENTUM CHANGES IN AN EXPLOSION

This experiment should be done as soon as definitions of impulse and momentum have been studied in the text (Volume 3, Chapter 4, Sec 1 and 2) and preferably before the students study momentum changes when two bodies interact. The experiment will help them to understand momentum changes and will introduce them to the law of conservation of momentum.

Spring exploders may differ in their strength and a weak spring acting on a heavy cart may not give it enough velocity to reduce the of slight variations in the table top. If the distance the cart must travel is too great, the cart may because of friction, come to a stop before reaching the bumper. On the other hand, if the distance traveled is too short and the velocities are reasonably high, the coincident timing method will not be accurate. The optimum distance between the two bumpers is about 1.5 m.

Both carts should be loaded with at least one brick since some of the force of the exploder applies torque to the wheels, making them rotate. This appears as a loss of linear momentum. As long as the mass of the wheels is small compared to the total mass, this effect is insignificant.

The student will discover for himself, as soon as he begins his first analysis, that the two carts (with the exploder attached to one of them) and the bricks must be weighed.

The exploder can be dangerous to fingers if it is released indiscriminately. It should not be cocked without the safety cord properly connected. Have the cord checked occasionally for fraying and stretching.

The starting positions of the carts can be marked on the table with chalk. The student should make sure he places the starting mark at the end of each cart closest to the bumper.

If both carts have the same load, friction tends to cancel out since the force of friction acts for the same time on both carts, giving equal and opposite impulses to the two carts and decreasing their momentum equally. There will, of course, be different forces of friction on two differently loaded carts but the effect of friction is small.

If the table is not level, one cart will gain momentum during its run and the other will lose momentum. If the table cannot be made nearly level, it may be necessary to run two trials for each loading of the carts, reversing the position of the carts for the second trial and comparing the average momentum change of each cart.

Answers to Questions

If the exploder is released when attached to a cart standing alone on the table, the cart jumps into the air but does not appear to move horizontally. The student will conclude that the total momentum of the cart and exploder was zero before the explosion and was again zero after the explosion.

When the student tries various loads on two carts and qualitatively observes their motions, he can see that the lighter cart moves faster as a result of the interaction. He may guess that the momentum of the carts have the same magnitude but are in opposite directions.

In calculating the ratio of momenta, the student needs no standard unit of momentum, but when asked to find the change in momentum of each cart, he may be at loss as to how to express the momenta. He can express them in kilogram-meters per "clank", (a "clank" is the unit time interval, the time taken by both carts to reach the bumpers in a given run). Of course, changing the loads on the carts will result in different speeds and a different time unit for another run but it will be the same for both carts. However, without trying to find a specific unit, the student can answer that the change in momentum of one cart is equal in magnitude and opposite in direction to the change in momentum of the other cart. (The ratio of the momenta is minus one to about 5%).

The total momentum before and after the explosion is zero and hence momentum is conserved.

If a stick of dynamite were exploded between the carts, the pieces would fly in many different directions at different speeds but we would expect the vector sum of their momenta to be zero. This is a hard question for the student to answer before he has become throughly familiar with momentum changes in two dimensions, studied later in Chapter 4 or learned in Experiment 540.

* * * *

Another way of doing this experiment eliminates the need for listening for simultaneous collisions of carts and bumpers. It requires more elaborate apparatus and raises more questions in the student's mind but some students may be interested in trying it. Place the carts on a smooth board about 1.5 meters long which has bumpers at its ends. The board should rest on 1 foot pieces of $\frac{1}{2}''$ or 3/4'' dowel. Use three pieces of dowel, one near each end and one in the middle. (Fig. a). Make an index mark on the board and a corresponding reference mark on the table. When the exploder sends the carts toward the ends of the board, one will strike its bumper first and move in the direction in which it is traveling. A moment later, the second cart strikes its bumper and stops the board. The movement of the index mark is a sensitive indicator of the relative order in which the carts strike. When the correct starting position of the carts has been found the carts will strike the bumpers simultaneously and the board will not move at all.

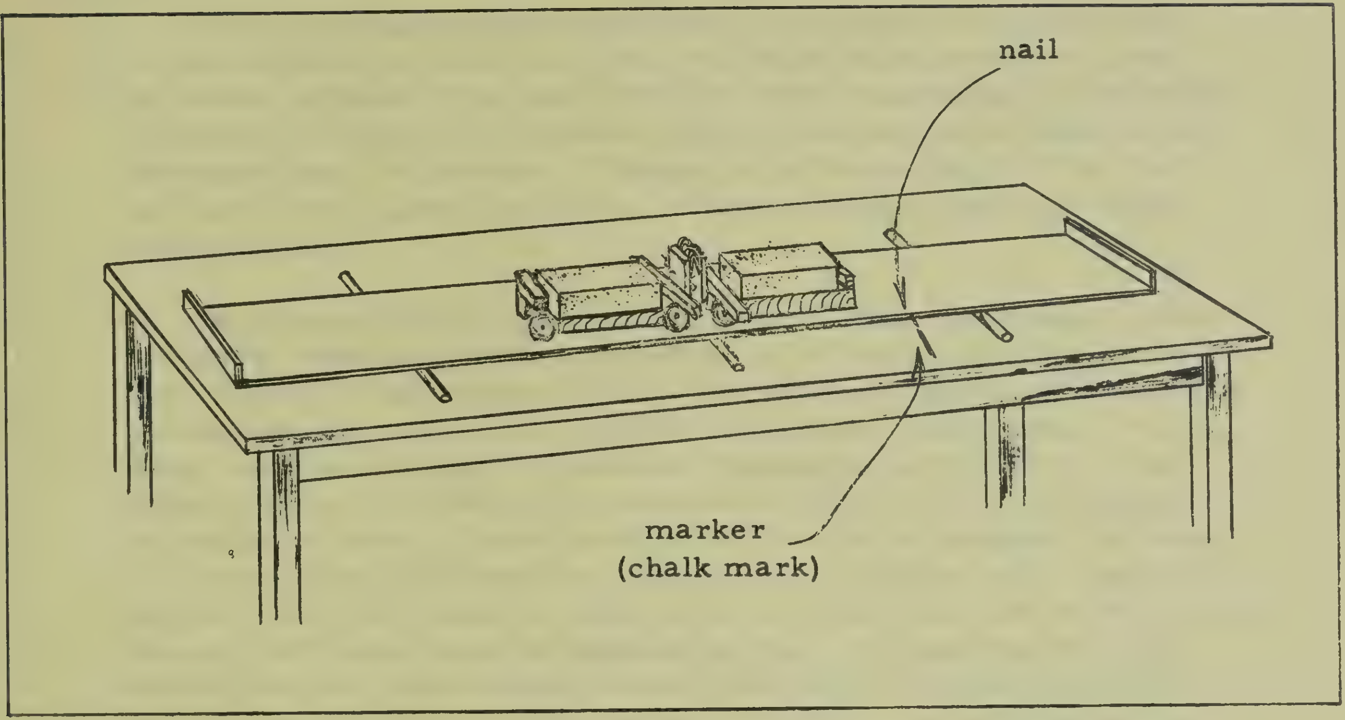


FIG. (a)

PSSC Teacher's Guide Experiment 530 Vol 3

THE CART AND THE BRICK

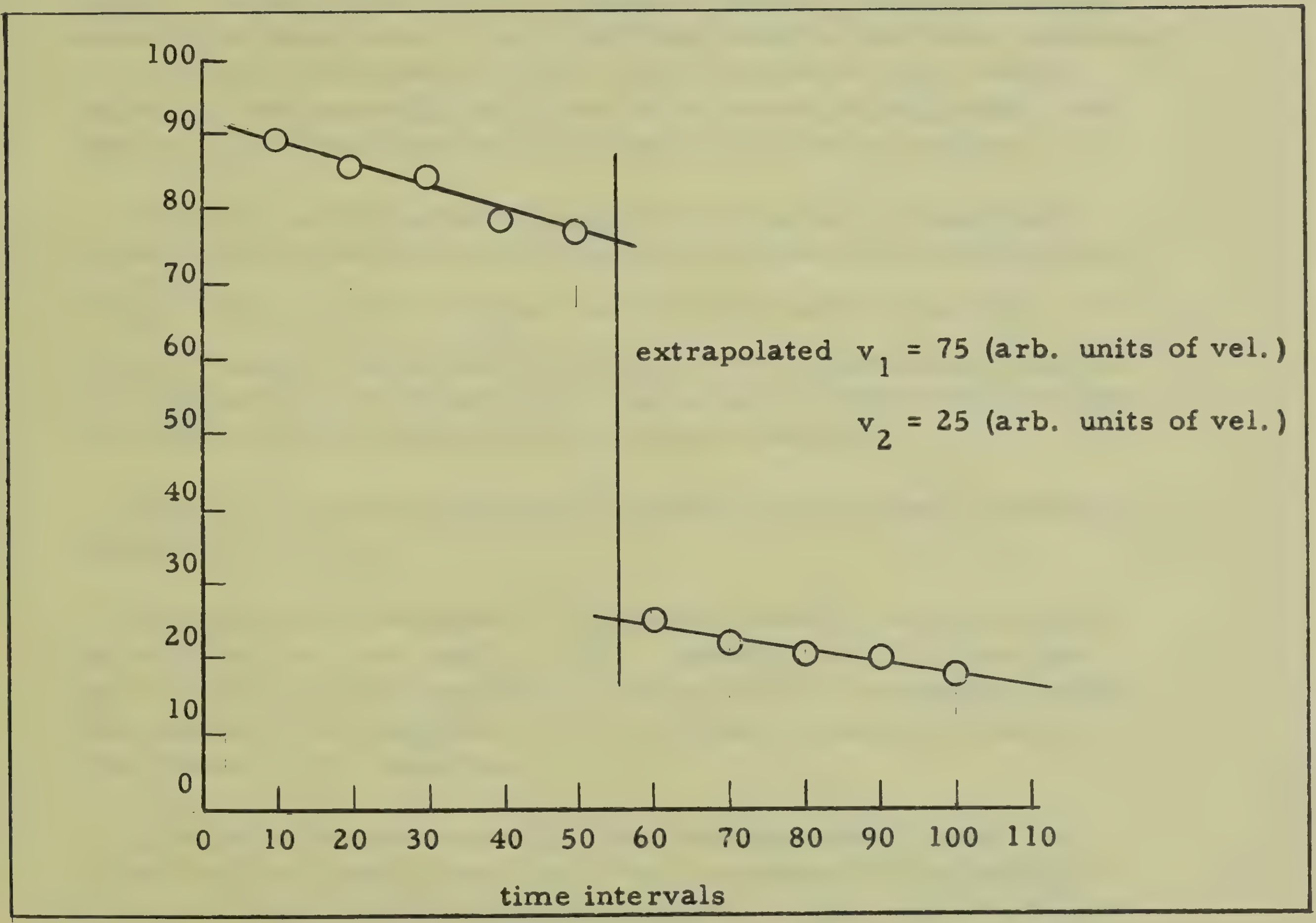
Since this experiment comes after the student has discovered conservation of momentum in the explosion experiment, it is not important whether it is done before or after momentum conservation is taken up in the text. A more complex example of momentum conservation than "Momentum Changes in an Explosion," "The Cart and the Brick" gives the student a deeper insight into momentum conservation.

The student learns from this experiment that momentum is also conserved when the initial momentum is not zero. He also learns that in calculating momenta before and after collision, momentum is conserved only when one is careful to include in the system everything that loses or gains momentum during the interaction. The experiment is not difficult to perform but there are several techniques to be noted. Heavy cord should be used to support the brick. It is hard to hold the brick by pinching a thin string to the support bar as shown in Figure 1. Since some friction will always be present, the cart will slow down throughout the run. To analyze the interaction properly, the velocity of the cart should be measured just before and just after the interaction. At least one brick should be on the cart when making measurements to prevent the final velocity from being so low that table irregularities will cause it to fluctuate too much. Good judgment is needed in finding the velocities before and after the interaction to choose an interval long enough to be meaningful and short enough to have fairly constant velocity.

An alternate analysis of the data can be made by plotting velocity as a function of time. By extrapolating the two lines which represent the slowly changing velocities before and after impact, the actual velocities can be determined at the time of impact (Fig a). On good, smooth tables practically no extrapolation is necessary. In any case, only the better students should be encouraged to use this method of analysis.

Answers to Questions

The decrease in the magnitude of the momentum of the cart equals the increase in the magnitude of the horizontal momentum of the brick but the changes are in opposite directions. The total horizontal momentum of the system of the cart and suspended brick before interaction is just that of the cart. Since, as a result


100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
110
time intervals
extrapolated v1 = 75 (arb. units of vel.)
v2 = 25 (arb. units of vel.)

of the collision, the cart loses the same momentum that the brick gains, the total momentum of the system of cart and brick after the interaction is the same as before the interaction. Momentum is conserved within experimental error (about 2%).

The horizontal impulse applied to the falling brick is equal to its change in horizontal momentum. The length of time of the interaction can be estimated by carefully examining the tapes and estimating how many intervals show a rapidly changing velocity. In general, the whole interaction will occur in one or two ticks.

To estimate the horizontal force applied to the brick, the time interval between ticks must be found by pulling a tape through the timer for a known time (4 or 5 sec) and counting the dots. Accuracy in calibrating the timer this way is not important since the inspection of the tape will only give, to an order of magnitude, the number of intervals during which the interaction took place.

Using the known impulse applied to the brick, the average force of the interaction can be found to an order of magnitude from $F = \frac{I}{\Delta t}$ where $\underline{I}$ is the impulse and Δt is the interaction time. This is a very rough determination and it is worth pointing out that this is an average force; in actuality, the force probably varies widely over the interaction time in some unknown manner.

The force applied to the cart is equal but opposite to the force applied to the brick.

The vertical momentum of the falling brick is transferred to the earth via the cart and the table. Momentum is a vector and none of the vertical momentum is transferred to horizontal motion. Theoretically the magnitude of the vertical velocity is of little importance in this experiment.

If the experiment is done by dropping sand slowly into a box on the cart, the cart slows down but the duration of the collision is greater than when the brick is dropped. If the sand was allowed to run out of the cart however, the velocity of the cart would not change. It must be kept clearly in mind that our system consists of cart and sand at all times. Only the sum of momenta of both remains unchanged. When we drop the brick on the cart there is a friction force acting between them until they both move with the

same velocity. When we let the brick or sand fall off the cart, both cart and sand have the same horizontal velocity and will continue to have the same velocity until the sand touches the table which is not part of our system. One may not include or exclude any object from the system during the interaction because this will lead to errors. Dropping the brick on the cart is misleading in this respect. Only because the brick has no horizontal momentum at the beginning can we say that the mass of the cart increased and hence its velocity had to decrease.

The reverse of the experiment of dropping the brick on the cart is to shoot the brick off the rear of the moving cart so its velocity relative to the table is zero; the cart then speeds up and the brick drops vertically.

A COLLISION IN TWO DIMENSIONS

The purpose of this experiment is twofold. The first objective is to impress upon the student that momentum is a vector quantity and that it is the vector sum of the momenta, not the arithmetical sum, that is conserved. Only by analyzing a two dimensional collision can this be made really clear. The second objective is to let the student discover that another quantity, mv^2, which is not a vector, appears to be conserved. The discovery of conservation of mv^2 in this experiment is only meant to stimulate the curiosity of the abler student and one should not go into a deep study of energy at this time. This experiment, though difficult, is important and should not be omitted. It is best done after Chapter 4, Section 4.

Tape timers can be used only to measure velocities and momentum in collisions occurring along a straight line. Collisions in two dimensions on a table can be analyzed by studying stroboscopic photographs and is done this way in the textbook. Lacking this equipment, some other method must be used to time the motions.

The constant horizontal velocity of a projectile, provides a simple way of measuring velocities in any horizontal direction. There are, however, drawbacks to this method. When a ball rolls over the edge of a table, the table gives it a small horizontal impulse; the slower the ball moves, the greater the impulse. For this reason the target ball is mounted on a nail away from the table edge, so that the incident ball will not be in contact with the table when the collision occurs. This complication is, however, intentionally kept from the student's attention. Another difficulty is explained in Figure 3 of the Laboratory Guide.

As shown in Figure 1, the bottom of the ramp is placed about three inches from the table edge, so the incident ball will not be bouncing when it leaves the table. The finishing nail must be mounted as accurately as possible. If it is in the wrong position so the target ball is struck too high or too low, the collision will not be in the horizontal plane as assumed.

Some students may be worried about this being a collision in two dimensions. They may feel that it is really in three dimensions since the balls move not only in different directions in the horizontal plane but also fall vertically. You may remind them that

they are finding the velocities just before and just after the collision when the objects are moving in the horizontal plane and that the following vertical motion is used merely as a means of measuring the velocities in the horizontal plane.

Some laboratory tables either have a lip on the edge or are not square. If such is the case, a launcher like that in Figure (a) can be made and clamped to the table so its square end projects well over the edge. The target support can be clamped near it in the proper position.

Answers to Questions

When the incident ball is rolled down the incline ten or fifteen times with the target removed, the points where it lands on the paper will be scattered over a small area that can be enclosed by a circle whose diameter is about 3 % of the distance traveled by the ball. Therefore, the initial velocities in the different trials vary from the average by about 1.5%.

The momenta in the case of the balls of equal mass are represented by the displacement vectors on the paper. The vector sum of the two final momenta should be equal in both magnitude and direction to the initial momentum of the incident ball. If the experiment is carefully done, the error lies within the experimental accuracy as determined by the scattering of the points in measuring the initial momentum of the incident ball. (A sample set of data and vector diagrams is shown in Figures b and c). Momentum is conserved within experimental error.

Though the vector sum of the momenta is constant, the arithmetic sum of the momenta magnitudes is not.

When the experiment is repeated with unequal masses, the heavier ball should be used as the incident ball. Otherwise, with head-on or near head-on collisions, the lighter incident ball will rebound and bounce off the table edge.

When unequal masses are used, the displacements on the paper representing velocities no longer represent momenta. The vector sum of the final velocities will not equal the initial velocity.

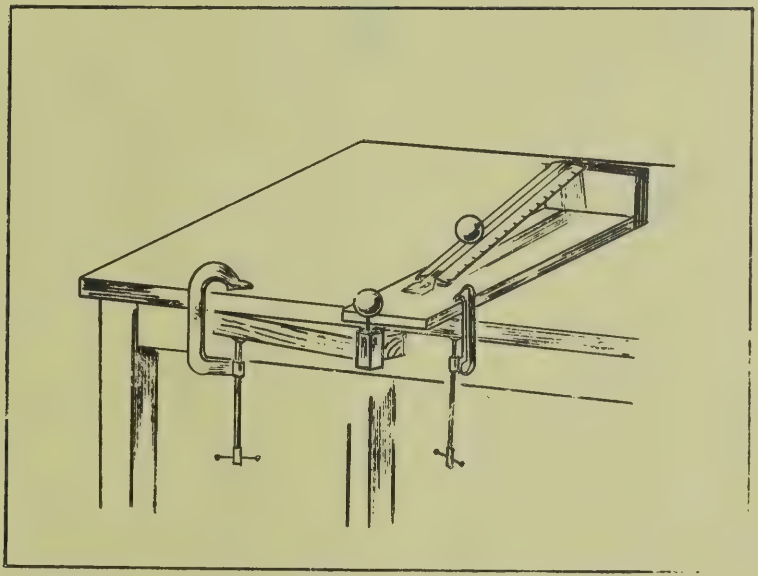
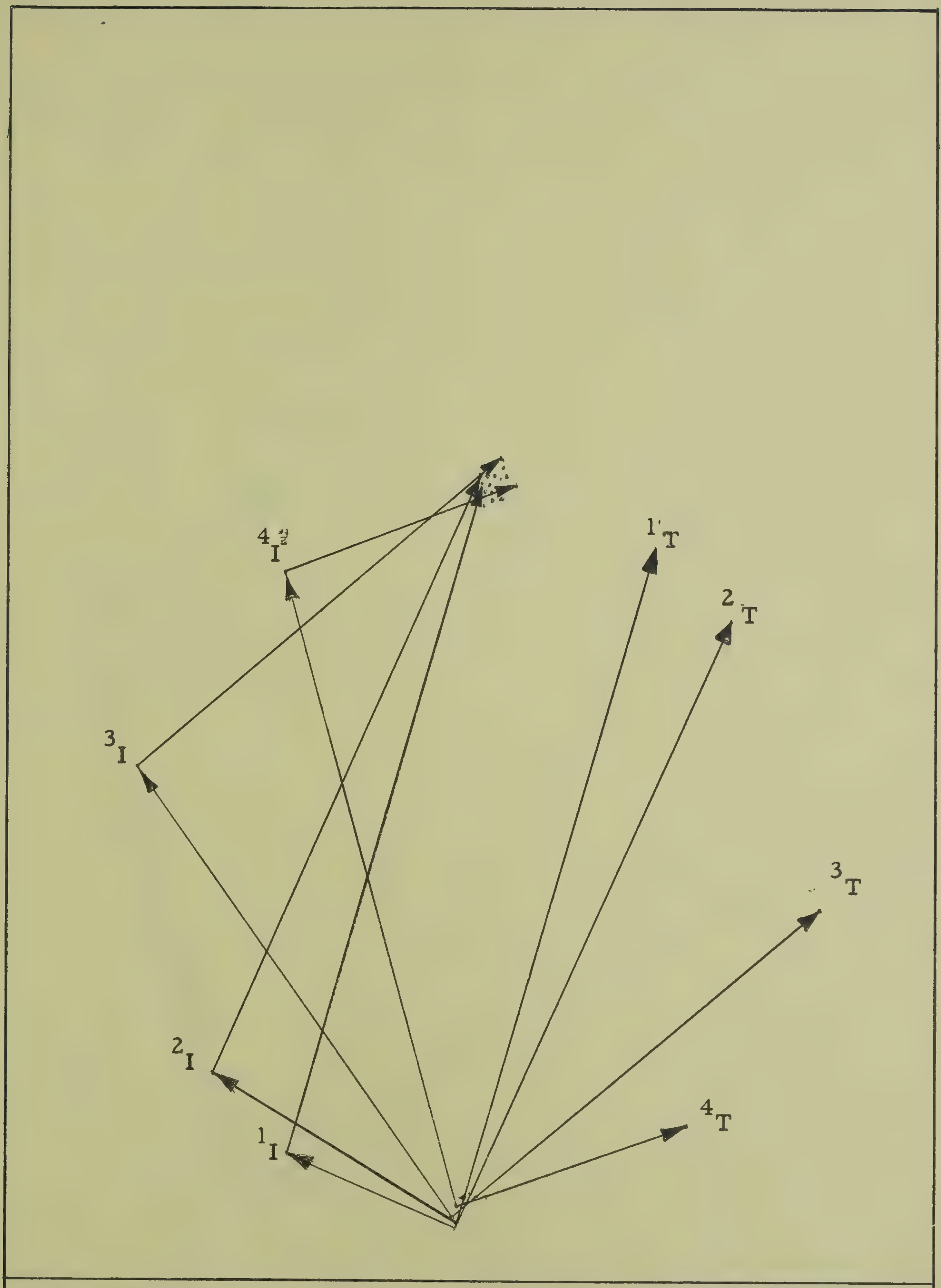


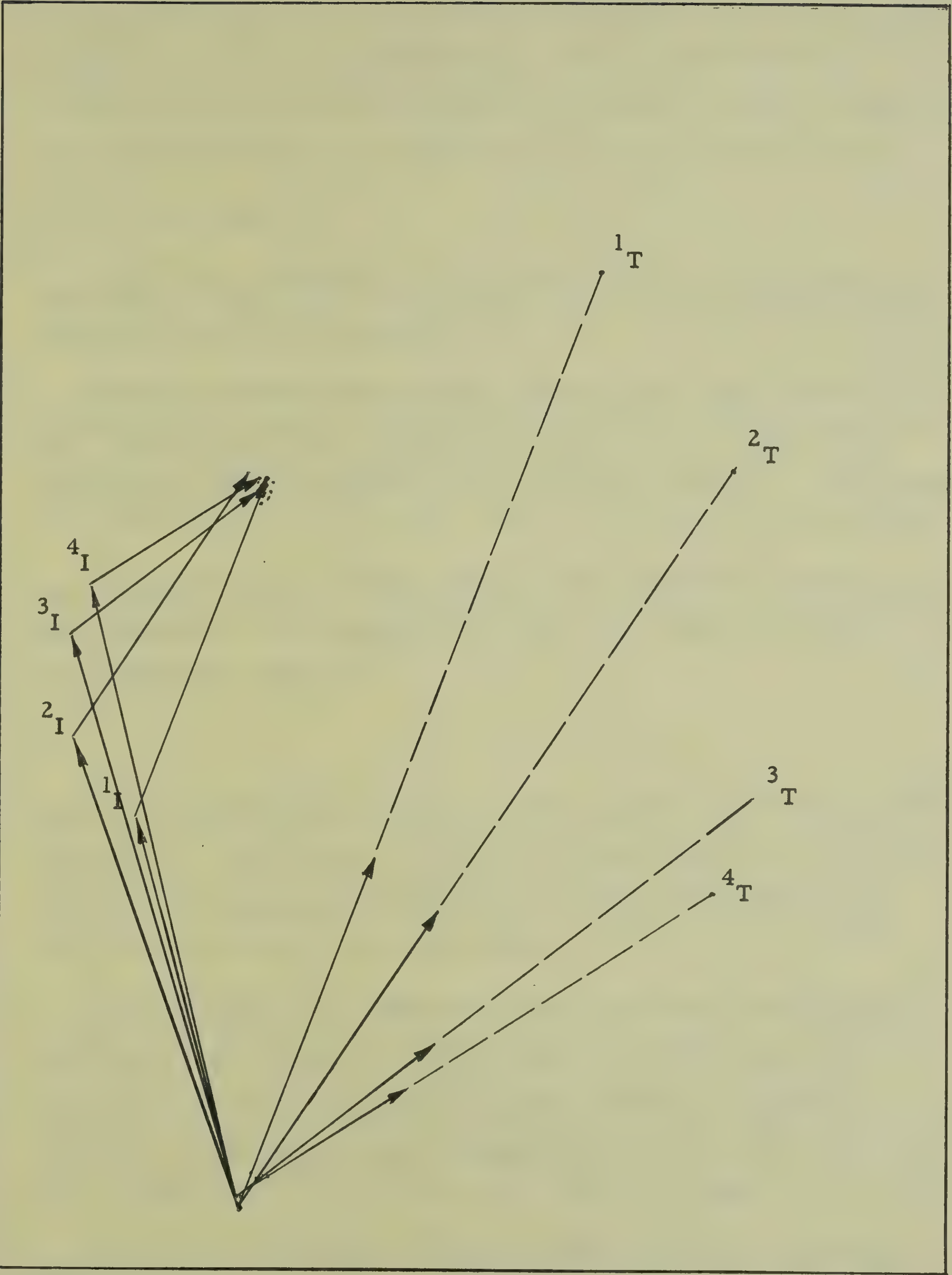
Fig. (a)



scale: 1'' = 3''

I = incident ball - 9/16'' steel ball bearing
T = target ball - 9/16'' steel ball bearing

Fig. (b)



scale: 1″ = 3″

I = incident ball - 9/16″ steel ball bearing
T= target ball - 5/8″ glass marble

Fig. (c)

To convert the velocity vectors to momentum vectors when unequal masses are used, the velocity vector of each ball must be multiplied by the ball's mass. For simplicity we may choose the mass of the incident ball as our unit of mass. The momentum vector of the incident ball then becomes identical with its velocity vector and the momentum vector of the target ball is $\frac{T}{m_i}\vec{T}$ (Fig c). Momentum is found to be conserved.

The vector components of the final momenta of the two balls in a direction at right angles to the initial momentum are equal in magnitude and opposite in direction.

When the student compares the square of the initial and final velocities for the case of equal masses, he will see that the square of the initial velocity of the incident ball is very nearly equal to the arithmetic sum of the squares of the two final velocities. This suggests that some quantity involving the square of the velocity is conserved.

In the case of the unequal masses, one finds that the square of the velocities is not conserved but when the squares of the velocities are multiplied by the appropriate masses, the sum of the final mv^2 equals the initial mv^2.

Once the apparatus is set up, a large amount of data can be taken quickly but the analysis is long and will probably have to be done as homework. Don't require too many collisions to be analyzed. (The calculations of the vector components of the final momenta in a direction at right angles to the initial momenta is interesting but not an essential part of the experiment.)

The last part of the experiment, the calculations showing the conservation of kinetic energy, may be left as optional. There is very little loss of kinetic energy in these collisions and a quick way of checking this in the case of equal masses is to examine the angle between the directions of motion of the two balls after collision. If there is no loss in kinetic energy, this will be a right angle; the greater the loss, the smaller the angle will be.

To see why this angle is 90° when kinetic energy is conserved and the masses are equal, note that since velocity is a vector $\vec{V}$,

the sum of the velocities $\vec{V}_a$ and $\vec{V}_b$ of the two equal masses is

$$\vec{V} = \vec{V}_a + \vec{V}_b$$

and the magnitudes are related by

$$V^2 = V_a^{\ 2} + V_b^{\ 2} - 2V_a V_b \cos \alpha$$

where α is the angle between $\vec{V}_a$ and $\vec{V}_b$. If energy is conserved and the masses are equal, the squares of the velocities are conserved and

$$V^2 = V_a^{\ 2} + V_b^{\ 2}$$

Hence, $-2V_a V_b \cos\alpha = 0$ and therefore $\alpha = 90^\circ$.

SLOW COLLISIONS

An interaction which results in the transfer of kinetic energy from one body to another is hard to visualize. The discussion in the text (Volume 3, Chapter 5, Section 5) is somewhat abstract but this experiment will demonstrate, in slow motion, what occurs in a similar interaction. It is best performed concurrently with the study of the text and it is a good idea to have the carts with hacksaw blade bumpers attached at hand during all the classroom discussions of Sections 5, 6 and 7 of Chapter 5.

A copy of Volume 3 is essential, and the student should follow it carefully while trying slow collisions. This experiment definitely falls into the classroom discussion-laboratory exercise category in which the text material is taught directly in the laboratory. A laboratory report is not necessary since this is a qualitative experiment designed primarily to clarify the interaction process.

Emphasize the parallelism of these slow collisions with billiard ball collisions which differ only in interaction time and distance. Energy is stored temporarily in the bent blades and released again as the carts move apart. In the case of billiard balls, energy is stored in their distortion during a collision.

The collision of the two carts is inelastic. About 30 percent of the energy is lost, mostly as a result of one hacksaw blade sliding over the other. Oiling the blades does not help but friction can be kept at a minimum by mounting one blade with its teeth up and the other with its teeth down. If the blade with the teeth up is adjusted so it is a little higher than the other blade when the two are in contact, friction from the teeth of one blade rubbing on the other blade will be eliminated. Frictional effects will be reduced if higher initial velocities are used. Collisions with several initial velocities should be tried.

Answers to Questions

It is easy to see that kinetic energy is lost by the incident cart and gained by the struck cart during the interaction. When the carts

are at minimum separation they both move very slowly. It looks as though the total kinetic energy is less at minimum separation than at any other time but one cannot be sure by just qualitative observations. It is easy to believe, that as stated in the text, a good deal of energy is stored in the blades when the carts are at minimum separation. One can see that the velocities of the two carts are small and equal at minimum separation.

The questions answered above simply call the student's attention to some of the details of an interaction as described in the text. In answering these questions, the student has finished the main part of the experiment. The experimenting that leads to the answering of the rest of the questions is an open-ended extension for students who are curious about types of collisions other than the one described in Section 5 of Chapter 5.

When the total mass of the carts is increased, the interaction time increases and the distance of minimum separation decreases for a given initial velocity. When the velocity of the incident cart is increased, the interaction time appears to be constant, as indeed it should, if the bending of the blades is proportional to the force. In this part of the experiment, the minimum separation must not become so small that the bodies of the carts come into contact.

If the carts are equally loaded and one is at rest before the collision, the incident cart would end up at rest and the struck cart would move off with all the energy, were it not for energy losses due to friction in the blades. In reality the incident cart keeps moving with a small velocity but is brought quickly to rest by friction with the table, sometimes before the interaction is over.

CHANGES IN POTENTIAL ENERGY

This experiment, like Experiment 570, "The Energy of a Simple Pendulum," presents a situation in which energy is transferred from one form to another with little loss. Here gravitational potential energy is converted into the potential energy of a stretched spring. It is desirable to do both experiments, but if time is short do at least one of them, preferably this one, after Chapter 6, Section 3.

A rigid ringstand about a meter high is needed to support the spring and masses. If a shorter ringstand is used, the masses on the spring are likely to strike the table when released.

The recommended release points for the falling masses will give a sufficient distance of fall for accurate results without the masses striking the table or stretching the spring permanently. A force of about 20 newtons is needed to permanently distort one of the roller shade springs. The forces used to calculate the work done should, of course, be expressed in newtons and the work in joules. Don't talk about "kilograms - force!"

If the laboratory period is short, the plotting of the extension of the spring as a function of the applied force may be deferred until all the data for the falling masses is obtained.

Answers to Questions

The graph of the spring extension x as a function of force F will be so close to a straight line passing through the origin (over the range investigated) that one may assume that x is proportional to F within this range. The slope $K = \frac{F}{x}$ of the line can be substituted in the potential energy function of a linear spring, $U_s = \frac{1}{2}kx^2$, to give the potential energy stored in the spring for any extension x within the elastic range of the spring.

If the graph is not a straight line, the energy stored in the spring can be found by graphical intergration (find the area under the force-extension curve from zero to different values of x).

A table or graph can then be made of U_s as a function of x for use later in the experiment. The roller shade springs, however, are linear. If a student gets a force-extension curve that is definitely non-linear, look for mistakes in technique such as a slipped spring support bar or careless errors in recording the extension of the spring or the force applied to the spring.

The loss in gravitational energy between the two extreme points equals the gain in spring energy between the same points within one or two percent. Energy is conserved and the interactions are elastic within experimental error. (When the mass is allowed to vibrate up and down a number of times, it will continue to rise to nearly the same height. This is additional evidence for both the conservation of energy and the elasticity of the interaction).

The sum of the potential energies when the mass is halfway down is less than the potential energy at the start and finish of the fall. The difference equals the kinetic energy of the falling mass.

Finding the sum of the two potential energies throughout the interaction is hard. It should be done for only one case and only by the most ambitious students. If a mass M, falls from a spring extension x, and the spring has a potential energy function given by $U_s = \frac{1}{2}kx^2$, one can calculate the loss in gravitational energy $\Delta U_g\downarrow$ and the gain in spring potential energy ΔU_s throughout the fall as follows:

Set the arbitrary zero for the gravitational potential energy at the lowest point, found either from experiment or calculated by assuming the conservation of energy and finding the extension x_2, where the change in gravitational potential energy ΔU_g is equal and opposite to the change in the potential energy ΔU_s of the spring. This gives

$$-\Delta U_g = \Delta U_s$$

$$-Mg(x_1 - x_2) = \frac{1}{2}k(x_2^2 - x_1^2)$$

$$= \frac{1}{2}k\,(x_2 + x_1)\,(x_2 - x_1)$$

$$Mg = \frac{1}{2}\,k\,(x_2 + x_1)$$

$$x_2 = \frac{2Mg}{k} - x_1$$

Setting $U_g = 0$ at x_2, we have for the sum of the two potential energies at any extension $\underline{x}$

$$U_g + U_s = Mg\,(x_2 - x) + \frac{1}{2}kx^2$$

$$= Mg\,(\frac{2Mg}{k} - x_1) - Mgx + \frac{1}{2}\,kx^2$$

For a spring whose force-extension ratio $\underline{k}$ is 40.6 newtons/meter and a mass of 1 kg released from a spring extension x_1 = 0.15 m, the sum of the two potential energies in joules is

$$U_g + U_s = 3.26 - 9.8\,x + 20.3x^2$$

Plotting the sum of these energies as a function of $\underline{x}$ gives a parabola as shown in Figure (a). From this graph it is possible to see how the kinetic energy changes as the mass falls. The kinetic energy is equal to the initial gravitational potential energy minus the gain in potential energy of the spring.

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The change in kinetic energy may be investigated experimentally by taking stroboscopic photographs of the falling mass or by having the heavy mass (1 kg) pull a timing tape as it falls.

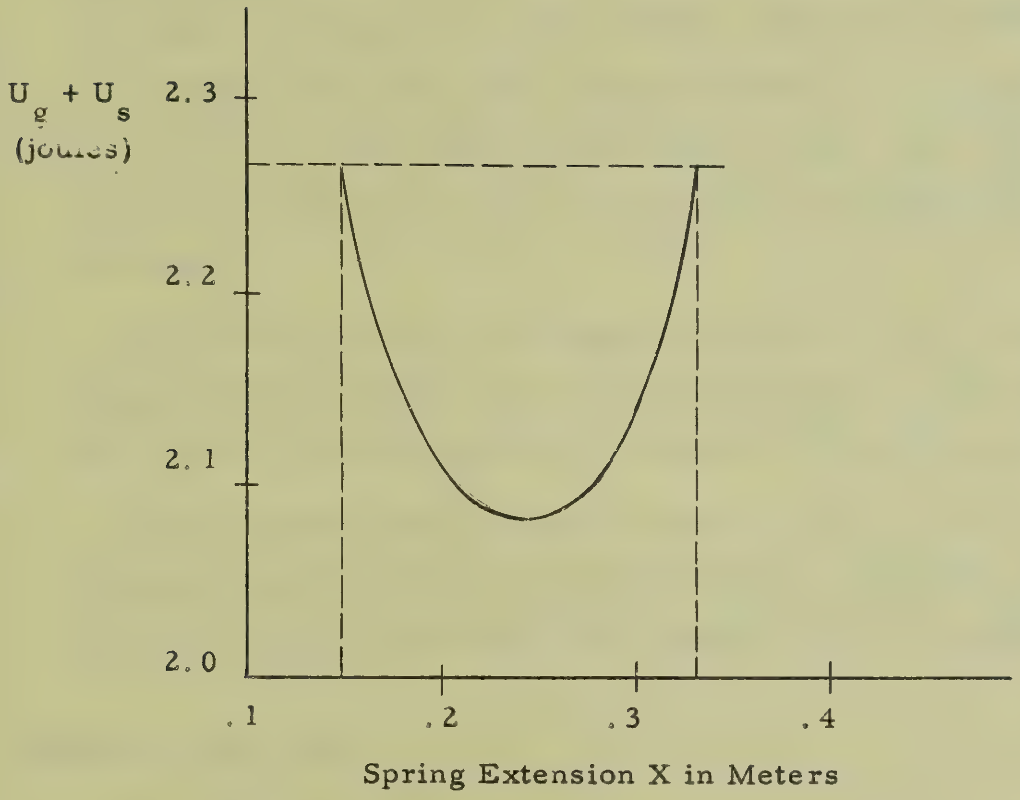

Fig. (a)

THE ENERGY OF A SIMPLE PENDULUM

This experiment shows how potential energy is transfomed into kinetic energy. Like Experiment 560, it is best done after the study of Section 3, Chapter 6 in the text.

A sturdy pendulum support with two or possibly three guys is needed. Let the student practice releasing the brick until he can give it a relatively smooth swing. The line of force of the release thread should extend through the center of the mass of the brick so the brick will not oscillate as it swings. Have the tape attached to the center of the bottom of the brick to minimize errors caused by the slow turning of the brick during the swing. It will aid the student in analyzing his tape if he marks the rest position of the brick on the tape before the run. When he does this, he must make sure that he does not move the timer after he has made the mark on the tape. A brick will help hold the timer in place as the measurements are being recorded.

The velocity of the bob at different positions can be obtained from the graph of position as a function of time or, if the laboratory period is short, the velocities can be determined directly from the tape and the graph of position as a function of time can be omitted.

The velocity of the bob must be found in meters per second if the kinetic energy is to be calculated in joules. Different ways of calibrating the timer are described in "Supplementary Notes on Construction and Use of Carts and Timers" (P. 11 of the Teacher's Laboratory Guide).

Answers to Questions

A comparison of the graphs for potential and kinetic energy clearly shows that, if both are expressed in the same units, one decreases by the same amount that the other increases (Fig. a). Their sum is constant within experimental error. A systematic error may be introduced by a small inaccuracy in the calibration of the timer. In Figure (a) this shows as a small rise in the total energy in the middle of the swing.

We limit the swing of the pendulum to 15° so we can use the approximate relation $h = x^2/2L$ instead of the exact relation

$H(2L - h) - x^2$. For larger swings, the second formula must be used. Also, corrections must be made because the tape follows only the horizontal projection of the motion of the bob, even if the timer is on the floor far from the bob. The height h can also be measured directly instead of using the formula $h\ (2L - h) - x^2$. The sum of the two energies for large swings will remain constant if the above geometric corrections are made.

If is unnecessary to measure the mass of the bob because both the gravitational potential energy and the kinetic energy are proportional to the mass.

Students who have had a course in trigonometry may also investigate simple harmonic motion by analyzing the graph of position as a function of time. To show that the motion for small angles is simple harmonic, the graph should be drawn with new scales as follows:

Set the origin for distance measurements at the rest position of the bob and measure the distances as positive and negative in opposite directions from this point. Call the maximum horizontal displacement x_{max} from this point $\pm$ 1.00 unit, converting the distance x in meters to this new unit $d = \frac{x_{max}}{x}$.

The time of the swing T_{max} is half a period and is therefore equivalent to 180°. The times can then be converted to an angular scale by calculating $\theta = \frac{t}{T_{max}} \times 180^{\circ}$. If d is now plotted as a function of θ, the graph will coincide very closely with that of $d = \cos\theta$. This result holds only for small angels of swing.

The ambitious student may investigate the more complicated, nonsinusoidal motion by releasing the bob from an angle much larger than 15°, making the necessary geometric corrections in analyzing his data.

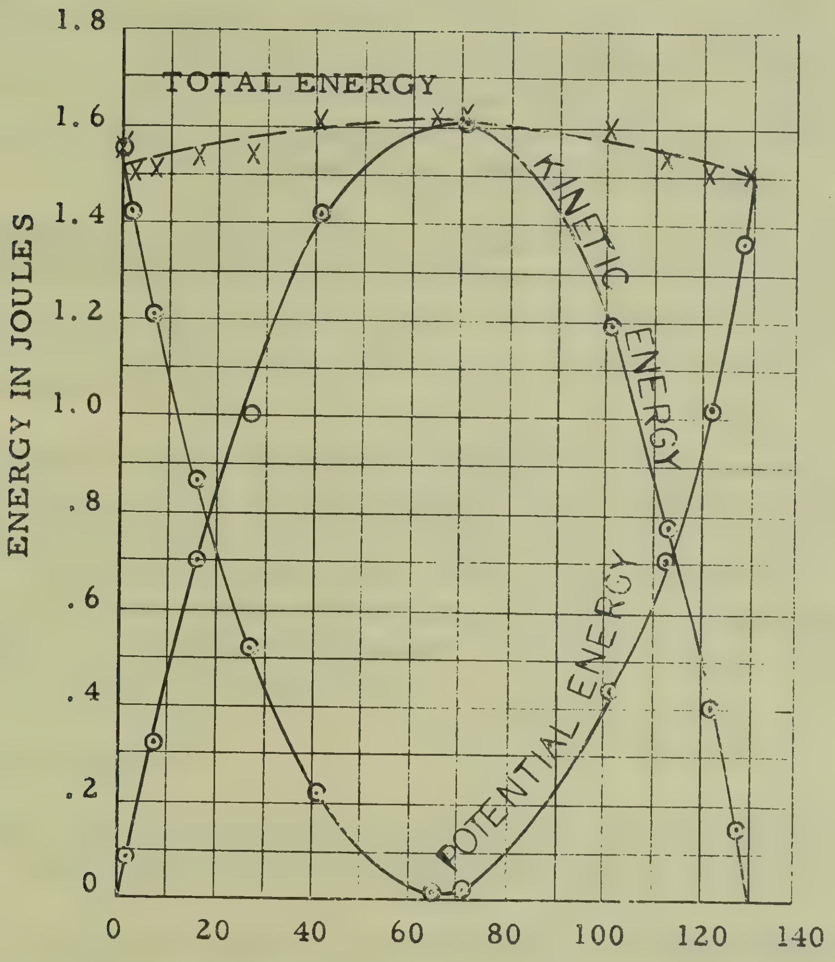
ENERGY IN JOULES
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
0
20
40
60
80
100
120
140
TOTAL ENERGY
KINETIC ENERGY
POTENTIAL ENERGY

Fig. (a)

A HEAD-ON COLLISION

This experiment can be done when Chapter 7 is discussed in class. Its purpose is to emphasize the difference between the conservation of momentum and conservation of mechanical energy. Here about 25% of the kinetic energy is converted into heat whereas momentum is conserved to within 3%. This experiment takes a long time to analyze and can be considered optional. Many of the conclusions can be obtained by restudying the results of Experiments 530 and 540. The collision in 530 is completely inelastic (i.e., the maximum of kinetic energy, consistant with the conservation of momentum, is transformed into heat) whereas 540 is almost completely elastic.

If only a short distance is available to set the first cart moving with constant velocity, it is important to give the cart a quick push. To make sure that both carts have a high enough velocity after the collision, the moving cart should have at least twice the mass of the stationary cart.

To save the time needed to calibrate the two timers, one timer can be used. Simply fold the carbon paper with the coated surfaces on the outside and pass the two tapes above and below the folded paper. If two timers are used, they can be calibrated by placing them next to each other and pulling two tapes through them simultaneously. The ratio of the number of ticks in a given length equals the ratio of their frequencies.

Answers to Questions

If the mass of the moving cart is less than that of the stationary one, it will bounce back thus making the tape useless.

The interaction time is several ticks. The momentum after the collision almost equals the initial momentum (within 3%). As in earlier experiments, some inaccuracy is introduced through the rotational motion of the wheels and some momentum is transferred to the table. The loss of kinetic energy, on the other hand, cannot be attributed to experimental error. A fair fraction is transformed into heat through friction.

If the mass of the incoming cart is greatly increased, it will push the standing cart with almost double its own velocity without

slowing down appreciably.

If the mass of the stationary cart is greatly increased, the incoming cart will bounce back with almost the initial speed and the heavy cart will remain practically at rest.

When the hacksaw blade bumpers are replaced by soft and inelastic material, more kinetic energy will be transformed into heat but momentum will still be conserved.

PHYSICS

VOLUME 3

MECHANICS

TEACHERS GUIDE
PRELIMINARY EDITION

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Physical Science Study Committee
of Educational Services Incorporated

PSSC Teacher's Guide Chapter 1 Vol 111

Chapter 1 - Newton's Law of Motion

Introduction

PURPOSE: To distinguish between kinematics and dynamics and show some of the applications of a study of dynamics.

CONTENT: Dynamics is a study of the causes of motion and of changes in motion.

EMPHASIS: Rather than spend time here to motivate the study of dynamics by showing its applications, the power of dynamical ideas can be shown through examples as the ideas are developed.

COMMENT: At the end of this Guide for Chapter 1, you will find a supplement which comments on frames of reference.

PSSC Teacher's Guide Chapter 1 Vol 111

Section 1 - Ideas About Force and Motion

PURPOSE: To prepare students for an appreciation of Galileo's intellectual achievement by having them consider the development of ideas about force and motion.

CONTENT: It is not easy to find the "natural" state of motion. Some objects tend to stop unless they are pushed; others accelerate toward the earth; still others (i.e., celestial bodies) move continuously without any apparent driving force.

EMPHASIS: Treat this section briefly if the students have already read the text. An alternative is to discuss the first few sections of the chapter with the students before you give any reading assignment in Volume III.

COMMENT: With your help in leading the discussion, your class will be able to give you examples of the three apparently different types of motion even though they have not read the text. An introductory class discussion of this type can add considerable freshness and student interest to this material.

* * *

It would be a mistake to dismiss as incorrect the Aristotelian approach to motion which included the three different categories. Despite the fact that our approach is different today, we still make use of the fact that there are indeed three different situations. In one, friction plays a very important role; in the second, falling objects tend to respond to an accelerating force, in the third, celestial bodies move in stable orbits.

If you discuss this in class, you will have an opportunity to illustrate some of the criteria which make one theory superior to another. Our current ideas of motion are superior to the Greek ideas because we explain more phenomena without introducing as many new, assumptions. Furthermore we can test our ideas experimentally. However, the Greek theory was neither foolish nor "wrong".

Some students misinterpret the last paragraph of section I and think that man-made satellites demolish the three categories by refuting the Greek idea that celestial objects are different from objects on earth. You might point out that the category of celestial objects can be saved if we merely reinterpret it. Instead of claiming that the objects are different, one can simply claim that the laws of motion are different if an object is out of the earth's atmosphere. (After all, there is no doubt that the absence of friction and the absence of the importance of the earth's pull makes outer space quite different from the surface of the earth.)

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Section 2 - Motion Without Force

PURPOSE: To point out Galileo's tremendous intellectual achievement, not only in the original enunciation of the Law of Inertia, but even more in his development of a new method of reasoning in which he combined the observation of real experiments with idealization of these to simpler and more useful situations.

CONTENT: 1) Since an object decelerates when it goes up an incline and accelerates when it goes down, ideally there should be neither acceleration nor deceleration along a horizontal plane.

2) An object which falls attains enough motion to enable it to rise to its original height (except for friction). Hence, an object which moves down an incline and onto a horizontal plane, tends to keep moving indefinitely since it never again will attain its initial height.

3) Quoting Galileo, ". . . . any velocity once imparted to a body will be rigidly maintained as long as there are no causes of acceleration or retardation . . ."

EMPHASIS: Stress this section with class discussion, emphasizing Galileo's reasoning. Appropriate Laboratory Experiment 450.

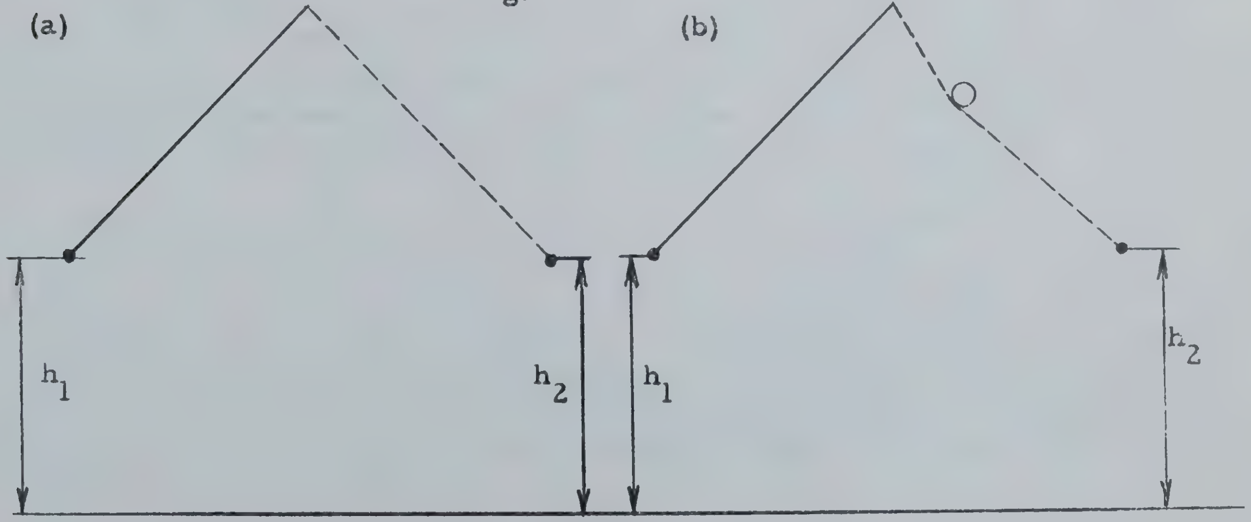
COMMENT: Many students are familiar with Galileo's Law of Inertia; most of them can learn it quickly and would be completely convinced by experiments with dry ice bearings or satellites. However, before you proceed to convince them through the use of modern techniques be sure that all the students understand how Galileo approached the problem. Note that exercise 1 is relevant.

DEMONSTRATION: 1) Show the students simple setups similar to Fig. 4 in the text. A handy material to use for the surface is a length of linoleum or plastic floor covering. The ends can be blocked up. Balls or carts can be kept on the "track" by building up the edges with tape or by slitting rubber tubing lengthwise and slipping a length over each edge. Use several slopes and several moving objects which have different amounts of friction. (See the laboratory manual for techniques useful in reducing friction). Be sure that the students understand that the objects tend to return to their original height as friction is reduced.

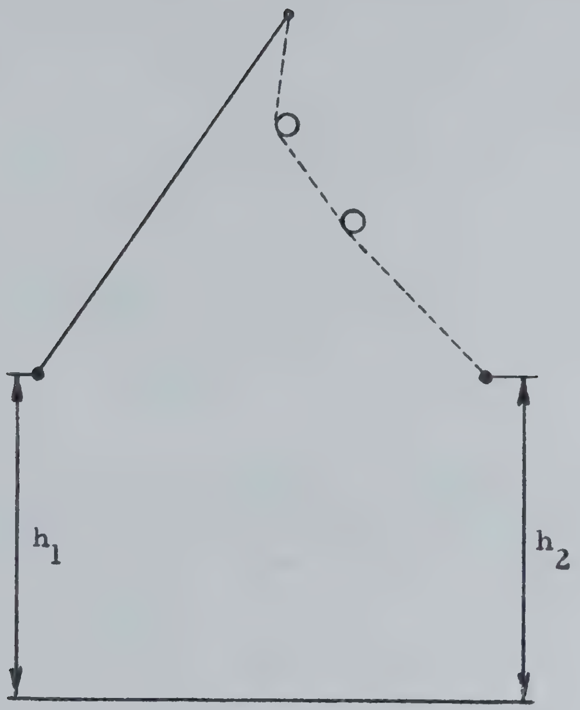
2) If you choose not to have the students do Experiment 450 in laboratory (See TG Laboratory guide on Exp. 450), you can perform the experiment or the following variant to illustrate these points. This is another of Galileo's experiments. A small but heavy weight and a strong non-stretching string or flexible wire are ideal.

Using at least two feet of string, hang the weight from some fairly rigid support near a wall. Make the support stand away from the wall enough so that it is easy to swing the weight parallel to but without bumping the wall. (One arrangement is to use a long nail driven into a wooden molding above a blackboard (or a piece of paper taped to the wall.) Draw the weight aside to a predetermined and marked position, release the weight, and have a student observe the extreme point of its swing. (It may take him several trials to locate it well. Do not let the system swing back and forth but start from rest each time so that the small friction effects do not accumulate. Avoid a loose loop tied around the nail because this will introduce frictional losses. The string should be tied tightly and wound several turns around the nail so that it will smoothly wind and unwind as it swings.) Compare, by measuring distances from the floor (if it is level), or by using a level, the heights of the starting and finishing positions (Figure la below).

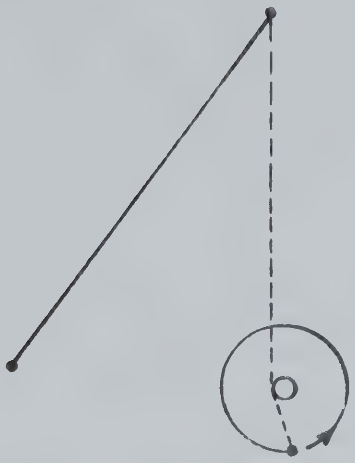
Fig. 1



(c)



(d)



You can emphasize the fact that it is the height, and not some other property of the geometry of the trajectory, which is of primary importance. To do this have another student hold a dowel stick firmly against the wall below the support but relatively close to it so that the dowel will intercept the string during its swing (Figure 1b). Be careful, at this point, not to choose too low a position for the dowel since this will make it impossible for the weight to reach its initial height (as will be discussed below). Do the experiment again, and again note the position to which the bob swings. Check the height. Try it again with two dowels if you like (Figure 1c).

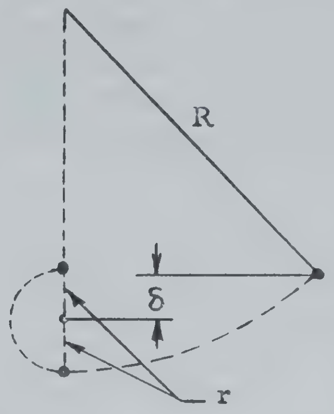
Finally you might propose to the class that a single dowel be held in a position such as Fig 1d below. Note that the weight now is constrained so that it cannot reach its initial height. It should be easy to stimulate guesses as to what will happen. Do the trial and observe the weight, unable to reach its original height, to continue in motion until physically stopped when the string has wound itself around the dowel. (The bob will be somewhat slowed as it strikes the suspending cord or wire as it goes around the dowel. However, it will usually slip past the suspending cord).

Note that in (d) it is important to have the weight moving fast enough as it goes around the top in its circular motion, that the string will remain taut. Otherwise frictional losses will be large. To satisfy this condition we note:

$$1/2\, mv^2 = mg\,(\delta - r)$$

$$mv^2 = 2\, mg\,(\delta - r)$$

$$\text{and } \frac{mv^2}{r} = \frac{2\, mg\,(\delta - r)}{r}$$



This is an expression for the centripetal force required to keep the weight in its circular trajectory at the top of its swing. If the force of gravity, mg, is equal to or less than this, then the string will be kept taut.

$$2\, mg\,\frac{(\delta - r)}{r} \geq mg$$

$$\text{giving } 2\,(\delta - r) \geq r$$

$$\delta - r \geq \frac{r}{2}$$

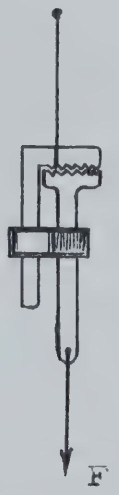
$$\text{and } \delta = \delta - r + r \geq \frac{3r}{2}$$

This problem is clearly not one for the students at this stage of the course, but might be interesting to go back to later on. For now, you should simply take care, without comment, to place the dowel in a position such that the demonstration will work smoothly. If you choose a point directly under the support and a distance r above the lowest point of swing of the weight, then you should start the bob swinging from a point which is more than $\frac{3}{2}$ r above the position of the dowel, where r is the difference between the length of the pendulum and the distance from its support point to the position of the dowel.

CAUTION: The standard demonstrations of the law of inertia concentrate on the tendency of an object which is at rest to remain at rest. No students, not even in Grecian Times, believe that an object at rest tends to move. When you pull a table cloth out from under some dishes or when you knock out the bottom block from a pile without spilling the others, the point that you make is simply that the frictional forces tending to move the upper objects are not large

enough to cause large accelerations. If you do wish to do such a demonstration, an amusing one which is somewhat more instructive is the following:

You need a weight of several pounds and a spool of thread, amply strong to support the weight. (The weight could be some reasonably heavy tool such as a pipe wrench.) Hang the weight from a support with a piece of thread. Tie another piece of the same thread below the weight. Now pull gradually, harder and harder on the lower piece of thread. (Let the students guess which thread will break.)



Eventually the thread above the pipe wrench will break. (Look out for your knuckles!). The upper thread will break first because the force on it will be your applied force plus the weight of the wrench.

Now, suspend the wrench as before, but this time starting with no tension, give the lower thread a sharp jerk. If you do this quickly enough, the lower thread will break and the top one will remain intact.

The explanation for this action lies in the fact that in order to transmit any force to the upper thread, it is necessary (since the thread is somewhat elastic) for the wrench to move, increasing the stretch of the thread. The tendency of the wrench to remain stationary prevents the force of your jerk from being transmitted instantaneously to the upper thread. Before the wrench has time to move far enough to transmit this force, the lower thread breaks.

COMMENT: Since the dry ice bearings (as in the text Figures 5a, 5b, 7, 9 and 16) will be used frequently to illustrate various aspects of dynamics, it is worthwhile to use some class time discussing the experimental setup and the proper interpretation of the pictures. For example, the students should realize that the view in Fig. 5a (p. 1-4) shows the dry ice (white) on which the discs (black) rest. The bent spring between the discs is important for Fig. 5b.

Have the students determine the speed of the disc in Fig. 5b. Using a compass (as a pair of dividers) they can easily find that the speed is about 12.7 cm. per flash or 127 cm per second, if you tell them that a centimeter scale is shown in the figure. They should be able to measure this to within 1% or 2%. Figure 5b is reduced from its real size by a factor of about 5.3. The disc has a diameter of about 10 cms. It is not possible to measure the speed in figure 5 with enough accuracy to measure the deceleration. However other experiments can be done from which the decelerations can be deduced.

Note that the deceleration implied by the last sentence in section 2 (p. 1-5) is about 0.02 ft/sec^2. (This number was not obtained from figure 5). During the seven intervals of the photograph, 0.7 seconds, the speed should have changed .014 ft/sec which is about 1/4 per cent of the measured 4 ft/sec. speed, therefore, unobservable in this observation. (The deceleration was actually obtained by measuring the damping of the oscillations of one of these discs driven back and forth along the table by a spring, with allowances for the losses in the spring.) This is the acceleration an object would get if it were on a perfectly flat 1 meter long table if one side of the table were 0.63 m m (1.063 cm) higher than the other. In order to do experiments which exploit fully the very low frictional force, one must use a very flat table and level it very carefully. Furthermore one must be sure that the table does not bend appreciably due to the arrangement of the points of support. Finally it is necessary to be sure that the table is extremely clean. One source of interference in the motion would be ice crystals formed from water vapor in the air. Using an aluminum table helps to avoid this because aluminum which is a good conductor of heat, prevents local freezing.

Section 3 - Changes in Velocity When a Constant Force Acts

PURPOSE: To show how a constant force affects motion.

CONTENT: A constant force changes the speed in direct proportion to the time the force acts. (A constant force produces a constant acceleration)

EMPHASIS: This material, although very important, will not take much class time, unless your class needs a review in kinematics. The main class discussion should center on Figure 7 and table 1.

APPROPRIATE LABORATORY: Experiment 460.

COMMENT: Note that the text emphasizes Δv and Δt rather than introducing the acceleration, a, in this section. The emphasis is intended to stress the measurements with which the students should be familiar. It also avoids confusion that can result when varying forces are considered (see section 8, p. 1-11). However, the language is much simpler when you can introduce the the term acceleration. It is quite satisfactory for you to start using the term acceleration after you are sure that your students understand table 1 (p. 1-6).

* * *

With reference to Table 1, some students wonder how such precise values as 5.68 cm and 9.48 cm can be determined. Such values could be obtained only by using greatly enlarged pictures and extremely careful measurements. It may be better to treat these two values as 5.7 cm and 9.5 cm respectively so that the students can feel that the limit of their capability in deriving data from the photograph is adequate. In this spirit it is better to treat table 1 as typical data rather than as correct data.

You should have all the students check table 1 carefully either at home or in class. (Note that Fig. 7 is reduced by a factor of about 6.1 from actual size.) They should be able to determine the distance increments to within about 0.5 cm. Inasmuch as an error of only 0.1 cm in the distance increments contributes 0.5 cm/sec to the speed, the values for change in speed might vary quite a bit from the average value of 19.6 cm/sec/1/5 second time interval.

For example, the following values were obtained by some very careful students:

Time Interval	Length	Ave. Speed	Change in Ave. Speed
2	9.8	49.0	
			20.5
3	13.9	69.5	
			18.0
4	17.5	87.5	
			18.0
5	21.1	105.5	
			19.5
6	25.0	125.0	
			$4\,\overline{)\,76.0}$
			19.0

Be sure that the students realize that although the experiment shown in Figure 7 was limited to about 5 per cent accuracy, Newton's law is actually known to a much higher accuracy from other experiments.

* * *

The final paragraph of section 3 states that the vector nature of acceleration is not considered until Chapter 2. The vector character of Newton's law appears explicitly in section 10, and is implied in section 9. However, no detailed problems involving a change in direction are done until Chapter 2.

Section 4 - Dependence of Change of Velocity on Magnitude of Force

PURPOSE AND CONTENT: To show that, for a given object, the change of velocity in a given time interval (i. e., the acceleration) is proportional to the force.

EMPHASIS: This material is very important, but it can be covered quickly if necessary. On the other hand the context provides an opportunity for a somewhat more extended treatment of forces as indicated below. You may wish to stimulate some of this kind of thinking if you have the time to devote to it.

APPROPRIATE LABORATORY: Experiment 470.

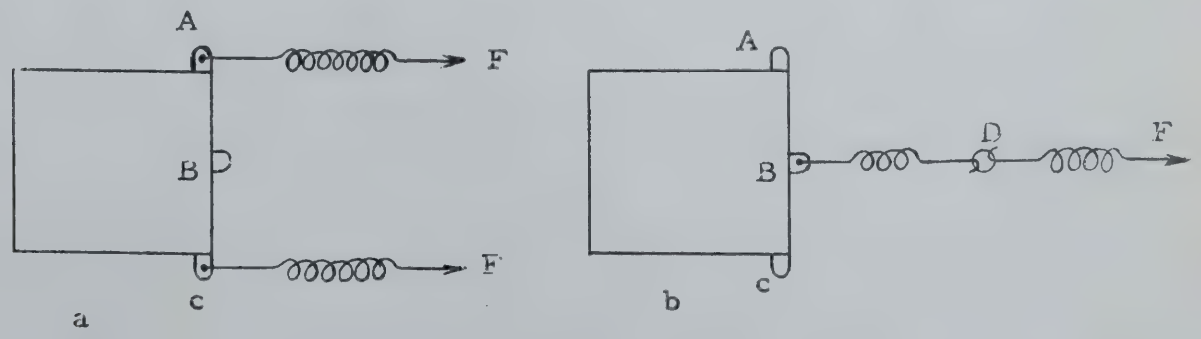
COMMENT: You can be sure that the students understand the crucial experiments of Figures 5, 7, and 9 by having them reconstruct Table 2 from Figure 9. (Figure 9 is reduced by a factor of about 6.1 from actual size.)

* * *

Note that the simple additivity of two parallel forces is assumed in the third paragraph of section 4 (bottom of column 1, p. 1-7). There are many ways in which this fact can be treated. Probably the most direct method is to treat it as a part of the definition of a force. $F + F = 2F$.

* * *

Some students may raise questions about the difference between the effects of hitching two forces to an object in parallel and in series. In the figure below these two arrangements are shown.



In a, a large crate is being pulled by two men, each exerting a force, F, on a spring which is attached to the crate. The two springs are identically constructed, thus stretch the same amounts, and each exerts a force F on the crate. The crate is pulled by two forces, F. F + F = 2F. In b, the same two springs are connected in series and attached to the crate at B. Now one of the men pulls the front spring with a force F. The spring stretches until it reaches the same extension that it had in a. It then exerts this force F on the second spring at their coupling point, D. Now, in turn, F stretches the second spring by the amount that it did in a, and that spring pulls on the crate at B with a force, F. Thus in b, although both springs are stretched the standard amount, one simply serves to transmit the one single force, F, to the other spring which in turn transmits it to the crate. Only one man is pulling. The force on the crate is only 1 x F.

* * *

Some students intuitively feel that a spring will exert twice as much force if it is extended twice as far from its equilibrium, non-extended, length. Although this is approximately true for many springs (and is known as Hooke's Law), it is only approximately true at best and is far from the truth in many cases. For this reason, it has not been used in this development, and should not be introduced as a significant factor in classroom discussion.

Section 5 - Inertial Mass

PURPOSE: To define inertial mass

CONTENT: (1) The inertial mass of an object is defined as the ratio of the force on the object to the acceleration produced by that force.

$$m = \frac{F}{a} \quad \text{or, since } a = \frac{\Delta v}{\Delta t}, \; m = \frac{F\Delta t}{\Delta v}$$

(2) Inertial masses are additive. The sum of the inertial masses of a system is not affected by a chemical reaction between any of the component parts.

EMPHASIS: This material, although quite important, requires very little class time.

COMMENT: Many students have heard about the conversion of mass and energy. Avoid discussion this in detail in class, but if you are asked, after class, you might tell the questioner that it is the sum of the inertial mass and the energy (if the energy is divided by the square of the speed of light) which is constant.

$$(m_1 + m_2 + \ldots + \frac{E}{c^2}) \text{ before} = (m_a + m_b + \ldots + \frac{E}{c^2}) \text{after}$$

Thus if light leaves a flashbulb the bulb will have less mass, however this mass difference would not be detectable by even the most precise balances. (See solution to problem 13).

COMMENT: Experiments which measure mass by measuring the acceleration and the force, illustrate the meaning of inertial mass. However, these experiments cannot be done with high precision and, whereever possible, they are replaced by other more precise measurements. For ordinary-sized objects, comparison of weights (gravitational masses) using an equal arm balance is preferred to an acceleration experiment. However the validity of such a procedure hinges upon a knowledge of the proportionality of inertial and gravitational mass. This is treated in the next section of this chapter.

Section 6 - Inertial and Gravitational Masses

PURPOSE: To show the relation between inertial mass and the pull of gravity.

CONTENT: The pull of gravity is found experimentally to be proportional to the inertial mass. Therefore inertial masses can be compared by using an equal arm balance.

EMPHASIS: This material is extremely important. Be ready to spend at least one whole period in presenting and untangling properly the concepts of gravitational mass, inertial mass and weight.

APPROPRIATE LABORATORY: Experiment 480.

COMMENT: Students may find it confusing to think of two different kinds of mass associated with any object. They may accept more easily the independence of gravitational force and inertial mass. The gravitational force between any object and the earth is what we call the object's weight. If object A weighs twice as much as object B, this signifies that the pull of the earth is twice as great on A as it is on B. We say, then, that the gravitational mass of A is twice that of B. We might equally well call this property gravitational charge. This might reduce the confusion. Now if we were to do another, completely independent set of experiments, in which we measured accelerations of A and B under the action of various forces, we would find that any given force would accelerate A just half as rapidly as it would B. From these acceleration experiments we could conclude that the inertial mass of A is one half that of B. This result could not have been predicted from the observations of the gravitational forces. No basic principle of classical physics relates the two sets of experiments. The fact that the two mass ratios are identical is a new experimentally-established relationship.

Actually the two kinds of experiments described above could not be performed very accurately. A more accurate method is one which is directly sensitive to the ratio $m_{\text{inertial}}/m_{\text{gravitational}}$. Such a method will be treated in detail in Chapter 2, section 2. There, it will be pointed out that an object in free fall is acted upon by a force proportional to $m_{\text{gravitational}}$; $F = Km_g$. Under the action of this force the object will undergo an acceleration, a, given by $F = m_i a$.

Then, $Km_g = m_i a$

or $\frac{m_g}{m_i} = \frac{a}{K}$ (See Exercise 2, Chap. 2)

Thus if m_g is proportional to m_i the accelerations of all objects in free fall will be equal, independent of the absolute values of their masses. This has been found experimentally to be true. Similarly, it can be shown that at any particular geographic location the frequency of oscillation of a pendulum depends only on its effective length and on the ratio m_g/m_i. Then Then if m_g/m_i is a constant for all samples of matter, two pendula of identical geometry but made of substances with different densities, will oscillate at identical frequencies. Again, experiments have verified this. The accuracy of current experiments[1] has established the constancy of m_g/m_i to three parts in 10^{10}. By defining both m_g and m_i in terms of the same standard kilogram, the value of the constant of proportionality has been arbitrarily established as unity.

This apparently coincidental relationship, the proportionality of m_g and m_i, is proposed, in Einstein's general theory of relativity, as a universal truth, an inherent property of matter. This "principle of Equivalency" is one of the premises upon which the theory is built.

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[1] Current techniques are to compare period of rotation of the moon or an artificial satellite with a time standard established by an atomic clock.

Section 7 - Newton's Law: Dynamical Measurement of Force; Units

PURPOSE: To introduce units of force

CONTENT: If Newton's Law is written as $F = ma$, when m is given in kilograms and a is given in m/sec^2, the force is given in newtons.

A newton of force is that force needed to give a one kilogram mass an acceleration of 1 m/sec^2 (or to give 2 kg mass an acceleration of $\frac{1}{2}$ m/sec^2, etc.)

EMPHASIS: The students should have some practice in the use of the units, but not much class time is needed for general discussion.

COMMENT: Note that the text does not enumerate what are commonly treated as Newton's First and Second Laws of motion. Newton's First Law was first stated by Galileo and has been presented as Galileo's Principle of Inertia. Actually this principle covers a situation which is simply a special case of the Second Law (i.e. when $F = o$, $a = o$).

* * *

Note further that the text makes use of the meter-kilogram-second (MKS) system of units. In this system the kilogram is the standard of mass. $Meters/sec^2$ is the unit in which an acceleration is expressed. Then Newton's Law, $F = ma$, defines a force unit, the newton, which is the only force unit presented in this course. To avoid confusion, it is the only system of units which should be discussed. However, if you wish to give your students a feeling for the magnitude of a newton, relative to more familiar unit of measure it may be useful to know that a one newton force is equivalent to a force of about one fifth of a pound. Thus most male adults will weigh between 600 and 1000 newtons.

* * *

The following table of units and equivalents may be useful to you as a reminder. However it should not be considered in class. It is much more important for students to understand the relationships in one system of units than to become adept at converting from one system to another.

	F	=	m	x	a
MKS	Newton		Kilogram		meters/sec^2
c. g. s.	dyne		gram		cm. /sec^2
English (a)	pound		slug		foot/sec^2
English (b)	poundal		pound		foot/sec^2

$$1 \text{ newton} = 10^5 \text{ dynes} = \frac{2.2}{9.8} \text{ pound} = \frac{32 \times 2.2}{9.8} \text{ poundals}$$

Section 8 - Forces That Change and Newton's Law

PURPOSE: To show how to apply Newton's Law if the force is not constant.

CONTENT: 1) If a force changes, its effect can be calculated by considering what happens in each of many very small successive time intervals. In each of these time intervals the motion is described by $F\Delta t = m\Delta v$ (The time intervals can be chosen so small that F is essentially constant during the interval).

2) $F\Delta t = m\Delta v$ is independent of the speed, v, with which an object may be moving.

Section 9 - How Forces Add; The Net Force

PURPOSE: To show the effect of several simultaneous forces.

CONTENT: (1) Forces add as vectors.

(2) Newton's law applies to the vector sum of the forces acting on an object.

EMPHASIS: Treat briefly. Students can learn to use these ideas best by doing some problems.

COMMENT: Avoid getting into a discussion of logic or semantics at this stage. Some very bright students may point out that we defined the force exerted by two identical springs acting in parallel as double the force exerted by one of them; instead of learning it as implied in the first paragraph of section 9.

Newton's Law has been established as a relationship between the acceleratio of an object and a single force acting upon that body. When several forces act, at any instant, there is still only a single observable acceleration. The single effective force which satisfies Newton's Law can then be found: $ma = F_{effective}$ This $F_{effective}$ must be related to the several independent forces which are acting. It is found that $F_{effective}$ is simply the vector sum of all of the forces that are acting.

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Section 10 - The Vector Nature of Newton's Law

PURPOSE: Based on Section 9, to suggest that Newton's law can be written as the relation between two vectors; the net force and the acceleration.

CONTENT: $F \Delta t = m \Delta v$, or $F = ma$

EMPHASIS: It is important for students to get used to the expression of Newton's law in vector form. Make sure they notice that it is the rate of change of velocity, not rate of change of speed, that appears in the equation.

COMMENT: A finer point: The relationship $F = ma$, as suggested in this section, has more content than has been shown up to this point. $F = ma$ implies that the acceleration is always proportional to the force, no matter what the angle between the force and the velocity, since velocity itself does not appear in the equation. For example, Newton's law should hold if we share something sidewise to its motion. That Newton's law really holds will be shown in the next chapter when we discuss motion in two dimensions.

Section 11 - Forces in Nature

PURPOSE: To show how Newton's Law helps us learn about nature.

CONTENT: We would like to know about the many forces in nature. Now that we have Newton's law we can do this by studying motions, finding the accelerations, and thereby deducing the forces. Once we have learned about the forces, we can predict any other motions produced by these forces when they act on different objects.

EMPHASIS: Treat briefly. Students can appreciate this section more fully after they have been through Chapter 3 which gives an exciting example of the application of Newton's Law.

SUPPLEMENTARY COMMENTS ON FRAMES OF REFERENCE AND ON RELATIVISTIC EFFECTS:

F = ma holds only if you measure F and a in a coordinate system which itself is not accelerating. For example if you are standing in a train and it starts abruptly, you fall back. If you measure position, velocity, and acceleration relative to the earth you recognize that the train accelerated but that you did not. However if you confined your measurements to the inside of the railroad car, it would appear that you had suddenly accelerated backwards. Therefore you might conclude that some strange force field had suddenly been turned on. In order to avoid this type of confusion, physicists say that F = ma holds only if the reference frame or coordinate system in which a is measured is not accelerating. This introduces the practical question of how to know whether the reference system is accelerating. It would be silly to say that F = ma applied if Newton's 2nd law was true because that means that F = ma if F = ma. However, it does make sense to say that F = ma if, in the absence of forces, a = o. In other words physicists often say Newton's 2nd law holds in a reference frame in which the first law holds. From this point of view, the first law (which is the name given to the special case when F = o) is used to determine whether or not the reference frame is itself accelerating.

For Home, Desk and Lab

Section	Easy	Medium	Hard	Laboratory	Class Discussion
1					
2	1*, 3	2		Exp 450	2
3		4*, 20		Exp 460*	4, 20
4	6*	7		Exp 470*	7
5	8*, 9, 13	5		Exp 480	
6	10*				
7	11*, 12*	14* (See note below)	16, 17, 21*		21
8					
9		23, 26	25		
10		18, 26	19, 24		18, 19
11		15, 22, 28	27		15

*Indicates problems or experiments which are particularly recommended.
Note: In assigning Problem 14, you may want to make some of the questions more explicit than they are stated in the problem. See NOTE in discussion under the solution to Exercise 14.

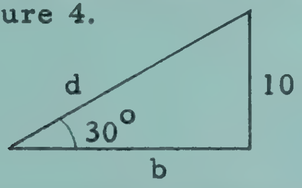
Note: In doing the following problems, it has been assumed that when a mass is given as 2 Kg or a time as 2 seconds, the intent is that this is an exact number and not a number known to only one significant figure, see particularly, 16, 23, 24, 25.

EXERCISE 1: A ball is released from rest on the left-hand incline of Fig. 4, at a height of 10 cm above the lowest point. When it reaches the bottom it starts up the righthand incline, which is at an angle of 30° with the horizontal. If there is no friction,

(a) To what height will it rise?
(b) How far to the right will it go along the second incline?

a) The ball will rise to a height of 10 cms as indicated in the caption of figure 4.

b)



$$d = \frac{10}{\sin 30^\circ} = 20 \text{ cms along the incline.}$$

Some students interpret the phrase "to the right" to mean horizontal component rather than distance along the plane; they should get an answer of

$$b = \frac{10}{\tan 30^\circ} = 17.3 \text{ cm.}$$

EXERCISE 2: Why is it dangerous to step from a moving vehicle onto the ground? In what direction would you tend to fall?

It is dangerous, because when your foot touches the ground it will be brought to rest. However, the remainder of your body will tend to maintain the motion it had in the vehicle. If the center of your body moves forward beyond your support, your foot, you tip over. The way to prevent yourself from falling is to set your other foot in front of you, thereby setting up a new support point beneath the center of your body. In order to continue to support yourself you should face in the direction of your motion and be ready to run in that direction when you step off the vehicle. You would tend to fall in the direction in which the vehicle is moving.

EXERCISE 3: Why is it particularly dangerous to drive fast on an icy highway?

Because the icy road cannot exert a large force on your tires; the force of friction between a tire and ice will be less than one tenth what it might be between the tire and a dry road. With such a relatively small force at your command, you can change neither the magnitude nor the direction of your speed very effectively. The car is "out of control".

EXERCISE 4: A certain force exerted for 1.2 sec raises the speed of an object from 2.8 m/sec to 4.2 m/sec. Later, this same force is applied for 2.0 sec. How much does the speed change in the 2-sec period? (In both cases, the force is applied in the direction of motion.)

In this problem we are, in effect, given a few bits of data like that in table 1. (This problem should follow a class discussion of table 1.) Since the force is the same in the two cases, the change in speed is proportional to the time.

$$\frac{\Delta t_1}{\Delta t_2} = \frac{\Delta V_1}{\Delta V_2}$$

$$\Delta V_2 = \frac{\Delta t_2}{\Delta t_1} \qquad \Delta V_1 = \frac{2}{1.2}\ 1.4 = 2.3 \text{ m/sec.}$$

EXERCISE 5: A body is pulled across a smooth horizontal surface by a spring that is kept stretched by a constant amount. It is found that the body is accelerated at 15 cm/sec^2. What will be the acceleration of the body if it is pulled by two springs, each just like the first spring, side by side and stretched by the same amount? (Note that the time acceleration is not used in the text until section 5. You can assign this problem whenever you have introduced F = ma).

The same mass is accelerated by two different forces. Since F = ma, the resulting accelerations will be proportional to the forces. Since the springs are next to each other and stretched by the same amount, $F_{total} = F_1 + F_2 = 2F_1$. Since the force has been doubled, the resultant acceleration must also be doubled.

$$a_2 = 2\,a_1 = 2 \times 15 = 30 \text{ cms/sec}^2$$

EXERCISE 6: An object sliding on a low-friction bearing is pulled with a constant force. In a time interval of 0.3 sec the speed changes from 0.2 m/sec to 0.4 m/sec. In a second trial, the body is pulled with another force. In the same length of time, the speed now changes from 0.5 to 0.8 m/sec.

(a) What is the ratio of the second force to the first?
(b) If the body is pulled with the second force for 0.9 sec what change in speed results?
(Note that the forces are in the direction of motion).

Both the mass and the time intervals are the same in these two experimental trials.

a) Thus the ratio of the forces is equal to the ratio of the changes of speed:

$$\frac{F_2}{F_1} = \frac{0.8-0.5}{0.4-0.2} = \frac{.3}{.2} = \frac{3}{2} = 1.5$$

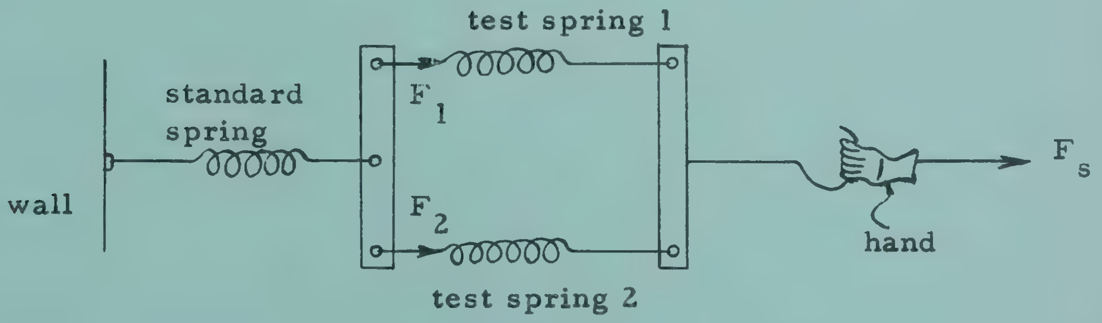
b) Since the time is 3 times as big, the change in speed is 3 times as big; this is fine because (for the same force and mass) the change in velocity will be proportional to the time of application of the force,

$$\frac{\Delta t_2}{\Delta t_1} = \frac{\Delta V_2}{\Delta V_1} \qquad \frac{0.9}{0.3} = \frac{\Delta V_2}{0.3}$$

$$\Delta V_2 = \frac{0.9}{0.3} \times 0.3 = 3 \times 0.3 = 0.9 \text{ meters/sec.}$$

EXERCISE 7: In section 4 we discussed a way in which a stretched spring could be calibrated so that it exerts twice as much force as a standard spring stretche a given amount. Outline a way to determine when a stretched spring exerts one-half as much force as the standard.

Take two identical springs and attach them to a yoke so that they can be made to pull together against one standard spring, as indicated in Figure 2.



Now pull with a force until the standard spring has its standard stretch. The total force exerted by the test springs must be equal to the standard force, since the connecting yoke has no net force unit (i.e. a = o)

$$F_s = F_1 + F_2$$

If the springs are truly identical, they will each have the same stretch, and F_1 and F_2 will be equal. Then $F_1 = F_2 = \frac{F_s}{2}$.

In order to test these half-standard forces, we can perform experiments in which the springs successively apply a force to the same body. Applying F_s, F_1 and F_2 in succession, each for the same time, observe the increase in velocity of the test body. If $F_1 = F_2 = 1/2\ F_s$, then when F_1 and F_2 act together additively on a disc they produce the same acceleration as does F_s acting along upon the same disc. Furthermore, F_1 and F_2, when acting separately on the same disc, should produce only half the velocity change.

EXERCISE 8: Suppose you accelerate an object with a steady force and find that the change in speed during a time interval Δt of 1 sec amounts to 2.4 m/sec. You now repeat the measurement using the same force with a second object. It gains 3.3 m/sec in 0.5 sec. Which body has the greater inertial mass? What is the ratio of the inertial mass of the second object to that of the first?

a) The change of speed of the first object is 2.4 m/sec. in one second. The change of speed for the second object is 3.3 m/sec in 0.5 seconds, or 6.6 meters in one second. Therefore the first object must have the greater inertial mass.

b) $$\frac{m_2}{m_1} = \frac{2.4}{6.6} = 4/11 = 0.37$$

EXERCISE 9: (a) Two blocks, made of different metals, identical in shape and size, are acted upon by equal forces, which cause them to slide across a frictionless horizontal surface. The acceleration of the second block is found to be 4.18 times that of the first. What is the ratio of the mass of the second body to that of the first?

(b) The first block is known to be made of lead. Using the densities given in Chapter 6 of Volume I, can you decide of what material the second block might be made?

a) Since the applied forces are equal, $m_1 a_1 = m_2 a_2$

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} = \frac{a_1}{4.18a_1} = \frac{1}{4.18}$$

b) Since the density of lead is 11.3, the density of the second object must be $\frac{11.3}{4.18} = 2.7$. From the table, the second block may be made of aluminum.

EXERCISE 10: You have two objects, A and B, which balance each other when placed on opposite sides of an equal arm balance. When you place both these objects on one side, they balance a third object C on the other side. Object A accelerated at 3.8 m/sec^2 when you apply a certain force. Suppose you now apply this same force to C. What is its acceleration?

Since $m_a = m_b$, and $m_a + m_b = m_c$, $m_c = 2\ m_a$. Since m_a has an acceleration of 3.8 m/sec^2, m_c will experience half that acceleration or 1.9 m/sec^2.

EXERCISE 11: A body with a mass of 0.5 kg is being accelerated at 4 m/sec^2. How large a force is acting.

Since the mass is given in kilograms and the acceleration in meters/sec^2, newtons will be the units in which the force is expressed.

$$F = ma = 0.5 \text{ kg} \times 4 \text{ m/sec}^2 = 2 \underline{\text{ newtons}}$$

EXERCISE 12: A force of 3 newtons is applied to a mass of 0.6 kg. How fast does it accelerate?

$$F = ma \qquad 3 \text{ Newtons} = 0.6 \text{ kg} \times a \text{ m/sec}^2 \qquad a = \frac{3}{0.6} = 5 \text{ meters/sec}^2$$

EXERCISE 13: Why is the flask in Fig. 10 closed?

The flask is closed so that there can be no possibility for any product of the chemical reaction to escape from the system. (Actually no gas is generated in this reaction.) Thus the mass of the flask plus contents remains constant. (Note that, rigorously, if heat, light or sound energy are given off during the reaction, there will be a corresponding decrease in mass as dictated by the mass equivalence of energy. However, this change is neglibibly small, and it is probably best not to discuss the point unless the question is raised by some student.) A change of mass Δm kg corresponds to a change of energy $E = \Delta m C^2 = \Delta m(3 \times 10^8 \text{ m/sec})^2 = 9 \times 10^{16}\ \Delta m$ joules. Very good heating coal gives about 1.5 joules of heat energy per kg. This corresponds to a mass change of only

$$1.5/9 \times 10^{16} = 1.67 \times 10^{-17} \text{ kg!!}$$

EXERCISE 14: In Fig. 16 measure Δx, the distance traveled in each interval, for intervals 5 through 10.

a) What is the speed $\Delta x/\Delta t$ in each interval?
b) What are the changes in speed, Δv, in each interval?
c) What is the acceleration, $\Delta v/\Delta t$, in each interval?
d) Has a constant force been acting?
e) Assuming the disc has a mass of 2 kg., what is the average applied force?

The values below were taken from a student's measurement. They are typical, not necessarily "correct".

Interval number	Δx (cm)	$\frac{\Delta x}{\Delta t}$ (cm/sec)	Δv (cm/sec)	a (cm/sec^2)
1	.9	4.5		
			10.0	50.0
2	2.9	14.5		
			9.5	47.5
3	4.8	24.0		
			10.0	50.0
4	6.8	34.0		
			10.5	52.5
5	8.9	44.5		
			9.5	47.5
6	10.8	54.0		
			10.0	50.0
7	12.8	64.0		
			10.5	52.5
8	14.9	74.5		
			9.5	47.5
9	16.8	84.0		
			9.5	47.5
10	18.7	93.5		

(d) A constant force has probably been acting
(e) $a_{\text{average}} = 49 \text{cm/sec}^2 = 0.49 \text{ m/sec}^2$ $F = 2\,(0.49) = 0.98$ newtons

Note that 1 cm on the scale in the picture is slightly less than 1/16 inch. Hence, even careful students may make errors of 0.3 cm in measuring Δx. This corresponds to an error of 1.5 cm/sec (or 15%) in Δv. If errors are made in adjacent Δv values, errors of 30% or 15 cm/sec^2 may be common in the acceleration, a.

Note also that certain errors are compensated through the procedure of taking successive differences. If one Δx were measured too large, this would result in too large a value for $\Delta x/\Delta t$ in that interval. This will affect the calculations of two of the Δv's, raising one value and lowering the other by the same amount. Then in taking a final average, the effects of such an error would tend to cancel out.

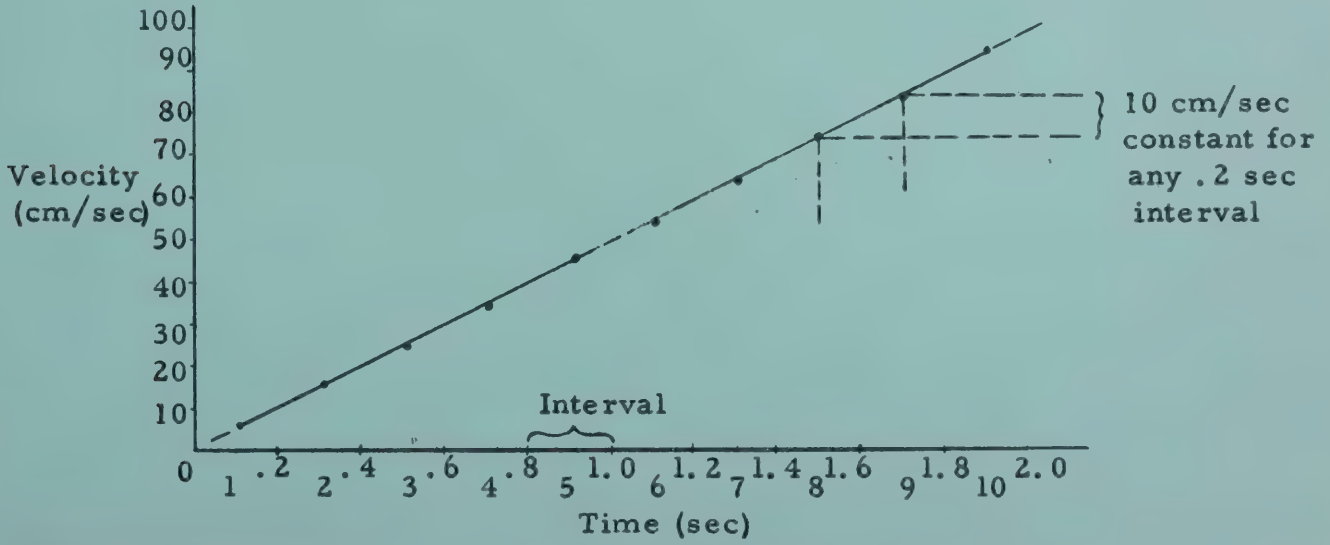
SPECIAL NOTE: In solving this problem, some students may have difficulty in seeing that they must deal with average speed in successive time intervals. For this difficulty, you may want to let them struggle, then straighten them out in class discussion, or you may want to give them a few pointers. If you want to provide hints, you can suggest that, in

Part a) "speed, $\Delta x/\Delta t$ in each interval" means average speed.
Part b) "changes in speed, Δv, in each interval" should be interpreted as changes in average speed between the successive intervals.
Part c) "acceleration $\Delta v/\Delta t$, in each interval" should be based on changes in average speed between the successive time intervals through assigning the average speed for the interval to the mid-point in time of each .2 sec interval.

In any case, it will be instructive for many students, after they have worked this problem, to graph on the blackboard the velocity-time data for this problem (see below), and remind them that

1) Δv for a .2 sec interval is constant (approximately 10 cm/sec) for any .2 sec interval no matter what instant is chosen for the start of that interval, and

2) that the acceleration $\Delta v/\Delta t$ in cm/sec^2 is simply Δv for a 1 sec interval instead of a .2 sec interval, and while its values were derived from changes in average speed between successive time intervals, it too is constant (the graph is straight) no matter where one chooses to "start" a one-second interval.



EXERCISE 15: A force of 3 newtons is exerted on an object and it speeds up at 1.5 m/sec^2.

(a) Assuming that this is the only force on the object, what is the mass?
(b) How would you make an independent measurement of the mass to see if you are correct?
(c) Suppose your second measurement indicates that the mass determined by the first method is too great. What might you conclude?

a) $F = m\,a$ 3 newtons = m kg x 1.5 m/sec^2

$$m = \frac{3}{1.5} = 2 \text{ kg.}$$

b) Place the object on an equal arm balance and balance it against standard masses.

c) You might conclude that some extraneous force (e.g. friction) was acting on the object in addition to the measured force that you intentionally applied.

If this were found not to be a reasonable explanation - you should suspect that one of the following errors had been made:

(1) One of the standard masses might be damaged.
(2) Your force standard (3 newtons) may be incorrectly calibrated.
(3) Your measuring stick might be incorrectly calibrated.
(4) Your timing device, used in measuring the acceleration, might be in error.
(5) You may have made a mistake in observing or recording your data.
(6) There may be some residual gravitational force acting if your surface is not completely level.

You could check each item (1), (2), (3), and (4) by comparing each standard used against a second standard. (2) is the one that is most likely to be in error. In order to check (5), you may repeat your experiment. If all those items check out correct, you would have to conclude that Newton's Law had failed. To investigate (6), you could do the experiment twice, in opposite directions. Taking the average of the two results obtained would eliminate the effect of a tilted surface.

EXERCISE 16: Two bodies, one of mass 8 kg and the other of mass 2 kg, stand at rest on a smooth table top side by side. The 8-kg mass is accelerated from rest by a force of .7 newton and the 2-kg mass is accelerated from rest in the same direction by a force of 1.4 newtons. Both bodies start accelerating at the same instant of time. How long does it take before the separation of the bodies is 5 m? What is the velocity of each at this instant of time?

We can first solve for the acceleration of each body using Newton's Law. Then, knowing the accelerations, we can solve the problem in kinematics which is involved in learning the relative positions of the bodies.

$$F = ma \qquad a_1 = \frac{.7}{8}\ \text{m/sec}^2 = 0.087\ \text{m/sec}^2$$

.875

$$.7 = 8\,a_1$$

$$a_2 = \frac{1.4}{2} = 0.7\ \text{m/sec}^2$$

$$1.4 = 2\,a_2$$

When a body moves from rest with a constant acceleration, (Chapter 6, Vol. 1)

$$d = \frac{1}{2}\,a_1 t^2$$

$$d_2 = \frac{1}{2}\,a_2\,t^2$$

For our problem the time of motion is the same for the two bodies.

We are interested in knowing how much time will elapse before the absolute value of the difference, $d_2 - d_1$, is equal to 5 meters. Since a_2 is greater than a_1, d_2 will always be greater than d_1. We wish to find the value of t under the condition, $d_2 - d_1 = 5$ meters. Subtracting the first equation from the second, above, we find:

$$d_2 - d_1 = 5 = 1/2\,(a_2 - a_1)\,t^2 = 1/2\,(0.7 - \frac{0.7}{8})\,t^2 = \frac{4.9}{16}\,t^2 = 5$$

$$t^2 = \frac{80}{4.9} = \frac{800}{49} = \frac{400 \times 2}{49}$$

$$t = \frac{20 \times \overset{1.414}{1.4}}{7} = \overset{4.04}{4.0}\ \text{seconds}$$

(We can check algebra by finding $d_1 = 0.7\boxed{1}$ meters and $d_2 = 5.71$m).

5.6?

These figures are right for 4.04 secs. But you can't get them from 40

Now to find the velocity of each body at this instant, knowing their accelerations and the time, we make use of the relation $V = a\,t_1$

$$V_1 = \frac{.7}{8} \times 4.0 = \frac{2.8}{8} = 0.35\ \text{m/sec.}$$

$$V_2 = .7 \times 4.0 = 2.8\ \text{m/sec.}$$

EXERCISE 17: A block of mass 3 kg is moving along a smooth horizontal surface with a velocity v_o at an instant of time $t = 0$. A force of 18 newtons is applied to this body opposite to the direction of its motion. This force reduces v to one-half its value while the body moves 9 m. How long does it take for this to occur? What is v_o?

A force of 18 newtons, acting on a body of mass 3 kg, produces an acceleration, $a = \frac{F}{m} = \frac{18}{3} = 6 \text{ m/sec}^2$. The change in velocity is given by: $\Delta V = a\,\Delta t = +\,6t$ m/sec. Since the force is directed backwards, the body slows down. Thus the residual speed, V_r, after the force has been acting for t seconds is:

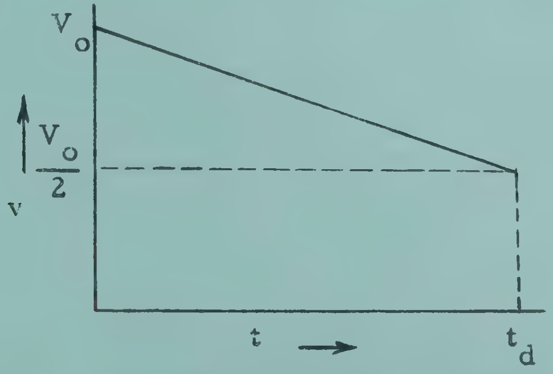
$$V_r = V_o - \Delta V = V_o - 6t$$

But it is stated that $V_r = \frac{V_o}{2}$, after the time interval of deceleration, t_d seconds

$$\frac{V_o}{2} = V_o - 6t_d$$

$$\frac{V_o}{2} = 6t_d \text{ and } V_o = 12\,t_d$$

We can plot the motion on a v vs. t graph:



The distance covered during the deceleration is given by the area under the straight line describing the body's motion, in units of v x t. This is the sum of the rectangle plus the triangle. Thus

$$d = \frac{V_o}{2}\,t_d + 1/2\,\frac{V_o}{2}\,t_d = 3/4\,V_o t_d$$

but $V_o = 12\ t_d$ therefore

$$d = 9 = 3/4\ 12\ t_d\ t_d = 9\ t_d^2$$

$$t_d^2 = \frac{9}{9} = 1 \qquad \underline{t_d = 1 \text{ second}}$$

and $V_o = 12\ t_d = 12$ m/sec.

(Note that the constant, 12, in the equation $V_o = 12\ t_d$, has the dimensions of an acceleration, m/sec^2. It is not a pure number.)

EXERCISE 18: If the distance covered by a moving object varies directly as the time, what conclusions could you draw about the motion and the forces?

If the distance covered varies directly as the time, this means that the increment in distance, ΔX, is proportional to the increment in time, Δt. Expressed analytically;

$$\frac{\Delta X}{\Delta t} = \text{constant}$$

But $\frac{\Delta X}{\Delta t}$ is simply the speed of the object. Therefore it must be moving with constant speed, and we may conclude that there is no net force acting in the direction of the motion. (A force might be acting perpendicular to the direction of motion and this would not affect the speed.) Were such a force constant, motion at constant speed in a circular path would result.

EXERCISE 19: If you observe an object covering distance in direct proportion to t^3 where t is the time elapsed, what conclusion might you draw about the acceleration and the forces?

Here, in a manner similar to that employed in problem 18, we can write:

$$x = Kt^3$$

Given, $s \propto t^3 = t \times t^2$

Know, $s \propto a t^2 = a \times t^2$

$\therefore a \propto t$

$F \propto m a$ $F \propto a$

m is constant, so $\therefore F \propto T$

We can now construct a table of time, distance covered, speed, change in speed and acceleration. To do this we must assume some arbitrary value for the constant, K, and choose an arbitrary time interval. For simplicity, we shall make $K = 1$ m/sec^3 and $\Delta t = 1$ sec. Then:

t(sec)	x(meters)	Δx	$v=\Delta x/\Delta t$(m/sec)	Δv(m/sec)	$\Delta v/\Delta t$ m/sec^2
0	0				
		1	1		
1	1			6	6
		7	7		
2	8			12	12
		19	19		
3	27			18	18
		37	37		
4	64			24	24
		61	61		
5	125			30	30
		91	91		
6	216			36	36
		127	127		
7	343				

We see, by inspection, that the acceleration is increasing linearly with the time. Therefore, we might conclude that the force (which must be proportional to the acceleration) must also be increasing linearly with the elapsed time.

EXERCISE 20: You pull a sliding disc with a constant force, starting from rest. The speed increases 0.1 m/sec in each Δt of 0.3 sec. How fast is the body moving after 1.2 sec? Now you begin pulling with the same force in the opposite direction. You continue pulling in the reverse direction for 0.9 sec. How fast is the body moving? In what direction is it moving?

First we must evaluate the acceleration.

$$a = \frac{\Delta V}{\Delta t} = \frac{0.1}{0.3} = 1/3 \text{ m/sec}^2$$

Since the disc started from rest at t = o, the instantaneous speed at any time, t seconds later, will be given by

$$V = at = 1/3\ t$$

After 1.2 seconds, the speed will be:

$$V_{1.2} = 1/3\ \ 1.2 = 0.4 \text{ m/sec.}$$

Applying the same force in the reverse direction produces a reversed (negative) acceleration. The change in speed (not the absolute speed now since the object is not starting from rest) is given by:

$$\Delta V = at = - 1/3\ t = - 1/3\ 0.9 = - 0.3 \text{ m/sec.}$$

The final resultant speed will be the vector sum of the initial speed at the time of the force reversal, and the change in speed that occurs in the 0.9 seconds thereafter:

$$V_f = V_{1.2} + \Delta V = 0.4 - 0.3 = + 0.1 \text{ m/sec.}$$

The positive sign of V_f indicates that the final speed is still in the direction of the motion at the time of the force reversal.

EXERCISE 21: A block of mass 2 kg is pulled on a frictionless table by a constant force of 6 newtons. The block starts from rest.

(a) What is the acceleration of the block expressed in meters/sec^2?
(b) What is the velocity of the block 3 sec after the force starts acting?
(c) How far does the block travel in 2 sec?
(d) If at the end of 3 sec, the block splits in two equal pieces - one piece still being pulled by the force of 6 newtons, and the other free - how far apart will the two pieces be 2 sec after the break occurs?

a) $F = ma$
$6 = 2a$
$a = \frac{6}{2} = 3 \text{ meters/sec}^2$

b) Since the block starts from rest, we have after 3 seconds

$$V = at = 3 \times 3 = 9 \text{ m/sec.}$$

c) Again, since the motion starts from rest, we have,

$$d = 1/2\ at^2 = 1/2\ 3 \times 2^2 = 6 \text{ meters}$$

d) At the end of the first 3 seconds the pieces both are moving with a speed of 9 m/sec. Then, after two more seconds, the unaccelerated half will still be moving at this speed and therefore will have traveled 9 x 2 = 18 meters further. The accelerated half has: $a = 6 \text{ m/sec}^2$ and therefore its speed at the end of the two additional seconds will be $6 \text{ m/sec}^2 \times 2 \text{ sec} = 12$ m/sec greater than at the time of the break. Thus it will be moving at 9 + 12 = 21 m/sec. Then its average speed during the 2 seconds period must have been

$\frac{9 + 21}{2} = 15$ m/sec. Traveling at this average speed for two seconds, the accelerated half of the block must cover a distance of 30 meters in the two second interval. Then the distance between the two pieces will be 30 - 18 = 12 meters.

EXERCISE 22: A block of mass 8 kg, starting from rest, is pulled along a horizontal table top by a constant force of 2 newtons. It is found that this body moves a distance of 3 m in 6 sec.

(a) What is the acceleration of the body?

(b) What is the ratio of the applied force to the mass?

(c) Since your answer to part (b) is not equal to that of part (a), (at least it shouldn't be), what conclusions can you draw about this motion? Give numerical results, if possible.

a) The block starts from rest and is acted upon by a constant force. Assuming that the net force acting upon the block was constant, the acceleration will also be constant. Then the distance travelled will be:

$$d = 1/2\ at^2$$

$$3 = 1/2\ a\ (6)^2 = 18\ a$$

$$a = \frac{3}{18} = \frac{1}{6}\ \text{m/sec}^2$$

STET

b) Since F = 2 newtons, and m = 8 kg, $\frac{F}{m} = \frac{2}{8} = \frac{1}{4}$ m/sec^2

c) The actual acceleration, $\frac{1}{6}$ m/sec^2, was less than the acceleration, $\frac{1}{4}$ m/sec^2, which would have been caused if the applied force had acted alone. Therefore some other force (retarding) must also have been acting. It could be assumed to have been a friction force. In order to evaluate it, we must assume that it also was constant. Then we can solve for the net effective force which must have been acting to produce the observed acceleration.

$$F_{\text{applied}} - F_{\text{friction}} = m\ a$$

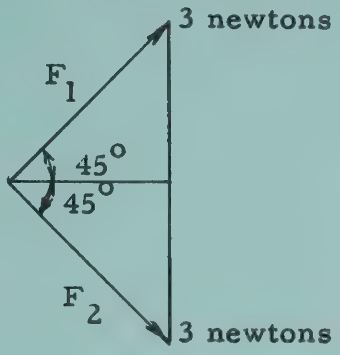
$$2 - F_{\text{friction}} = 8(1/6) = 4/3 \quad \therefore F_{\text{friction}} = 2/3 \text{ newtons}$$

The friction force is, of course, opposed to the motion of the block.

EXERCISE 23: A 6 kg object is acted upon by two forces of 3 newtons each.

(a) If these forces act at an angle of 90° with each other, what is the magnitude and the direction of the acceleration?

(b) What is the acceleration if the forces act in the same direction? In opposite directions?



the

a) Remember that Newton's Law applies to the vector sum of all forces acting on a body. F_1 and F_2 may each be resolved into two components, one horizontal and one vertical. The horizontal components will both be forces directed toward the right. The vertical component of F_1 will be directed upward, whereas that of F_2 is directed downward. Since the angles are all 45° the magnitude of each individual component is $\frac{3}{\sqrt{2}} = \frac{3}{1.41} = 2.12$. The

(2)

net vertical force, above, will be zero. The horizontal force will be

$F_h = 2 \frac{3}{\sqrt{2}} = 4.24$ newtons to the right. Therefore,

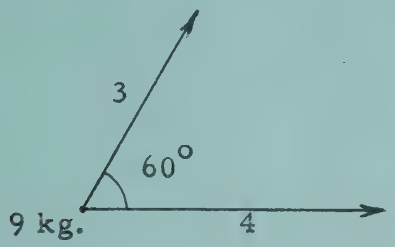
$$a = \frac{F}{m} = \frac{4.24}{6} = 0.707 \text{ m/sec}^2$$ along the bisector of the 90° angle.

b) $a = \frac{6}{6} = 1 \text{ m/sec}^2$ Since F = 6 newtons if the forces act in the same direction

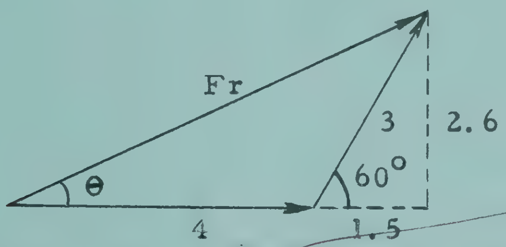
$a = \frac{0}{6} = 0$ Since F = 0 if the forces act in opposite directions

EXERCISE 24: A 3-newton force and a 4-newton force act on a 9-kg mass. The two forces act at an angle of 60° with each other.

(a) If the object starts from rest, how fast will it be moving after 3 sec?
(b) In what direction will it be moving?



The first problem here is to obtain from the two forces which are given, one single resultant force of known magnitude and direction. This can be done graphically with protractor and ruler.



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25.5°

Such a construction gives $F_r \doteq 6.1$ newtons directed at an angle $\theta \doteq 26°$ from the 4 newton force. The 6.1 newton force, will produce a constant acceleration of:

$$a = \frac{6.1}{9}\ \text{m/sec}^2$$

The resultant speed after 3 seconds will be $V = at = \frac{6.1}{9} \times 3 = 2.03$ m/sec.

This velocity will have the same direction as that described above for the resultant force.

This vector problem can also be solved analytically by making the construction indicated in the second figure. Thus we can see that the 3 newton force has a component of $3 \times \cos 60° = 3 \times 1/2 = 1.5$ newtons in the direction of the 4 newton force (the x direction). Its perpendicular component is $3 \sin 60° = 3 \times .866 = 2.6$ newtons. Then the components of acceleration are

$$a_x = \frac{F_x}{m} = \frac{4 + 1.5}{9} = \frac{5.5}{9}\ \text{m/sec}^2$$

$$a_y = \frac{2.6}{9}\ \text{m/sec}^2$$

and after 3 seconds time

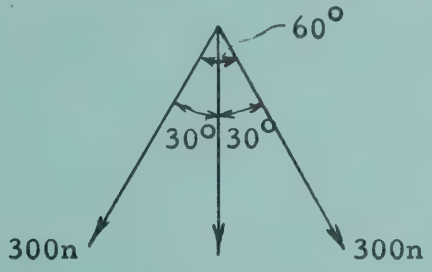
$$v_x = 3\,a_x = \frac{5.5}{3}\ \text{m/sec}$$

$$v_y = \frac{2.6}{3}\ \text{m/sec}$$

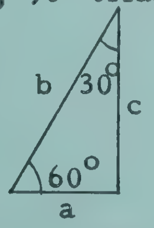
$$v = \sqrt{v_x^{\,2} + v_y^{\,2}} = 1/3\sqrt{30.25 + 6.76} = 1/3\sqrt{36.01} = 6/3 = 2\ \text{m/sec.}$$

EXERCISE 25: Two men wish to pull down a tree by means of a rope fastened near the top. If they use one rope, the tree will come down on top of them. To prevent this, they tie two ropes 10 meters long to the same point and stand on the ground 10 meters apart when they pull. If each pulls with a force of 300 newtons, what is the net force exerted on the tree?

Since each rope is 10 m. long and the two men stand 10 m. apart, the three lines form an equilateral triangle, and the angle between the ropes is equal to 60°.



The two forces are equal, so the resultant must be along the line which bisects the 60° angle. To find the component of each of the forces along this line, note that we have a 30°, 60°, 90° triangle.



The ratio of the sides a: b: c: = 1: 2:$\sqrt{3}$

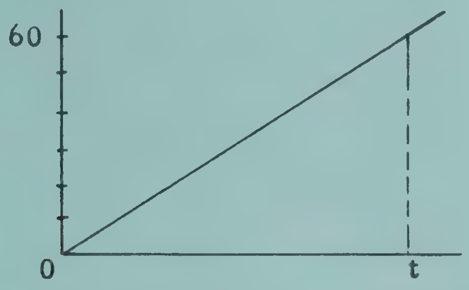
Therefore $\frac{b}{c} = \frac{X}{300} = \frac{\sqrt{3}}{2}$ and $X = 300\,\frac{\sqrt{3}}{2} = 150 \times 1.73$

$= 260$ newtons

There are two ropes pulling on the tree, and each exerts a force which has a component of 260 newtons along this direction. Thus the net force on the tree is 520 newtons.

EXERCISE 26: Find the total force required to accelerate a 450-kg rocket from a standing start to a velocity to 60 m/sec along a 100-meter horizontal track. The retarding force of friction is 93 newtons.

We assume that "total" force means "applied" force. In order to do this problem we must assume that the thrust of the rocket is a constant and therefore that the acceleration is a constant. Then on a speed vs. time graph, the speed will increase linearly from 0 to 60 meters/second, as shown:



To find the time that is required for this acceleration process, note that the area of the above triangle, the distance traveled, is given as 100 meters.

$$d = V_{average} t$$

$$100 = 1/2 \; 60 \; t$$

$$t = \frac{200}{60} = 3.33 \text{ seconds}$$

Then $$F_{net} \Delta t = m \Delta V$$

$$3.33 \, F_{net} = 450 \times 60$$

$$F_{net} = 135 \times 60 = 8100 \text{ newtons}$$

$$F_{net} = F_{total} - F_{retarding} = F_{total} - 93$$

$$F_{total} = 8193 \text{ newtons}$$

EXERCISE 27: The retarding force of air resistance on a balloon is proportional to the square of the velocity. For a certain balloon, inflated a certain amount, this force is given in newtons by $F_r = .2v^2$ where v is the velocity in m/sec. The balloon has a mass of 10 gm.

(a) Draw graphs of the balloon's acceleration as a function of velocity when you pull it with a 1.8-newton force and with a 7.2-newton force.

(b) What is the maximum velocity that the balloon will reach in each case?

(c) If the mass were 5 gm, how would this affect the maximum velocity?

(d) What do you think would be the effect on the maximum velocity if you inflated the balloon to a larger volume?

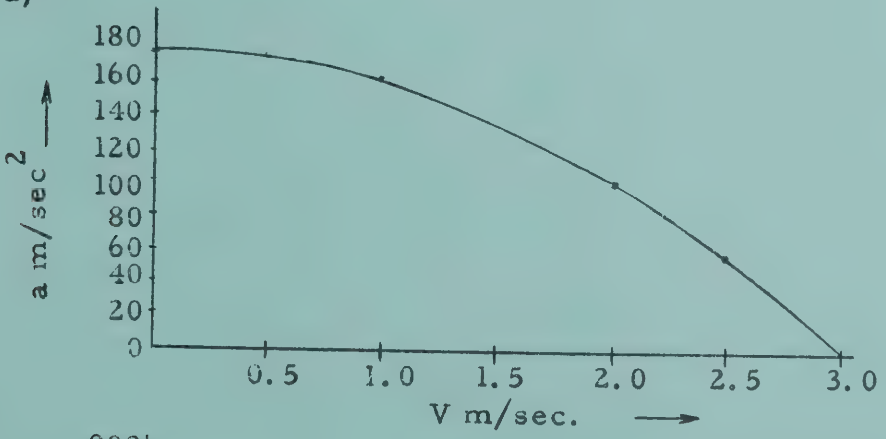
(We shall assume that the given mass is the effective mass of the balloon plus the air inside it.)

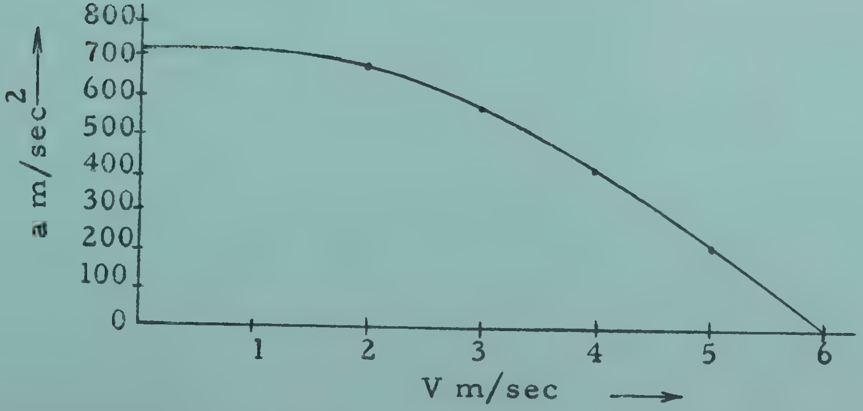
$$a = \frac{F_{net}}{m} = \frac{F_{applied} - F_{resistance}}{m} = \frac{F_{applied} - .2v^2}{m}$$

For an applied force of 1.8 newtons: $a = \dfrac{1.8 - 0.2v^2}{10^{-2}} = 180 - 20v^2$

For a 7.2 newton force, $a = \dfrac{7.2}{10^{-2}} - \dfrac{.2v^2}{10^{-2}} = 720 - 20\,v^2$

a)





b) The maximum velocity is that velocity for which $a = 0$. In the first case, the balloon reaches a terminal velocity of 3.0 m/sec. In the second case, it reaches a velocity of 6 meters/sec.

c) It would not affect the velocity at all. The resistive force of the air depends on the velocity only, not on the mass. Thus the resistive force will equal the applied force at some particular velocity, independent of the mass. We can demonstrate this analytically as follows:

$$a = \frac{F_{\text{applied}} - F_{\text{resistance}}}{m}$$

$$m\,a = F_{\text{applied}} - F_{\text{resistance}} = F_{\text{applied}} - 0.2v^2$$

To satisfy the maximum velocity condition we must set $a = 0$. Thus

$$0 = F_{\text{applied}} - 0.2\,v_{\text{max}}^2$$

$$v_{\text{max}}^2 = 5\,F_{\text{applied}}$$

$$v_{\text{max}} = \sqrt{5\,F_{\text{applied}}} \qquad \text{(independent of m)}$$

Where v is in m/sec and F is in newtons.

d) If the balloon were larger, it would present a greater perimeter to the air, and the resistance force would be larger at any given velocity. (This means that the coefficient of v^2 would be larger.) The result would be that the resistive force would become equal to the applied force at a lower velocity than before. Therefore v_{max} would be decreased.

EXERCISE 28: Aristotle is said to have taught that a constant force was required to produce a constant velocity and from this he concluded that, in the absence of force, bodies would come to rest.

(a) Name several situations where a constant force produces a constant velocity.

(b) How do you explain each of the situations in (a) in the light of Newton's law of motion?

a) A horse pulls a wagon with a constant force at constant velocity.

A car attains a constant speed for any given gas pedal position (constant force).

An airplane flies at a constant air speed when its engine speed and propeller pitch remain constant.

The balloon of problem 21 would move with a constant velocity if a constant force were applied to it.

b) In each example of this kind, careful inspection shows there to be another force, in addition to the applied force, which opposes the motion.

For the horse and wagon, there are frictional forces of wheels on bearings and wheels on pavement.

For the car there are similar frictional forces, (and air resistance at high speeds).

For the airplane there is the force of air resistance.

For the balloon, the air resistive force equalizes the applied force.

EXERCISE 29: How would you define a unit of mass if people had placed a standard spring at Sevres instead of a standard mass?

If a spring were the standard unit of force, the unit of mass would have to be defined by means of an acceleration experiment. That is, the unit of mass would be the mass of an object which would be accelerated at a certain specified rate by application of the standard force (and that force only). Actually, this is a rather unrealistic answer to the problem, because the condition that no other force act on the body is a very difficult one to satisfy. A practical experimentalist would be more likely to establish his unit of mass through a gravitational force experiment, recognizing that the result was sensitive to his position relative to other massive objects. That is, his position on the earth would have to be standardized. For great accuracy, so also would be his position relative to the sun + moon. He would also have to recognize that an appreciable change in earth configuration would destroy his standard. This still would be a more practical standard that determined by an acceleration experiment.

A still more sophisticated procedure would be to define the mass standard in terms of a frequency of oscillation of that standard when set into motion hanging from the standard spring, in a vacuum. The frequency of an oscillating mass varies inversely as the square root of the oscillating mass. This method would be by form the most practical. Furthermore it is a direct measure of inertial mass. Unfortunately it is not within a student's comprehension at this stage of his studies.

Clarification of Figure 20 and related text.

The implication of the text concerning the motion of the puck as seen in a rotating room may not be clear from the graph of Figure 20. It may be that there will be no question asked about this material by any of your students. To avoid undo emphasis it would be better to say nothing, unless they ask. These few words may help you to clarify the difficulties of relative motion in case a question should arise.

Suppose we trace the path of the puck on a piece of paper that is pinned at just one point to the rotating table. The paper is held so that it moves with a circular motion, but it does not change its direction of orientation in space. If the point of pinning the paper is the release point of the puck, the path will be the cycloid of Fig. 20, --the release point being one of the cusps.

If, on the other hand, the paper is fastened to table at two points so that it turns about with the table as well as following its circular displacement in space, the path of the puck traced on the paper will be a spiral-like curve called an involute. It is the involute of the circular ground path of the release point. If one would hold a spool of thread firmly on end on a piece of paper and unwind the thread, keeping it taut, a point on the thread would trace out on the paper the involute of the circle that is the cross-section of the spool.

The accompanying figures indicate the orientation of the paper by means of x and y coordinate axes. The dashed parts of the figures are supposed to represent the situation shortly after the puck has been released. The figures are like double exposure photographs taken from the stationary reference frame, the ground.

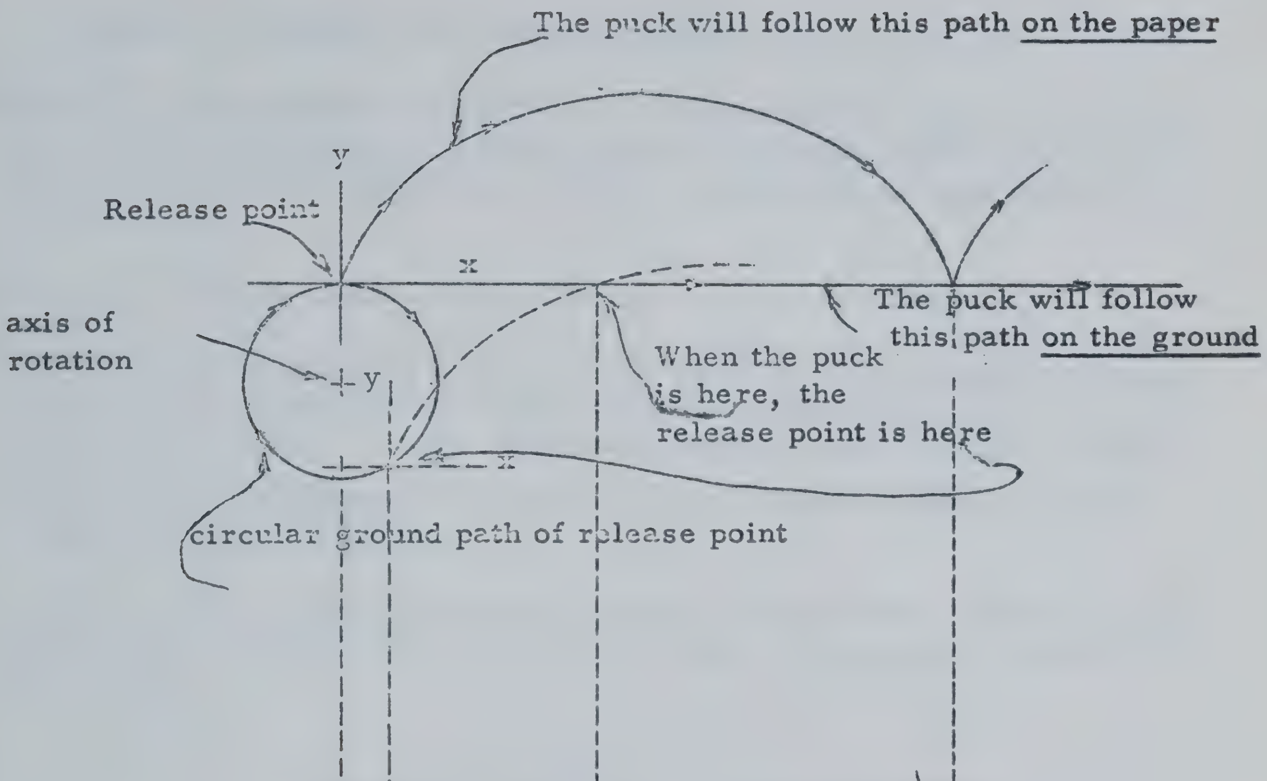

The puck will follow this path on the paper
y
Release point
x
axis of rotation
y
x
The puck will follow this path on the ground
When the puck is here, the release point is here
circular ground path of release point

Figure showing how cycloid is generated.

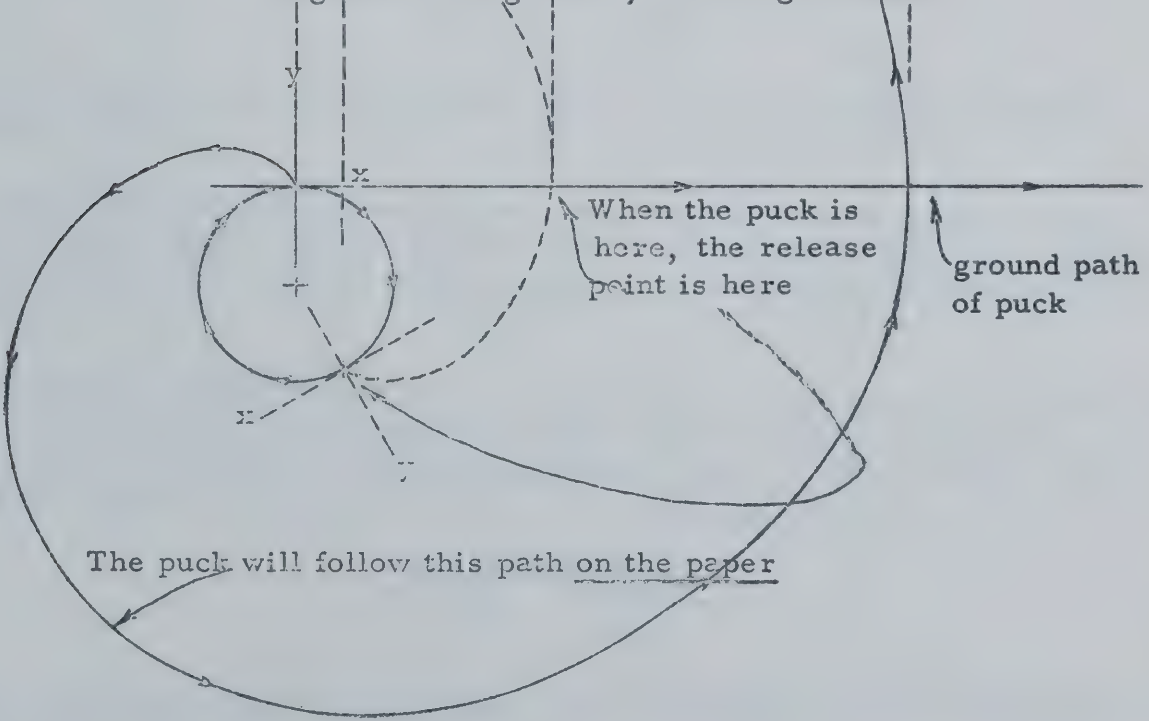

y
x
When the puck is here, the release point is here
ground path of puck
x
y
The puck will follow this path on the paper

Figure showing how involute of the circle is generated.

PSSC Teacher's Guide Chapter 2 Vol III

Chapter 2 - Motion at the Earth's Surface

PURPOSE OF THE ENTIRE CHAPTER: To formulate the equations of motion for a point mass moving in a plane, for the particular cases of projectile motion in a gravitational field, uniform circular motion, and simple harmonic motion.

CONTENT OF THE ENTIRE CHAPTER:

a) Any body at or near the surface of the earth is pulled toward the center of the earth by a force equal to its weight and directly proportional to its gravitational mass; $\vec{F} = m_g\vec{g}$. The gravitational field, $\vec{g}$, varies slightly in magnitude at different places on the earth's surface, but has a value of approximately 9.8 nt/kg.

b) The net force acting on a body in free fall is the vector sum of its weight and the frictional force $\vec{F}_a$, acting on the body. The acceleration of the body is given by

$$\vec{a} = F/m = \vec{g} + \frac{\vec{F}_a}{m},$$

if m = inertial mass = gravitational mass. In the absence of frictional forces, $\vec{a} = \vec{g}$.

c) The gravitational force acting on a body is independent of the state of motion of the body. Hence, motion of projectiles can be found by considering separately the vertical motion (under an acceleration $\vec{g}$), and the horizontal motion.

d) All bodies which move in a curved path must be influenced by a force which has a component perpendicular to the direction of motion (centripetal force). For a body, of mass m, moving at constant speed, v, in a circular path of radius R (uniform circular motion), the centripetal force is

$$F_c = \frac{mv^2}{R} = \frac{4\pi^2 mR}{T^2},$$

where T is the period of revolution. The centripetal force is directed toward the center of the circle.

e) A body moving subject to a force, $\vec{F} = -k\vec{x}$, which is directly proportional to its displacement, $\vec{x}$, from a point, and directed opposite to the displacement, executes simple harmonic motion with a period $T = 2\pi\sqrt{\frac{m}{k}}$.

A simple pendulum of length L, displaced from the vertical through a small angle, executes simple harmonic motion with a period $T = 2\pi\sqrt{\frac{L}{g}}$.

f) While accurate kinematic descriptions of motion can be devised for arbitrary frames of reference, Newton's Law of motion is not valid in an arbitrary reference frame. In particular, one must introduce a fictitious (centrifugal) force in order to make Newton's Law apply in a rotating frame of reference. A frame in which Newton's Law is valid, without invoking extraneous forces, is called an inertial frame. The surface of the earth is only approximately an inertial frame, and it is possible to show (e.g., Foucault pendulum) that the earth is in fact rotating with respect to an inertial frame.

EMPHASIS: This chapter presents extremely important material, and should be fully developed through extensive class discussion and a large number of numerical exercises. Much of this material is difficult, both mathematically and conceptually, and the pace cannot be geared to that of the slowest students, without spending an unwarranted amount of time on this chapter. If you are pressed for time, the material on simple harmonic motion and the discussion of frames of reference can be treated somewhat superficially. Because of conceptual difficulties, these particular sections may not be fully appreciated by other than your best students, even with extensive discussion.

DEVELOPMENT: It is important that you review the earlier material on kinematics prior to or during the treatment of this chapter. Many of the problems will be concerned with the motion of bodies under a constant acceleration along the path. Progress will be considerably aided if the relevant equations are familiar to most students:

$$d = d_o + v_o t + \frac{1}{2} at^2$$

$$v = v_o + at$$

$$v = \sqrt{2a\,(d - d_o) + v_o^2}$$

where d is the position (initial position d_o), v the velocity along the path (initial velocity v_o), a the acceleration along the path, and t the time.

COMMENT: This is not an easy chapter. The present material is very much at the same level of sophistication as that usually presented in a college-level course. Here we begin to look deeply into motion, and try to use our apparently "trivial" laws to make quantitative predictions for a number of definitely non-trivial cases. Here is the real "meat" of mechanics. Treat it with enthusiasm, and you may awaken the latent scientific potentialities of your students; treat it as a sequence of dull and routine algebraic exercises, and your students will surely miss an opportunity to discover the true nature of physics as a science.

PSSC Teacher's Guide Chapter 2 Vol III

Section 1 - Weight and the Gravitational Field of the Earth

PURPOSE: To redefine weight as the force with which gravity acts on a body. (c.f. Vol 1 Chapter 7). To define the earth's gravitational field, $\vec{g}$, as the force per unit mass exerted by the earth on any object.

CONTENT: a) The gravitational force acting on a body is directly proportional to its gravitational mass, m_g, $\vec{F} = m_g \vec{g}$. b) The factor $\vec{g}$ = varies slightly in magnitude at different locations on the earth's surface.

EMPHASIS: This section reviews previous material with a slightly different emphasis. A brief treatment should suffice.

COMMENT: Defer discussion of why $\vec{g}$ varies from one place to another until the next chapter, which deals with the law of universal gravitation.

Probably no single subject in physics has been confused by more students than the distinction between weight and mass. It is important that students clearly understand the distinction between these quite different properties (see Vol. 1, Ch. 7) and (Vol. 3, Ch 1)

Since the weight of an object is the force, F, exerted by gravity on its mass, m, it is most convenient, when thinking in these terms, to express $\vec{g}$ as 9.8 newtons/kg. (32 pounds/slug in the British system). On the other hand, when dealing with the acceleration in a free fall problem, $\vec{g}$ is most simply considered in the equivalent units 9.8 m/sec^2 (or 32 ft/sec^2 in the British system).

In this section the concept of a force field is introduced. While a brief definition of what is meant by a field is certainly in order, do not dwell on the subject here. This subject will be further explored in the next chapter and in Volume 4.

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Section 2 - Free Fall

PURPOSE: To show that a body falling vertically in a gravitational field has an acceleration $\vec{a} = \vec{g}$ in the absence of frictional forces.

CONTENT: a) The net force acting on a body in free fall is the vector sum of the gravitational force of attraction, $m_g\vec{g}$ and the frictional force due to air resistance, $\vec{F}_a$; $\vec{F} = \vec{F}_a + m_g\vec{g}$. b) If we equate the inertial mass, m_i, with the gravitational mass, m_g, the acceleration of a freely falling body is given by

$$\vec{a} = \vec{g} + \vec{F}_a/m.$$

c) In the absence of frictional forces, $\vec{F}_a = o$; and $\vec{a} = \vec{g}$.

Under these conditions, a body released from rest achieves a vertical speed $v = gt$, in a time interval t, and falls a distance $d = \frac{1}{2} gt^2$.

d) If air resistance is present, a falling body ultimately achieves a constant maximum speed known as its terminal velocity.

EMPHASIS: Important material; treat fully with ample class discussion and exercises. It is best if students fully understand the relatively simple material of this section before attempting a study of projectile motion. Much of the present section is similar to material covered in chapters 5 and 6, Volume 1.

DEVELOPMENT: This is a good point to review the material of Volume 1 on kinematics. Students will have less trouble working exercises if they are familiar with the equations of motion for a body moving with constant acceleration along a path. The vector character of the motion can be treated separately in the next section, if desired.

While the phenomenon of air resistance to the motion of bodies through it is familiar (in walking against a strong wind, or putting one's hand out of the window in a rapidly moving car) is is very difficult to make quantitative predictions about it. The fact that bodies achieve a terminal velocity if allowed to fall far enough is also reasonably familiar, in falling snow-flakes, raindrops, and we hope in parachutes, or even in a toy balloon. But for the

systems that will be considered in detail here, air resistance is a small effect which we shall neglect. For example, although one generally imagines that air resistance affects considerably the motion of a ping-pong ball, you cannot easily detect the difference in time of fall over, say, two meters between it and a steel ball. So don't try this as a class demonstration! For systems for which it is important, the problem is beyond the scope of this course.

DEMONSTRATION: The film on free fall opens with a convincing demonstration of the constancy of the acceleration of all objects, light or heavy, when falling under the force of gravity alone. Start out by dropping, simultaneously, a book and a sheet of paper (smaller in area than the book), releasing them side by side. For the piece of paper, F_a is relatively larger than for the book. The paper quickly reaches its terminal velocity and is left behind by the accelerating book. To show that this is, indeed, the effect of air resistance, crumple the paper into a tight ball and again drop the paper and book together. They will now reach the floor very nearly together. To eliminate F_a entirely in the falling sheet of paper, take another identical sheet and now place it on top of the book so that it does not protrude beyond the edges of the book. Now drop the book, and the paper will remain lying closely on top of the book during the fall. This demonstrates that the acceleration, in free fall, of the piece of paper is at least as great as that of the book falling through the air. (Actually, of course, it is greater, since the book is retarded, somewhat, by the force of air resistance on it.)

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Sections 3, 4 - Projectile Motion

PURPOSE: To derive the equations of motion for a body moving with an arbitrary initial velocity in a uniform gravitational field.

CONTENT: (a) The (vertical) acceleration of a freely falling body due to gravity is experimentally found to be independent of the state of motion of the body, thus, the vertical motion of the body is independent of its horizontal motion.

(b) The horizontal motion of a freely falling body is independent of its vertical motion.

(c) We may solve for the motion of bodies projected with an arbitrary initial velocity $\vec{v}_o$, by considering separately the vertical (y) and horizontal (x) components of the motion. Denoting the vertical component of $\vec{v}_o$ as v_{oy}, and the horizontal compondent by v_{ox}, and taking the initial position as $y = o$, $x = o$, (y is positive for upward displacements)

$$y = v_{oy}t - \frac{1}{2} g t^2,$$

$$x = v_{ox}t,$$

in the absence of air resistance. From the second equation, $t = \frac{x}{v_{ox}}$ Substituting in the first equation,

$$y = \frac{V_{oy} X}{V_{ox}} - \frac{1}{2} \frac{g X^2}{(V_{ox})^2} ,$$

and the path of the projectile is a parabola.

APPROPRIATE LABORATORY: Experiment 490

EMPHASIS: Important material, treat fully. It is best to consider first cases with only an initial horizontal velcoity ($v_{oy} = 0$), before treating the general case. Stress the vector nature of forces, velocities and displacements.

COMMENT: Students sometimes have trouble believing the results of this section. They often feel intuitively that a bullet fired horizontally will reach the ground much later than one simultaneously dropped from the end of the gun barrel. There is nothing quite so convincing to drive home the validity of the fact (and the failure of the student's intuition!) as a demonstration. If you are equipped with one of the standard demonstrations to illustrate this fact, now is a good time to use it. Also the PSSC movie on free fall illustrates this phenomenon. You can repeat the movies simple demonstration "spontaneously"--try placing two coins next to one another at the edge of a table; push one into the other with a flick of the finger. They will probably go off the edge with very different initial velocities, but should hit the floor simultaneously. (Caution-be sure you practice this "spontaneous" demonstration in advance of class!)

It is important that the students appreciate the quantitative aspects of the equations of projectile motion. Exercises 10, 13 and 14 are well suited, and you will probably wish to supplement these with others of your own choosing. Any standard college textbook is a good source of additional exercises.

Students are usually interested in the fact that, in the absence of air resistance, a projectile will travel a maximum horizontal distance if initially projected at an angle of 45° above the horizontal plane, assuming that the magnitude of V_o ($V_o = \sqrt{V_{ox}^2 + V_{oy}^2}$) is fixed. The proof of this is not too difficult. With a fixed initial speed v_o, the initial horizontal and vertical velocities are, for angle of projection θ, $v_{ox} = v_o \cos \theta$ and $v_{oy} = v_o \text{Sin } \theta$. Suppose the projectile takes a total time T to hit the ground, the range is X. The horizontal motion is given by

$$X = v_o \cos \theta \cdot T$$

and the vertical motion (since the total vertical distance travelled is zero) by

$$0 = v_o \sin \theta \cdot T - \frac{1}{2} gT^2.$$

The T = 0 root of this is not of interest (it is just the statement that $y = o$ at $t = o$ as well as at $t = T$, which is not news!). So

$$T = \frac{2v_o \sin \theta}{g}$$

and if we substitute for T in the equation for X,

$$X = v_o \cos \theta \cdot \frac{2\, v_o \sin \theta}{g}$$

$$= \frac{v_o^2 \sin 2\, \theta}{g}$$

The maximum value that this can have is $X = \frac{v_o^2}{g}$ which occurs when $\sin 2\, \theta = 1$. This happens when $2\theta = 90^\circ$, i.e. $\theta = 45^\circ$.

The equations of motion become more complicated if air resistance is taken into account. For many bodies in which we are interested, (e.g. ping-pong balls or bullets), the force of air resistance varies approximately as the square of the velocity,

$$F = -\, kv^2$$

directed backwards along the vector v. The horizontal and vertical components of the total force on the body are thus

$$F_x = -\, kv^2 \cdot \frac{V_x}{v} = -\, kvv_x$$

$$F_y = mg - kv^2 \frac{v_y}{v} = mg - kv\, v_y \, .$$

Since the F_x depends on v, and thus on v_y as well as v_x, the motion is no longer separable into horizontal and vertical motion. The solutions are very difficult. That is the reason why mathematicians work so hard to make up ballistics tables.

PSSC Teacher's Guide Chapter 2 Vol III

Section 5 - Deflecting Forces and Circular Motion

PURPOSE: To show that a body which moves along a curved path must be subjected to a force which has a component perpendicular to the direction of motion, and to derive the equations for uniform motion in a circular path.

CONTENT: a) A body moves along a curved path only if acted on by an external force, perpendicular to the direction of motion. b) The speed of motion of a body is unchanged by a force perpendicular to the direction of motion. c) A body of mass m, moving with constant speed, v, in a circular path of radius R, with a period $T = 2\pi R/V$, must be subjected to a perpendicular force always directed toward the center of the circle (centripetal force) of magnitude

$$F_c = \frac{mv^2}{R} = \frac{4\pi^2 mR}{T^2} .$$

If $\vec{R}$ is the radius vector from the center of the circle to the position of the body, the centripetal force, in vector form, is

$$\vec{F}_c = - \frac{4\pi^2 m \vec{R}}{T^2}$$

LABORATORY: Experiment 500 should be performed before the formula is derived (See TG Lab. sections on Exp. 500)

EMPHASIS: This is important material and should be treated fully. However, most of the present section reviews material treated earlier in Volume 1, Chapter 6, so it should not require too much class time for discussion.

COMMENT: In the previous discussion of circular motion, in Volume 1, emphasis was placed on the perpendicular acceleration of a body moving in a curved path. In the present section, while the algebraic treatment closely parallels that used earlier, we have now shifted our inquiry to the determination of the cause of the observed motion, that is, to the force which must be acting on the body to effect the observed motion.

To emphasize this viewpoint, ask students to isolate the centripetal forces acting in commonplace situations. What is the force which keeps the moon rotating about the earth? (the gravitational attraction of the earth for the moon). What are the forces acting on a car moving around a curve? (the force of the road on the tires.) What are the forces acting on the driver? (the force of friction exerted by the seat and floor of the car, and if these are insufficient, the force exerted by the steering wheel on the driver's hands).

CAUTION: Be very careful to use the term centripetal, not "centrifugal", to describe the forces producing circular motion. The latter term is used to describe "fictitious" forces, as discussed later in this chapter, while the centripetal force is a real force which must be present if an object is moving along a curved path.

Students should realize that a centripetal force is acting when any curvilinear motion is involved. A constant centripetal force, acting alone, causes uniform circular motion. If the body moves, even in a circular path, with a varying speed, the centripetal force must change in magnitude since, in the case of varying speed, mv^2/R is not constant.

Section 6 - Earth Satellites

PURPOSE: To apply the results of the previous section to a case of popular interest.

EMPHASIS: The specific problem considered in this section should be treated as an illustrative exercise, not as an exercise for an extensive discourse on earth satellites, missiles, or rockets. Treat briefly. The factors which determine the variation of the acceleration of gravity, g, with altitudes will be discussed in Chapter 3 in a general treatment of gravitation.

Section 7 - The Moon's Motion

PURPOSE: To apply the previous results to the motion of the moon.

EMPHASIS: Treat briefly, as another illustrative example. The reason for the difference in centripetal acceleration between this case and that treated in section 6 is considered in detail in Chapter 3.

Section 8 - Simple Harmonic Motion

PURPOSE: To derive the equations of motion for a body under the action of a linear restoring force.

CONTENT: a) An object subjected to a force proportional to its displacement, and opposite to the displacement, ($\vec{F} = k\vec{x}$), executes oscillatory motion of a particularly simple kind, called simple harmonic motion. b) Uniform circular motion can be resolved into two perpendicular simple harmonic c) The period of oscillation of a body of mass m subjected to a restoring force $\vec{F} = - k\vec{x}$, is $T = 2\pi\sqrt{m/k}$ d) A simple pendulum of length L, executes simple harmonic motion with a period $T = 2\pi\sqrt{L/g}$.

EMPHASIS: This is important material, but a full treatment may be difficult for many students. Unless you have ample time available, you may wish to treat this section somewhat superficially. A full mathematical development is given below, but is beyond the scope of the present course, since it involves differential equations.

DEMONSTRATION: Many students may have trouble visualizing the resolution of circular motion into simple harmonic motions.

Simple harmonic motion can be demonstrated by observing the shadow of an object moving in a circle at constant speed. The shadow will move with simple harmonic motion back and forth on a screen. A comparison of the motion of a pendulum with the motion of the shadow of the object moving in a circle shows that a pendulum with small amplitudes also performs simple harmonic motion. The apparatus in Figure (a) can be used to demonstrate this.

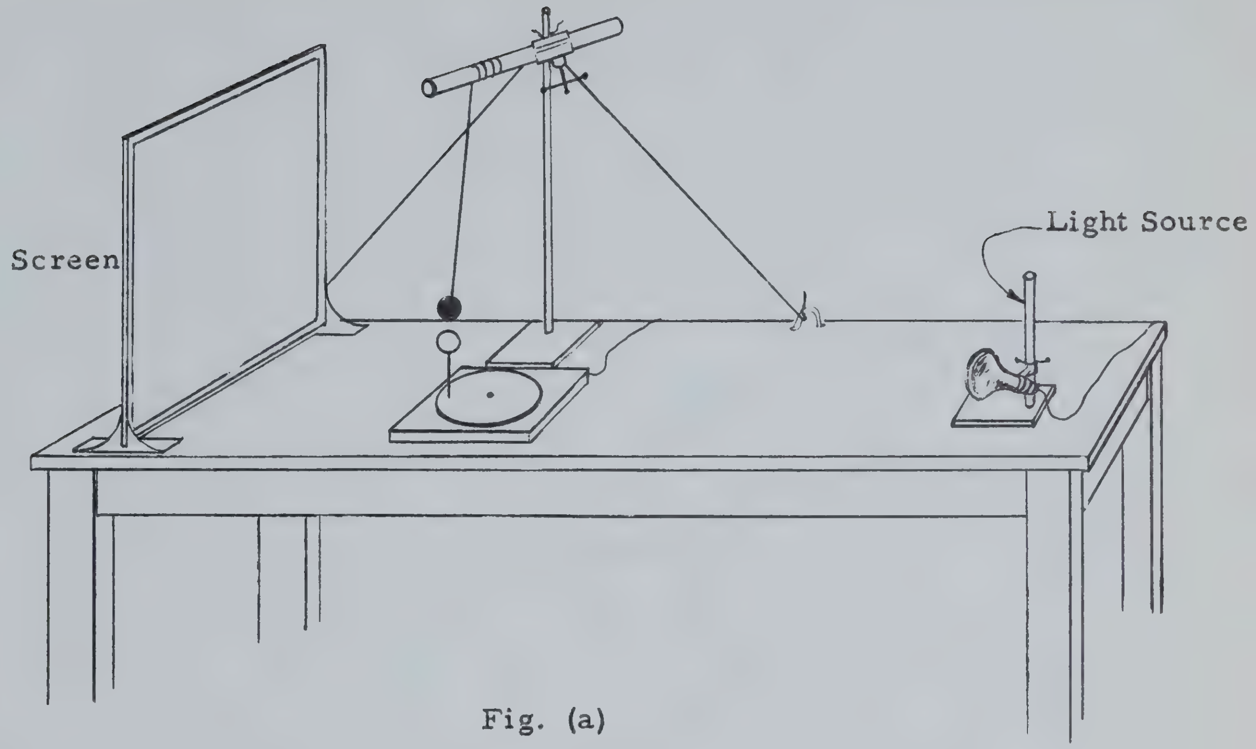


Fig. (a)

A ping pong ball is glued to one end of a thin stick while the other end of the stick is taped to a magnet which adheres to a phonograph turntable (Figure b). From a tall, sturdy support, suspend a simple pendulum with its length adjusted so that it will swing with the same frequency as the rotation of the turntable. The length of the pendulum should be about 80 cm for 33 1/3 r.p.m.'s. This turntable frequency is best since a higher frequency necessitates a short pendulum which swings through too large an angle. Hang the bob above the turntable so that the bob is slightly higher than the ping pong ball (as shown in Figure b).

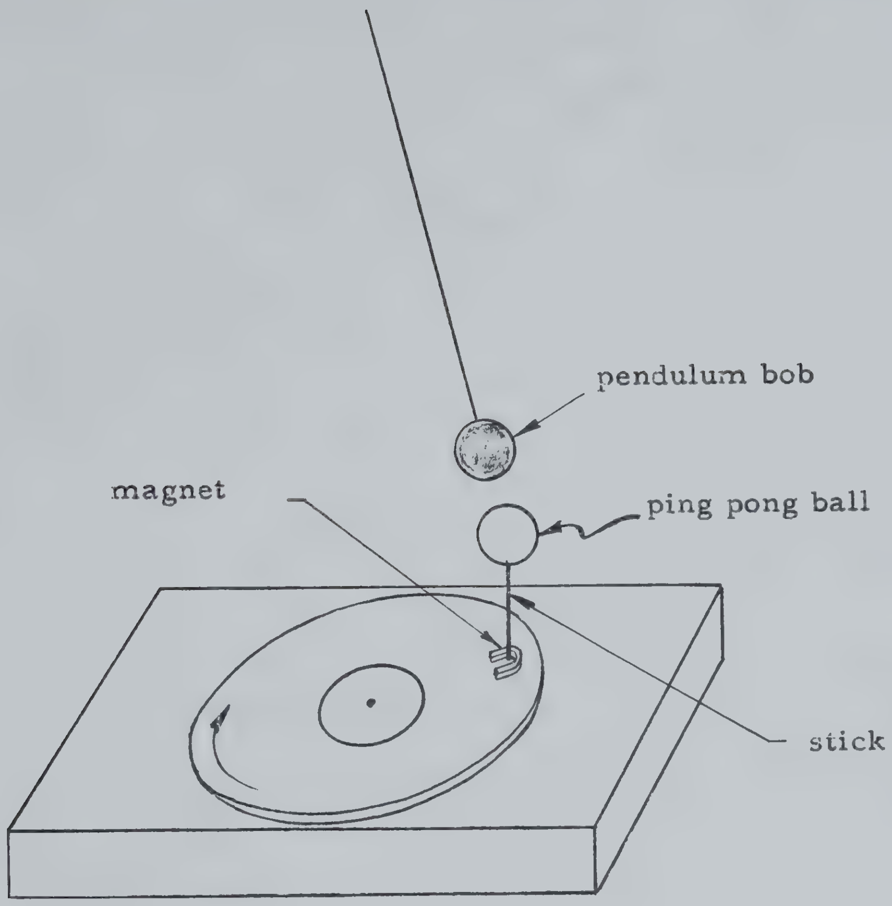


Fig. (b)

A dowel or rod clamped to a ringstand provides the pivot for the pendulum. This pivot can be rotated (Figure c) to adjust the length of the pendulum unitl the period is the same as that of the turntable.

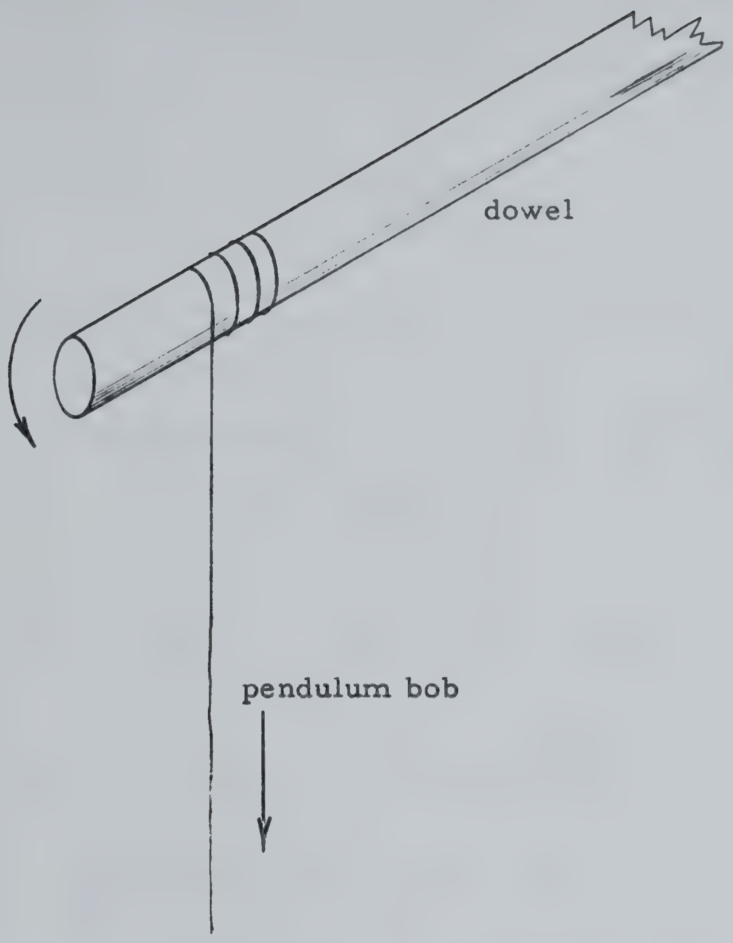


Fig. (c)

The light source should be as far from the pendulum and turntable as possible to reduce the effects of parallax due to the changing distance between the light and the rotating ball.

Initially, the amplitude of the pendulum should be slightly greater than the radius of the turntable so the amplitudes will be the same after the bob and ball have been synchronized.

Release the bob at an instant so that its moving shadow lags a little behind the shadow of the ball. The motions may then be synchronized by holding a finger against the rim of the turntable, slowing it down until the two shadows coincide.

MATHEMATICAL DEVELOPMENT: (For those teachers whose calculus is fresh and for whom a review of the differential equation treatment will be helpful). We can represent the conditions imposed by the restoring force by noting that

$$\vec{F} = - k\vec{x} = m\vec{a} = m \frac{d^2x}{dt^2}$$

$$\text{Then } m \frac{d^2x}{dt^2} + kx = 0. \qquad (1)$$

To solve this equation, we use for a trial solution

$$x = A \cos \left(\frac{2\pi t}{T} + \phi\right).$$

$$\text{Then } \frac{dx}{dt} = - \frac{2\pi A}{T} \sin \left(\frac{2\pi t}{T} + \phi\right), \text{ and}$$

$$\frac{d^2x}{dt^2} = - \frac{4\pi^2 A}{T^2} \cos \left(\frac{2\pi t}{T} + \phi\right)$$

Substituting into equation (1),

$$- m \frac{4\pi^2 Am}{T^2} \cos \left(\frac{2\pi t}{T} + \phi\right) = - KA \cos \left(\frac{2\pi t}{T} + \phi\right)$$

$$\text{Thus } T^2 = 4\pi^2 m/k$$

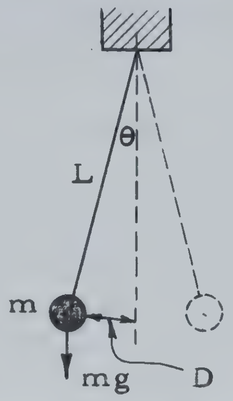
$$T = 2\pi\sqrt{m/k}.$$

The amplitude of motion A, and phase factor ϕ can only be determined if the initial conditions are specified. For example, if at time $t = 0$, x is at its maximum displacement of 5 cm, we could set $\phi = 0$ ($\cos(0 + \phi)$ is a maximum for $\phi = 0$), and $A = 5$ cm. Then

$$x = 5 \cos\left(\sqrt{\frac{k}{m}}\, t\right) \text{ cm.}$$

The mathematical treatment of the motion of a simple pendulum is somewhat more complicated, but is given below for completeness.

A massless string of length L supports a point mass, m. It is deflected from the vertical by a small angle θ, as shown. The displacement of m from its equilibrium position is $D = L\theta$.



Due to the weight of the bob, there is a force $m\vec{g}$ acting downward on the mass. We resolve this force into components $mg \cos\theta$ in the direction of the string and $mg \sin\theta$ perpendicular to the string. The force along the string is merely a constraining force and does not enter the problem. The perpendicular force is the restoring force.

So we can write $F = -mg\sin\theta = ma = m\dfrac{d^2D}{dt^2}$,

$$\text{or } mL\frac{d^2\theta}{dt^2} + mg\sin\theta = 0.$$

This equation does not describe a simple harmonic motion, if θ is a large angle. However, if θ is very small, we may write $\sin\theta \approx \theta$, and the equation becomes

$$mL\frac{d^2\theta}{dt^2} + mg\theta = 0.$$

Simplifying, $$L\frac{d^2\theta}{dt^2} + g\theta = 0 \qquad (2)$$

Equation (2) is identical with our previous equation (1), except for relabeling the variables, and has a similar solution. It should be apparent that a solution is $\theta = \theta_o \cos\left(\dfrac{2\pi t}{T} + \phi\right)$, with $T = 2\pi\sqrt{L/g}$.

Section 9 - Experimental Frames of Reference

PURPOSE: To point out that while a choice of reference system for a kinematic description of motion is arbitrary, the laws of motion are valid only in special ("inertial") frames of reference. Introduction to the next section.

EMPHASIS: Treat briefly, as part of a longer assignment which includes Section 10.

COMMENT: Students should not misinterpret the sentence in the final paragraph that "a frame of reference rigidly attached to the earth is a good one." As noted later in this chapter, we can easily do experiments to show that the earth is not an inertial frame. It is "good enough" for most motions which we commonly measure in a laboratory.

Section 10 - Fictitious Forces in Accelerated Frames

PURPOSE: To show that the laws of motion are not valid in accelerated frames of reference, and that motion observed from an accelerated frame can only be explained by introduction of fictitious forces.

EMPHASIS: Treat in sufficient detail that students realize the fictitious nature of the "centrifugal" force, but do not belabor the issue.

COMMENT: Some students may raise embarrassing questions about the fact that we frequently invoke "fictitious" forces to explain familiar phenomena, such as gravity and electric and magnetic forces, none of which are directly observable as pushes or pulls. We do not know enough about the causes of gravity or electricity to know whether these "action-at-a-distance" forces are truly real or "fictitious". Einstein, in his theory of general relativity (see supplement to Guide for next chapter), has formally accounted for the force of gravity in terms of the properties of space and time, but we do not yet have a so-called "unified field theory" to account similarly for electric and magnetic effects. Thus, at least at the present stage of development in physics, we treat these action-at-a-distance forces as real, applied forces. There are no transformations to other frames of reference which we can apply (as we can with the "centrifugal" force) which will make electric and magnetic forces vanish.

Section 11 - Newton's Law and the Rotation of the Earth

PURPOSE: To show that the earth, because of its spin, is not an ideal Newtonian frame of reference, and that this fact can be determined experimentally.

EMPHASIS: Treat briefly, but be certain that all students appreciate the conclusive results of Foucault's experiment.

COMMENT: Although for many experiments and calculations, we find it convenient and acceptable to consider the earth to be a Newtonian frame, we can demonstrate experimentally that this is not the case. The fixed stars, in their turn, although we take them to be a "good" reference frame, are not really fixed relative to one another, but are moving at enormous speeds. Imaginary lines between them may rotate in a true Newtonian frame. However, their distance from the earth is so vast, that on the time scale of a human life, they can be considered "fixed".

For Home, Desk and Lab.

It is essential, in covering this chapter, that students receive considerable drill in numerical and algebraic problems both in class discussion and as homework. The present set of exercises is well suited to this purpose, but you may wish to supplement these particularly with simpler problems. A number of possible supplements are suggested. In the table below, the primary category of the problem is indicated, and that of the suggested supplements.

Section	Easy	Medium	Hard	Class Discussion	Laboratory
1	1	2		1, 2	
2	3	4, 5, 6, 15, 16, 19	7, 8, 9	3, 5, 6, 7, 8, 9, 16, 19	
3, 4	11, 12	10, 13, 14, (18 Sup)		13, 14	Exp 490
5, 6, 7	17, 20, 24	18, 19, 23		17, 18, 20	Exp 500
8	25	4, 4a, 21, 22, 26	27, 28	21, 22, 25, 27	
9, 10, 11		4a, 29, 19		29, 19	

NOTES: Problem 11 is simpler if it is considered after Problem 10.
Problems 19, 20, 21, and 22 should be considered together.

EXERCISE 1: If the gravitational field at the surface of the moon is one-sixth of that at the surface of the earth, how much would a 70-kg man weigh on the moon? What would be his mass on the earth? On the moon?

This exercise is concerned with the difference between mass and weight. The weight of a 70kg. man (i.e. a man whose mass is 70kg.) is 70 g newtons, where g is the gravitational constant at that point. On the surface of the earth g is approximately 9.8 nt/kg, while on the surface of the moon it is only one sixth as large (i.e. g_{moon} = 1.6 nt/kg). The man's weight on the earth is thus 70 x 9.8 = 686 newtons. (His weight on the moon is 1/6 of this, i.e. 114 newtons.) The man's mass is always the same, 70 kg, on the earth,

on the moon, or anywhere in the universe. But his weight is proportional to the strength of the gravitational field at the place where he is.

You may start a profitable class discussion about the characteristics of everyday life for earth people living on the moon, using the above result. Assume that our man is adequately supplied with food and oxygen so that his bodily strength on the moon will be the same as on earth; the forces he can exert will also be the same. Those activities in which these bodily forces are used to overcome gravity will now be considerably altered: a man will be able to jump six times as high from the moon's surface as he can from the earth's. He can lift bodies whose masses are six times those he could lift on earth. This means that lifting equipment, ladders, etc; could all be much flimsier than they would have to be to do the same job with objects of the same mass on earth. On the other hand, processes in which gravity does not enter (for example, in the acceleration of a body in a horizontal direction by pushing on it) would not be affected. An interesting example of an act involving both types of process is throwing a baseball, The maximum velocity that a man can impart to the baseball (a process unaffected by gravity, and governed only by the masses of the baseball and the man's arm, and his muscles) will be the same as on earth. The distance it will travel after it leaves his hand before hitting the moon's surface does depend on g, however. Thus for a given throwing angle, the distance the ball can be thrown will be now six times what it is on earth. Imagine a lunar baseball game!

A "pound" of butter, that is, a mass which weighed a pound on the earth's surface, will now weigh only 1/6th pound, but will cover just as much bread and provide just as much nurishment as before.

A fire of a particular size will require the same mass of fuel as on earth (and the same mass of oxygen), and the same mass of water will be needed to put it out.

Students will undoubtedly be able to think of other examples. Be sure that it is clearly understood that all properties relating only to mass are unchanged--such as inertia, chemical reactions, heat content, collisions, etc.

EXERCISE 2: If the ratio of the gravitational masses of objects to their inertial masses were different for different objects, would they all accelerate at the same rate in the gravitational field of the earth? Explain.

The purpose of this exercise is to emphasize the fact that the equality of inertial mass and gravitational mass is an experimental result, and not at all a necessary consequence of other assumptions we have made in developing dynamics. Up to the present time their equality has been verified experimentally to within one part in 10^8. To see the consequences of different gravitational and inertial masses, assume that gravitational and inertial mass are not equal. Suppose, for example, that

$$\frac{m_i}{m_g} = 1 + c/m_g$$

where c is some constant. A body of mass m_i will fall freely under gravity with an acceleration a given, according to Newton's law, by

$$m_i a = m_g g$$

Substituting for m_i,

$$m_g (1 + c/m_g) a = m_g g,$$

$$a = \frac{g}{1 + c/m_g} = g\left(\frac{m_g}{m_g + c}\right)$$

The acceleration of a freely-falling body would then not be a constant, but would depend on m_g (or m_i, the two are assumed related by our initial equation).

We chose this particular relation between m_i and m_g as an illustration, because it becomes approximately the same as our expected relation, $m_i = m_g$, for very large masses. We need this to be able to explain the motion of the planets in the solar system. As a hypothesis, we would test it by performing free-fall experiments for light masses, since this relation departs from $m_i = m_g$ when the masses are of order c, where c is the constant

mass introduced in our hypothetical relationship between m_i and m_g. The lightest body on which a free-fall experiment has actually been made is the neutron. (Its mass is about that of a hydrogen atom). The law $m_i = m_g$ has been verified in this case to within ten percent, so we have very good evidence that it holds for all bodies. Thus, the basic experiment which demonstrates the identity of gravitational and inertial masses is the measurement of the acceleration of a body under free fall. As noted in the Guide for the next chapter, this identity means that it is inherently impossible to distinguish between the effects of an acceleration and those of a gravitational field.

EXERCISE 3: What happens if you throw a ping-pong ball down faster than its terminal velocity? Try it.

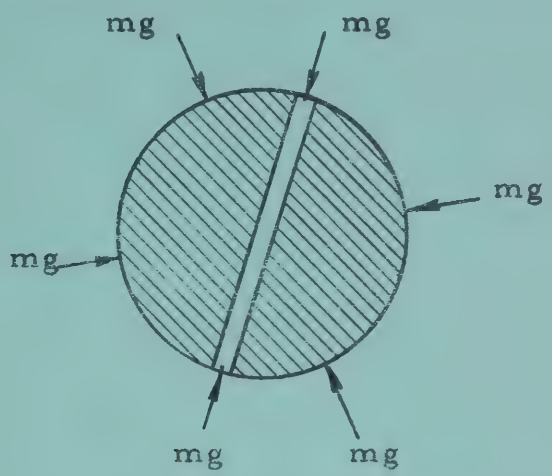
A home exercise for cliff-dwellers. A body thrown downward with an initial velocity greater than its terminal velocity will be decelerated until it reaches its terminal velocity. However, the terminal velocity of a ping-pong ball is so high (about 8 m/sec) that students would never observe the effect if the distance traveled was much under 40 feet. Students may observe an entirely unrelated effect due to spin of the ball, which will tend to turn it onto a curved trajectory in the manner of a boomerang. The present effect can be easily observed with an inflated balloon, and intrepid experimenters would be well advised to turn in their ping-pong balls for balloons.

EXERCISE 4: What would happen if a stone were dropped into a hole clear through the center of the earth from the North Pole to the South Pole?

This exercise can be discussed early in the chapter, in connection with free fall, but is more appropriate to section 8, as an example of oscillatory motion.

Students may have to be reminded, at the outset, that the earth is round, and that "down" is not the same direction for a man in this country as for a man in Australia. "Down" is defined as the direction in which a plumb bob hangs, or in which a freely-falling body moves, and is essentially a vector pointing to the center of the earth (but not exactly-see later discussion.)

We may solve this problem in a straight forward manner, by making use only of the fact that the earth's gravitational field has spherical symmetry with respect to the center of the earth.



The force on the body at the north pole (mg "down") is then equal and opposite to that at the south pole (still mg "down"), and must be zero at the center of the earth. At any two points, at equal depths from the North pole and South pole, the restoring forces will be equal in magnitude, and will point to the center of the earth. Thus, the body moves under a restoring force, relative to the center of the earth, and the motion is oscillatory. This is probably as much of an "answer" as can be expected of students. However, it is relatively simple (with knowledge of calculus, Newton's law of universal gravitation, and a few simplifying assumptions) to solve the present problem exactly. This solution is given below.

(4a) The information given so far about gravitational forces does not tell us quantitatively how the magnitude of the force on the stone in the hole will vary with depth. It requires some calculus to show, from Newton's law of gravitation, that when the stone is a distance x from the center, only the matter inside the sphere of radius x actually contributes to the gravitational force on the stone. The force acts as though this matter were concentrated at the center of the earth. The amount of matter (assuming constant mass density in the earth) is proportional to the volume of this sphere, i.e. to x^3, and the force due to this amount of matter, concentrated at the center, will thus be proportional to

$$x^3 \cdot \frac{1}{x^2} = x$$

where the $1/x^2$ comes from Newton's law of universal gravitation discussed in the next chapter. The force is proportional to the distance from the center, so that the oscillatory motion is actually simple harmonic. Since the force is

known to be mg at the earth's surface, we can write the equation of motion as $ma = -mg(x/R)$ where R is the radius of the earth. The period of the oscillation is thus

$$T = 2\pi\sqrt{\frac{R}{g}} .$$

R is about 4000 miles, i.e. 2×10^7 feet, and g is 32 ft/sec^2, so T is about 5×10^3 sec, i.e. 1 1/2 hours. The velocity of the stone at the center of the earth is $\sqrt{gR} = 2.5 \times 10^4$ ft/sec. = 5 miles per second!

(4b) The question may arise: why choose the North Pole-South Pole axis? The whole problem is hardly a practical one, but if it were, there would be a very good reason to use this axis, since it is the axis of rotation of the earth. If the hole were in any other direction (but still running through the earth's center) the stone would not fall straight down, but would veer to one side. This is because the frame of reference fixed with respect to the hole (this is the one in which we calculate) is now rotating with respect to the frame fixed with respect to the earth's axis. In the rotating frame there are "fictitious" forces (the centrifugal force is one) which are no longer parallel to the hole, and which would move the stone toward the side of the hole. If we adopt a frame of reference fixed with respect to the inertial frame of the fixed stars, where no "fictitious" forces enter, we would explain the same result by saying that the earth had rotated, while the stone was falling, so that the stone which keeps the traverse velocity it had when at the earth's surface, catches up with the side of the hole and hits it. (It is a rather complicated way to look at the problem!)

EXERCISE 5: A stone drops from the edge of a roof.

(a) How long does it take to fall 4.9 meters?
(b) How fast is it going at the end of that fall?
(c) How much longer does it take to fall an additional 3 meters?
(d) How fast is it going at the end of this 7.9-meter fall?
(e) Through what distance does it move during the third second?

NOTE: Problems 5, 6, 7 are exercises concerned with free fall. It is helpful in solving problems of this kind to standardize the procedure somewhat, so that there is never any uncertainty about which of the kinematic equations are most appropriate in the particular problem being attacked.

If we define our measured quantities so that at zero time, the distance coordinate is zero, and the velocity is v_o; we have the following equations for d and v at later times, t: -

$$v = v_o + at \quad (\text{relates } \underline{v}, \underline{t}) \qquad (1)$$

$$d = v_o t + 1/2\, at^2 \quad (\text{relates } \underline{d}, \underline{t}) \qquad (2)$$

and from these we can deduce

$$v^2 = v_o^{\,2} + 2\, ad \; (\text{relates } \underline{d}, \underline{v}). \qquad (3)$$

On the information given in each problem, we decide which of these three equations is appropriate. In all of exercises 5, 6, 7, the motion is vertical motion under gravity, so that $a = -g$, if the distance d is considered positive in the upward direction. $g = 9.8\ \text{m/sec}^2$.

In the present exercise, the stone drops, i.e. it starts with zero initial velocity: $v_o = o$.

(a) We want a relation between distance fallen, d, and time, t, so we use equ. (2). With $v_o = o$, and $d = -4.9$ m (the stone will go down, i.e. in the negative direction) -

$$d = -1/2\, g\, t^2$$

$$-4.9 = -1/2 \cdot 9.8 \cdot t^2$$

$$= -4.9\, t^2$$

$$t = \pm 1 \text{ sec.}$$

We of course need the positive root, i.e. $t = 1$ sec. (It may be of interest, however, to note to the inquiring student that the root $t = -1$ is not meaningless: it is the solution to the physical problem, at what time will the stone be 4.9 meters below the top of the roof so that at time $t = 0$ it is just level with the roof, with zero velocity? The answer, as we see, is that it could have been there at time $t = -1$ sec, and moving upward with the appropriate velocity. This is a valid alternative to the solution we have obtained, but is not appropriate to our problem.)

(b) We need v at that point. We can either say: we need v when $d = -4.9$m, in which case we use equ. (3),

$$v^2 = -2gd$$

$$= -2 \times 9.8 \times 4.9$$

$$= -(9.8)^2$$

$$v = -9.8 \text{ m/sec},$$

i.e. 9.8 m/sec. downwards; or we can use the answer to part (a), i.e. we need v at $t = 1$, so we use equ. (1),

$$v = -gt$$

$$= -9.8 \text{ m/sec.}$$

(c) Most simply, we ask for the time at which $d = -4.9 - 3 = -7.9$m. Use equ. (2),

$$-d = -gt^2$$

$$-7.9 = -\frac{1}{2} \cdot 9.8 \cdot t^2$$

$$= -4.9\, t^2$$

$$t = \left(\frac{7.9}{4.9}\right)^{1/2} = 1.27 \text{ sec.}$$

The extra time taken to fall the additional 3 meters is $1.27 - 1 = 0.27$ secs.

(d) It is simplest to use equ. (1) for v after t = 1.27 sec.

$$v = -gt$$

$$v = -9.8 \cdot 1.27$$

$$= -12.4 \text{ m/sec.}$$

The velocity is 12.4 m/sec downwards.

(e) To see what distance the body moves between t = 2 seconds and t = 3 seconds, we calculate d at these two times, using equ. (2).

$$\text{After 2 secs, } d_2 = -\frac{1}{2} \cdot 9.8 \cdot 2^2$$

$$= -19.6\text{m}$$

$$\text{After 3 secs, } d_3 = -\frac{1}{2} \cdot 9.8 \cdot 3^2$$

$$= -44.1 \text{ m.}$$

Thus during the third second the distance traveled is $d_3 - d_2 = -24.5$ m, i.e. 24.5 m downwards. [$d_3 = -44.1$ m. Quite a high roof!] .

EXERCISE 6: A ball is thrown straight up with a velocity of 15 m/sec.
(a) How fast will it be going after 1.2 sec?
(b) How far above the ground will it be at that time?
(c) How fast will it be going after 2.3 sec?
(d) How far above the ground will it be at that time?
(e) What is the stone's acceleration at the top of its rise?

This exercise is similar to (5), but a little more complicated, since there is an initial velocity v, of +15 m/sec. Still $a = -9.8 \text{ m/sec}^2$.

(a) We want v at t = 1.2 secs. Use equ. (1):

$$v = v_o - gt$$

$$= 15 - (9.8 \cdot 1.2)$$

$$= 3.2 \text{ m/sec.}$$

This is 3.2 m/sec. Upwards.

(b) Because we now have v as well as t, and we need d, we could use either equ. (2) or equ. (3). (See note, Exercise 5, p. 29)

Using equ. (2),

$$d = v_o t - \frac{1}{2} gt^2$$

$$= 15 \cdot 1.2 - \frac{1}{2} \cdot 9.8 \cdot (1.2)^2$$

$$= 10.9 \text{ m.}$$

Using equ. (3),

$$v^2 = v_o^2 - 2 g d$$

$$\text{i.e. } d = \frac{v_o^2 - v^2}{2 g}$$

$$= \frac{(15)^2 - (3.2)^2}{2 \cdot 9.8}$$

$$= 10.9 \text{ m.}$$

(c) Use equ. (1):

$$v = v_o - gt$$

$$= 15 - 9.8 \cdot 2.3$$

$$= - 7.5 \text{ m/sec.}$$

i.e. 7.5 m/sec. downwards.

(d) As in (b), we could use either equ. (2) or equ. (3). It is a little easier to use equ. (2). We have

$$d = v_o t - \frac{1}{2} at^2$$

$$d = 15 \cdot 2.3 - \frac{1}{2} \cdot 9.8 \cdot (2.3)^2$$

$$= 8.6 \text{ m.}$$

It is 8.6 m. above the point of projection.

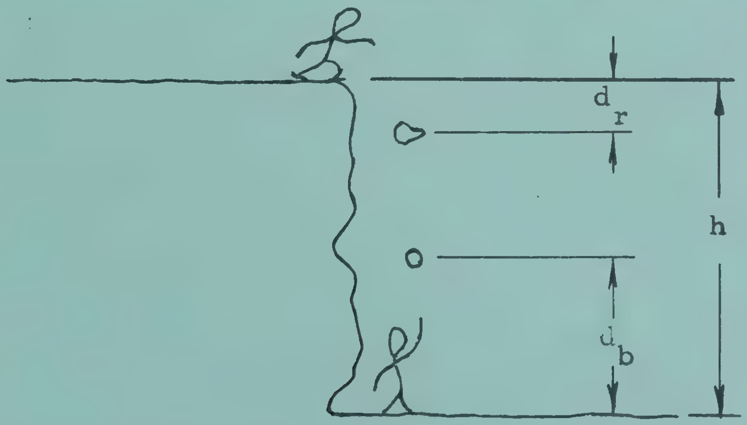
(e) The ball's acceleration is constant, and equal to -g everywhere along its path. At the top of its rise the velocity is zero, but this does not affect the acceleration, of course.

It is good practice for the students to check their answers by rough qualitative arguments, where possible. For example, since the answer to (c) was -7.5 m/sec., which is smaller in magnitude than the initial velocity, the ball must still be above its point of projection. [It will have a downward velocity of 15 m/sec. just as it passes this point of projection.] Thus the answer to (d) had to be a positive distance. If there is time, it may be helpful even to graph d and v as functions of t, but these simple order-of-magnitude estimates can be made very quickly, and are a useful check on arithmetic, as well as the student's understanding of the problem.

EXERCISE 7: A stone is dropped off a cliff h meters high. At the same instant a ball is thrown straight up from the base of the cliff with initial velocity of v m/sec. Assuming the ball is thrown hard enough, at what time t will stone and ball meet? When they meet, is the ball still rising?

This exercise in free fall is a little harder than Exercises 5 and 6, since it involves two projectiles, and consequently, two simultaneous equations.

The stone is dropped, and the ball is thrown, at t = o. In order to be able to use our equation for both rock and ball, we indicate the distance the rock has fallen after time t by a negative quantity (since it is downward). Our equations allow us immediately to write down d_r and d_b as



functions of t: using Equ. (2),

$$d_r = -\frac{1}{2}gt^2$$

$$d_b = v_o t - \frac{1}{2}gt^2$$

The rock and ball will meet if and when

$$-d_r + d_b = h$$

We subtract the two equations in t to get

$$d_b - d_r = v_o t$$

and this is just equal to h, i.e. $t = h/v_o$

This apparently always has a solution; we can always calculate h/v_o. Yet there was a qualification in the question: v_o must be large enough. It arises because with this value for t, we may find that $(-d_r)$ at the point where they collide is greater than h! Since, in our problem, this is presumably not possible (since the rock will just hit the ground and stop), we have

$$-d_r < h$$

$$\text{i.e. } \frac{1}{2} g \, (h/v_o)^2 < h$$

$$\text{i.e. } v_o^2 > \frac{1}{2} gh.$$

(This condition automatically ensures that $d_b > 0$, which our problem also requires, of course: since $d_b = h + d_r$, if $-d_r < h$, $d_b > 0$.)

The velocity of the ball at the time they meet is obtained by using equ. (1):

$$v_b = v_o - gt$$

$$= v_o - gh/v_o$$

$$= \frac{1}{v_o} \, (v_o^2 - gh).$$

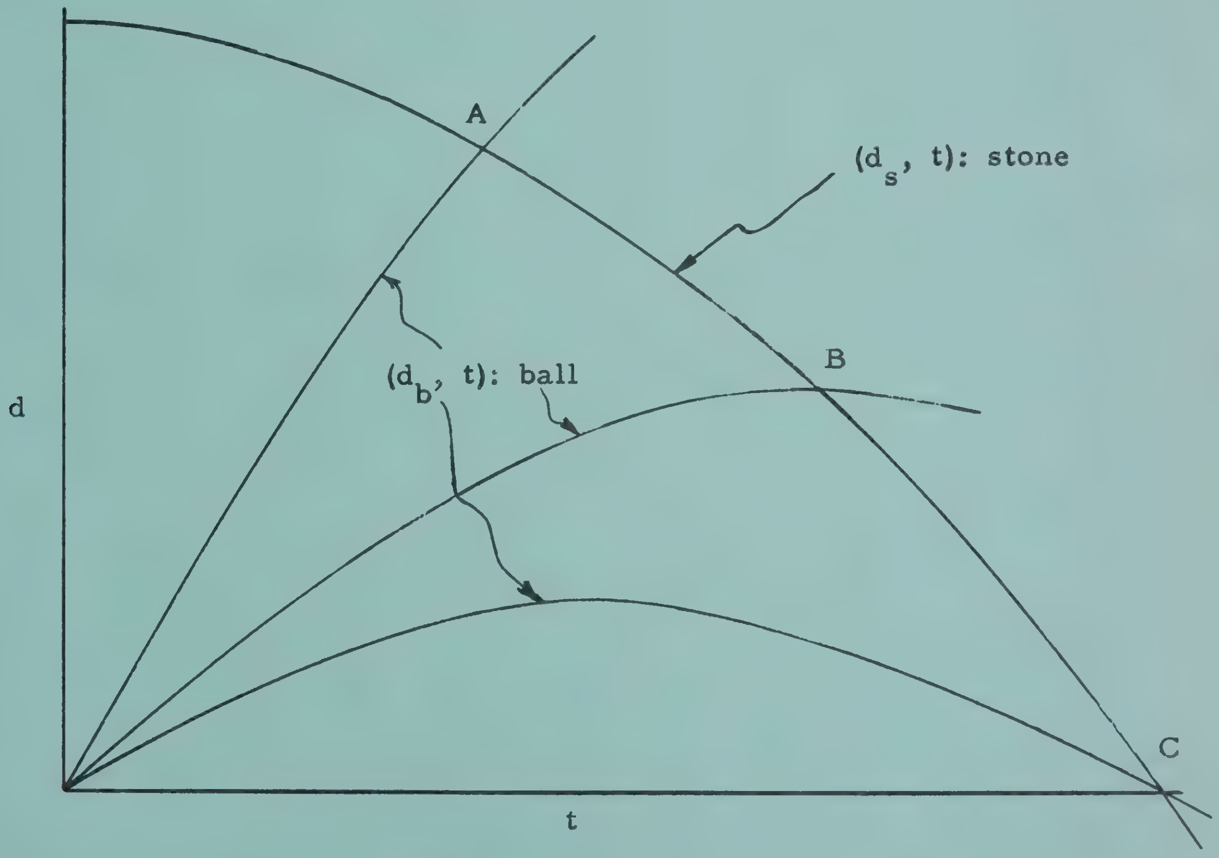
As to whether or not is is positive (i.e. upward), all that our condition on v_o, obtained above, tells us is that

$$v_b > \frac{1}{v_o}\left(\frac{1}{2}gh - gh\right)$$

$$\text{i.e. } v_b > -\frac{gh}{2v_o}$$

It can be either positive or negative.

The situation is complicated, and is made much clearer by drawing a graph of d_r and d_b versus t. The curve of (d_r, t) is unique, but that for (d_b, t) depends on v_o, the initial velocity, and we have drawn several cases.



The intersection A corresponds to an initial velocity of the ball high enough that the ball is still moving fast when it meets the stone. B corresponds to the case where $v_b = o$ at collision. C illustrates the minimum possible v_o: the ball and stone collide just as each reaches the ground.

Remember that we are plotting d versus t; these are not the trajectories of projectiles in two dimensions, although they look very similar.

Additional problems involving special cases of this can be set: e.g.

(i) What is the maximum height reached by the ball, and how far is it above the rock at this point? (This assumes that they pass, not colliding.)
(ii) How long after the rock hits the ground does the ball hit?
Answers:

(c) At its maximum height, the ball is stopped, so that $v_b = 0 = v_o - gt$. this occurs at a time $t = \frac{v_o}{g}$.

From equation (3) ($v_b^2 = v_o^2 - 2gd$) when $v_b = 0$, $d_b = \frac{v_o^2}{2g}$.

To find d_r at this time, we use equation (2), setting $t = \frac{v_o}{g}$,

$$d_r = -\frac{1}{2} gt^2$$

$$d_r = -\frac{1}{2} g\left(\frac{v_o}{g}\right)^2 = -\frac{v_o^2}{2g}$$

The distance between the rock and ball at this instant is evidently

$$d_b - (h + d_r) = \frac{v_o^2}{g} - h$$

(ii) the rock hits the ground when $d_r = -h$, that is when

$$h = -\frac{1}{2} gt_r^2 ; t_r = \sqrt{2h/g}.$$

The ball hits the ground when $d_b = 0$, that is when

$$0 = v_o t_b - \frac{1}{2} gt_b^2 ; t_b = \frac{2v_o}{g} .$$

thus the difference in times $t_b - t_r = \frac{2v_o}{g} - \sqrt{\frac{2h}{g}}$.

EXERCISE 8: Two 0.5-kg dry-ice discs on a flat table are tied together by a string.

(a) What is the total gravitational force on these discs?
(b) At what rate will they speed up if pulled across the table by a horizontal force equal to the gravitational force found in (a)?
(c) If one disc remains on the table and the other hangs over the edge, what will be the acceleration of the discs? Neglect friction.

A problem involving the application of the notion that the gravitational force on a body at the earth's surface is m_g, directed vertically downwards. The masses are each 0.5 kg, and we take g to be $9.8\ \text{m/sec}^2$.

(a) Total gravitational force: mg vertically downwards on each, i.e. $2\ mg = 2 \times 0.5 \times 9.8 = 9.8$ nt. [There is also a mutual gravitational attraction $\frac{Gm^2}{r^2}$, (see next chapter) where r is the distance between their centers, but it is quite neglibible: with $r = 0.1$ m, it is in this case 2×10^{-9} nt!]

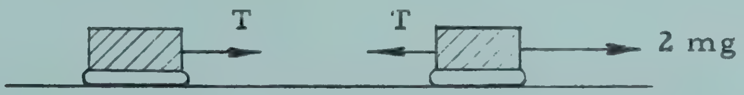
(b) If friction is neglected, then in this case the two discs have a total horizontal force on them of 2 mg, which is just the vertical force they would experience in free fall. Their horizontal acceleration is thus g. In mathematical terms,

$$F = ma$$

$$2\ mg = (2m)a$$

$$a = g.$$

In preparation for part (c), it is useful to consider a special case of this problem in which one of the discs is acted on by an external horizontal force of 2 mg. It transmits a force to the other by means of the string, and we can ask about the tension in the string. We were able to ignore it in the solution we just obtained because it is an internal force: its contribution to the motion of the whole system sums to zero. We investigate its magnitude by considering the discs separately.



We already know that the acceleration of each disc is g. Hence for the first disc

$$F = ma$$

$$T = mg,$$

and for the second,

$$2\ mg - T = mg.$$

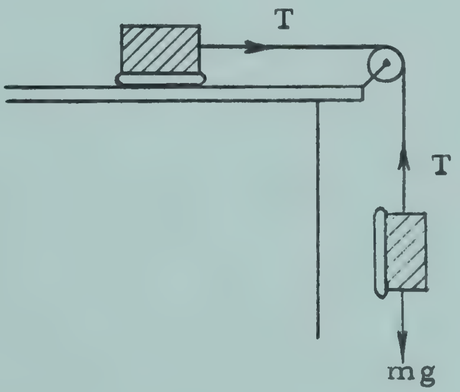
Thus T = mg from considering the motion of either disc. We could of course have started the problem from this approach. in which case we would not know the acceleration. The equations of motion for the discs are then

$$T = ma$$

$$2\ mg - T = ma$$

We can add the two equations to get a = g. The cancellation of T illustrates rather simply the whole idea of being able to neglect internal forces when calculating the motion of the whole body.

(c)



The most direct way to solve this problem is to consider each system separately, assuming a tension T in the string. There is also the gravitational force mg on the hanging disc. (We consider, of course, only forces acting in the direction in which the motion takes place. In the perpendicular direction, for the disc on the table, the gravitational force is balanced by the reaction from the table, since there is no vertical motion.) For the two discs, each accelerating with an acceleration a, Newton's law gives us

$$T = m\ a$$

$$mg - T = ma$$

and adding these, we get $a = \frac{1}{2}\ g$. This then gives us that $T = \frac{1}{2}\ mg$.

The simplest way to solve the problem is to realize that although the discs actually move in perpendicular directions, we can still use the principle that only external forces affect the motion of the system, if we are careful in obtaining the external forces acting in the direction of motion. In this case the only force is mg on the hanging disc, so we have (since the total mass is 2m)

$$F = ma$$

$$mg = (2m)a$$

$$a = \frac{1}{2} g.$$

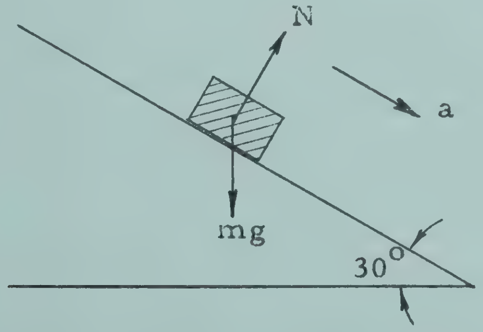
[Don't emphasize this last principle too much: its applicability is somewhat limited. For the general problem it is much safer to isolate each body, and consider all forces acting on it.]

EXERCISE 9: An object of mass m is moving on a frictionless inclined plane that makes an angle of 30 degrees with the horizontal.

(a) What is the net force on the object?
(b) What is its acceleration down the plane?

A slightly more complicated problem involving gravitational attraction.

In problems of this kind a force diagram, i.e. a sketch of the system with all forces drawn in, is invaluable.



What we have drawn are the forces on the object. (The object exerts a force equal and opposite to N on the plane.)

(a) We resolve mg along and perpendicular to the plane. In the perpendicular direction there is no motion i.e. no acceleration, so in terms of Newton's law,

$$F_N = N - mg \cos 30^\circ = m \times o$$

$$\text{i.e. } N = mg \cos 30^\circ.$$

(The net force perpendicular to the plane is zero. It must be, since there is no acceleration perpendicular to the plane. The net force is along the plane, since this is the direction in which motion takes place.)

(b) Down the plane, $mg \sin 30^\circ = ma$, $a = g \sin 30^\circ = \frac{1}{2} g$.

We have resolved mg perpendicular to, and parallel to the plane since those are clearly the "natural" axes of the problem (since there is then no motion along one of them.) Some students may wish to consider the motion in the vertical and horizontal directions instead. It is somewhat more messy, but of course leads to the same answer. If we call these the y and x directions, then we have

$$F_x = N \sin 30^\circ$$

$$F_x = N \cos 30^\circ - mg,$$

$$\text{and } a_x = a \cos 30^\circ$$

$$a_y = - a \sin 30^\circ$$

$$\text{thus} \qquad N \sin 30^\circ = a \cos 30^\circ$$

$$N \cos 30^\circ - mg = - a \sin 30^\circ$$

Multiplying these by $\cos 30^\circ$ and $\sin 30^\circ$ respectively, and subtracting, we obtain $mg \sin 30^\circ = ma (\cos^2 30^\circ + \sin^2 30^\circ) = ma$, $a = g \sin 30^\circ = \frac{1}{2} g$.

EXERCISE 10: A baseball thrown horizontally starts at a height of 2 meters with a velocity of 30 m/sec.

(a) Plot the trajectory of the ball, using 1/10-sec intervals.
(b) How long before the ball strikes the ground?
(c) Determine both the magnitude and direction of the velocity of the ball at the instant it strikes the ground.

A graphical problem in projectile motion.

In setting out to plot the curve, it does not hurt to know what to expect. Using equ. (2) of our kinematics, we see that the time t for the

ball to reach the ground, is given by

$$-d = -\frac{1}{2}gt^2$$

$$-2 = -\frac{1}{2}gt^2$$

$$t = (4/g)^{1/2} = 0.64 \text{ sec.}$$

and the horizontal distance traveled is R where

$$R = 30\,t$$

$$= 19.2 \text{ m.}$$

(a) If the horizontal and vertical components, referred to the point where the ball starts, are x and y, then as functions of t,

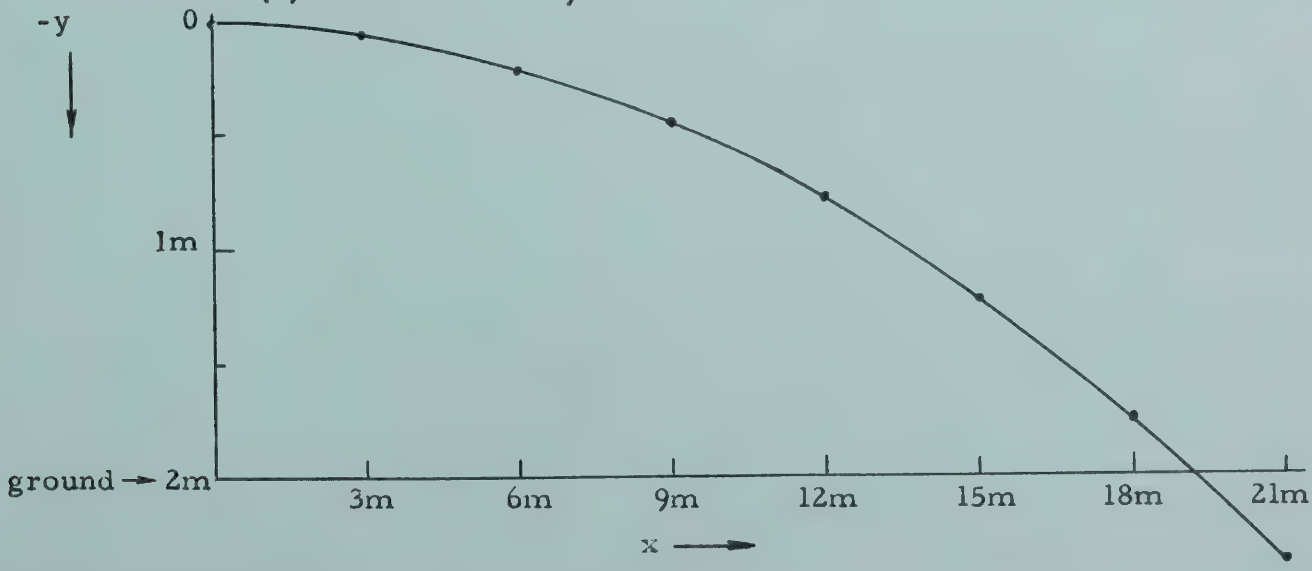
$$x = 30\,t$$

$$y = -\frac{1}{2}g\,t^2 = -4.9\,t^2$$

where y is positive upwards, and so is negative here. Thus equal intervals of t give equal intervals of x. We obtain the following table:

t	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
x	0	3.0	6.0	9.0	12.0	15.0	18.0	21.0
-y	0	0.05	0.20	0.44	0.78	1.22	1.76	2.40

We give y to two decimal places because we shall need a larger scale in y than in x. We also continue it to t = 0.7 (i.e. below ground) to be able to answer (c) more accurately.



(b) By inspection, the time taken to hit ground is between 0.6 and 0.7 secs. A more careful interpolation in x (since x is proportional to t) then gives us results as accurate (in principle) as we like. $t \approx 0.65$ secs. (Actually, = 0.64.)

(c) Remember that the slope of this graph is not a velocity. It is in fact the ratio of vertical to horizontal velocities. Since we know that the horizontal velocity is constant, and equal to 30 m/sec, we can then get the vertical velocity from the measured ratio.

A crude interpolation (between t = 0.6 and t = 0.7 from the table) gives this ratio as

$$\frac{v_y}{v_x} = \frac{2.40 - 1.76}{21.0 - 18.0}$$

$$= 0.21$$

$$\text{i.e. } v_y = .21 \times 30 = 6.3 \text{ m/sec.}$$

We can obtain a more accurate result by drawing a careful curve, and measuring the slope at the point where it crosses the line y = 2. Actually, by our kinematic equation, $v_y = 6.6$ m/sec.

In anticipation of problem (11), it should be pointed out once again that the vertical motion is exactly the same as that of a body falling vertically from 2 meters, with no horizontal motion.

EXERCISE 11: A bullet is dropped at exactly the same moment that a bullet is fired horizontally. Discuss the motion of each of the bullets.

This problem in projectile motion should follow exercise (10). The answer is then almost obvious, so long as we assume that there is nothing in principle different between the motion of baseballs and the motion of bullets. (A discussion of the actual differences - air resistance, swerving (of the baseball) etc. - should be avoided at this stage, as it will not contribute anything useful.)

As was remarked at the end of the last problem, the vertical motion of a bullet fired horizontally is identical to that of a bullet dropped vertically. Thus the vertical heights of the two bullets will be the same, although the fired bullet may travel a large distance horizontally.

EXERCISE 12: Try rolling one marble slowly off the edge of a table at the same instant that you snap another marble off the table horizontally with your finger. Do they hit the floor at the same instant? Explain.

A demonstration experiment for students to do at home to convince themselves of the independence of horizontal and vertical motions. The technique is shown in the film on free fall, and the experiment is very suitable for a class demonstration.

EXERCISE 13: A baseball is thrown from center field to home plate. If the ball is in the air 3 seconds and we neglect air resistance, to what vertical height must it have risen?

An application of the result discussed in problem (10), and used in (11) and (12), that vertical motion and horizontal motion are independent. It is significant that we are not given the horizontal distance covered (for the benefit of non-baseballers, it may be assumed to be about 80 meters): It does not matter in this particular problem.

Call the vertical component of the initial velocity v_{oy}. For simplicity, assume that the ball is caught at the same level as thrown. After 3 seconds, the ball is caught, and the total vertical displacement from its initial position is zero. Using our kinematic equ. (2),

$$d = v_{oy}t - \frac{1}{2}gt^2$$

$$0 = v_{oy} \cdot 3 - \frac{1}{2}g(3)^2 ,$$

$$v_{oy} = \frac{3}{2}g$$

$$= 14.7 \text{ m/sec.}$$

~~Since up~~

Since $-a_{up} = a_{down}$,

$t_{up} = t_{down}$

t_{down} (or up) $= 1.5$ s

$\therefore$ In a time t, $s = \frac{1}{2}at^2$

$s = \frac{1}{2}\,9.8 \times 1.5^2$

$s = 11$

If the maximum height reached is h, then because the vertical component of velocity at that point is zero, we have using kinematic equ. (3),

$$0 = v_{oy}^2 - 2gh$$

$$h = \frac{v_{oy}^2}{2g} = \frac{(14.7)^2}{2 \cdot 9.8} = 11.0 \text{ m.}$$

Although that is all the question requires, the interest of baseball enthusiasts among the class may be aroused by further calculation of this particular throw. If we assume that the horizontal distance traveled is 80 m, then since this took 3 seconds, the constant horizontal velocity of the ball, v_{ox}, must have been 80 m/3 sec = 26.7 m/sec. The resultant initial speed must be

$$\sqrt{(v_{ox})^2 + (v_{oy})^2} = \sqrt{(26.7)^2 + (14.7)^2} \approx 30 \text{ m/sec.}$$

Perhaps not extraordinary for a center fielder, but some throw for a student!

EXERCISE 14: Plot the trajectory of an object that is thrown into the air with an initial velocity of 10 m/sec at an angle of 45 degrees with the horizontal.

(a) How high does it rise?
(b) How far does it go?
(c) At what velocity and angle does it strike the ground?

This is an exercise in plotting a trajectory, and is similar to problem (10). The calculation of the table of x and y, the horizontal and vertical coordinates of the object, can now be done a little more elegantly. Since the horizontal and vertical components of the initial velocity are both $10\sqrt{2} = 7.07$ m/sec., the kinematic equ. (2) gives x and y as function of t

$$x = 7.07\ t$$

$$y = 7.07\ t - 4.9t^2$$

We could, as in problem (10), calculate x and y for various values of t. It is a little simpler, however, to eliminate t from the above equations, and to obtain one equation connecting x and y.

$$t = x/7.07$$

$$y = 7.07\ (x/7.07) - 4.9\ (x/7.07)^2$$

$$= x - 0.098\ x^2$$

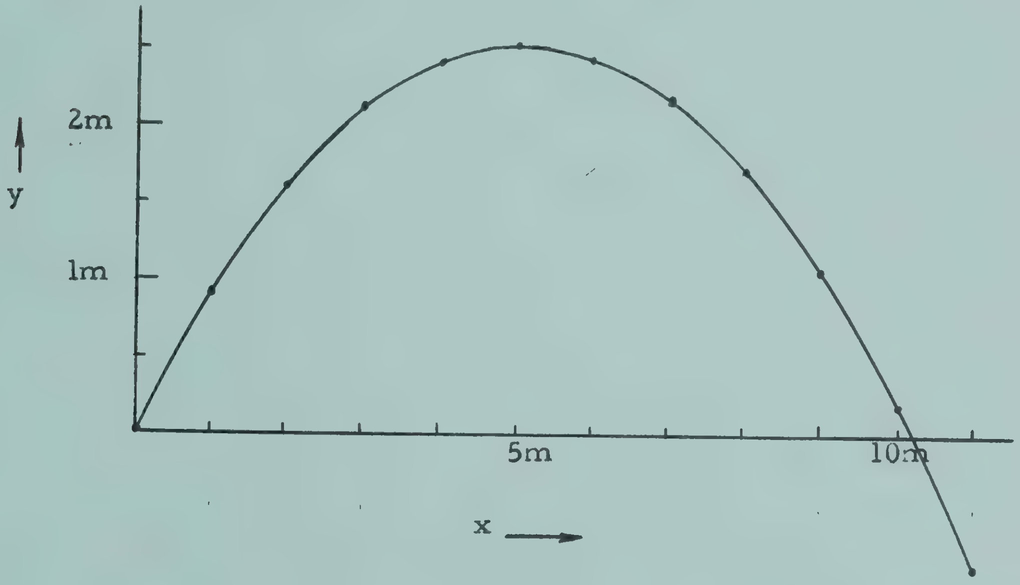
We can now plot y against x. [The curve is a parabola, and we can calculate its salient features. The curve crosses the x axis at x = o and x = 1/.098 = 10.2m. The maximum value of y is at the x for which

$$\frac{dy}{dx} = 1 - 0.196\,x = 0$$

i.e. x = 5.1 m; here y = 2.55 m.]

In tabular form,

x	0	1	2	3	4	5	6	7	8	9	10	11
y	0	0.90	1.61	2.12	2.43	2.55	2.48	2.20	1.73	1.06	0.20	-0.86



The slope of this curve gives the ratio of vertical to horizontal velocities. We can measure it, and using the fact that the horizontal component is constant, and equal to 7.07 m/sec., we can obtain the vertical component. Crudely, from the x values of 10 and 11 meters, we have from the table

$$\frac{v_v}{v_h} = \frac{-0.86 - 0.20}{11-10}$$

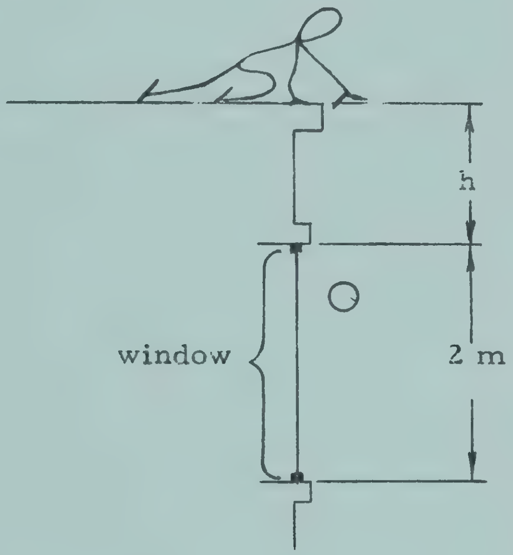
$$- 1.06$$

The actual result is of course 1 at this point, since that is the value at the starting point, and there is a complete symmetry between these two points. In fact, of course, since the curve is a parabola with its axis vertical, it must be symmetrical about some x (in this case x ≑ 5.1 m). The plotted curve shows this, and it is a point which may usefully be remembered, since it simplifies the working of some problems.

As noted earlier in the guide, the choice of an angle of 45° with the horizontal will give the maximum range. Students will be able to see this readily by making graphs for other angles such as 40 degrees or 50 degrees.

EXERCISE 15: A stone drops from the edge of a roof and is observed to pass by a window which is 2 meters high in 0.10 second. How far is the roof above the top of the window?

An exercise in free fall.



A diagram is needed. Suppose the roof is a height h above the top of the window. We are given that the stone, which has already dropped a distance h by the time it comes into view through the window, takes 0.1 secs. to pass the window, which is 2 m high.

The pedestrian way to do this problem is as follows: at time t = 0 the stone drops from the roof, at $t = t_1$ it is seen at the top of the window, and at $t = t_1 + 0.1$, it gets to the bottom of the window. Using the kinematic equ. (2),

$$- h = - \frac{1}{2} g t_1^2$$

$$- (h + 2m) = - \frac{1}{2} g (t_1 + 0.1)^2$$

To solve, first subtract them to get

$$2 = gt_1\ (0.1) + \frac{1}{2}\, g\ (0.1)^2$$

$$2 = 0.98\ t_1 + 0.049$$

$$t_1 = \frac{1.95}{0.98}$$

$$\approx 2.0 \text{ sec.}$$

Substituting in the first equation,

$$h = 4.9\ t_1^{\ 2}$$

$$= 19.6 \text{ m.}$$

The quick way to do the problem is to say that the average velocity of the stone while in view is 2/0.1 = 20 m/sec. Because the velocity varies linearly with time, this is also the actual velocity of the stone when it is in the middle of the window, i.e. when it has fallen h + 1 m. Using our kinematic equ. (3),

$$v^2 \doteq -\ 2g\ d$$

$$(20)^2 = 2g\ (h + 1)$$

$$h = 20.4 - 1$$

$$= 19.4 \text{ m.}$$

EXERCISE 16: A boy on a cliff 49 meters high drops a stone and one second later throws a second stone after the first. They both hit the ground at at the same time. How fast did the boy throw the second stone? Neglect air resistance.

A problem involving free fall, similar to problem (7).

At time t = 0 the first stone is dropped. Let it take t_1 seconds to reach the ground. Then, using equ. (2),

$$-\ d = -\ \frac{1}{2}\, gt_1^{\ 2}$$

$$-\ 49 = -\ \frac{1}{2}\, gt_1^{\ 2}$$

$$= -\ 4.9\ t_1^{\ 2}$$

$$t_1 = \sqrt{10} \text{ secs.}$$

$$= 3.16 \text{ secs.}$$

The second stone is thrown downwards; let it have an initial velocity $-v_o$ (downwards). It takes $t_1 - 1 = 2.16$ secs to reach the ground. Using equ. (2) again,

$$-d = -v_o t - \frac{1}{2} gt^2$$

$$-49 = -v_o (2.16) - 4.9 (2.16)^2$$

minus

$$v_o = \frac{49 \textcircled{+} 4.9 (2.16)^2}{2.16}$$

$$\text{i.e., } v_o = 12.0 \text{ m/sec.}$$

EXERCISE 17: Show that the expression $\frac{v^2}{R}$ for centripetal acceleration has the units of acceleration.

An exercise in dimensional analysis related to circular motion.

The units of velocity are (length/time), so the units of v^2/R are $(\text{length/time})^2 \frac{1}{\text{length}} = \frac{\text{length}}{(\text{time})^2}$ the same as those of acceleration.

For more complicated situations, it is useful to have a notation for the calculation of dimensions. We frequently denote the dimension of a quality by enclosing it in square brackets. The dimensions of all quantities we have met so far can be expressed in terms of length, time and mass, which we denote by L, T, and M. Thus $[v] = LT^{-1}$, so that

$$[\frac{v^2}{R}] = L^2T^{-2} \cdot L^{-1} = LT^{-2} = [\text{acceleration}].$$

Dimensional analysis is extremely useful as a check on derived formulae, to ensure that no factors have been lost. You may want to illustrate by considering additional examples. For instance, does the equation

$$d = v_o t + \frac{1}{2} at^2$$

check dimensionally?

$$[d] = L,$$

$$[v_o t] = LT^{-1} \cdot T = L,$$

$$[at^2] = LT^{-2} \cdot T^2 = L,$$

so it clearly does.

Also, although the law of universal gravitation does not occur until the next chapter, consider the dimensions of G in the expression

$$F = -\frac{G\, m_1 m_2}{r^2}$$

$$[F] = MLT^{-2}$$

$$\left[\frac{G\, m_1 m_2}{[r^2]}\right] = [G] \cdot M^2 L^{-2}$$

$$\therefore\ MLT^{-2} = [G]\, M^2 L^{-2}$$

$$[G] = M^{-1} L^3 T^{-2}$$

EXERCISE 18: (a) What is the centripetal acceleration of a hammer thrower's hammer if his axis of rotation is 1.5 m from the hammer and he turns around once every 3 seconds? (b) If the hammer weighs 196 newtons, what force must the hammer thrower exert to maintain this centripetal acceleration?

This is a straightforward exercise concerned with centripetal acceleration. It can easily be enlarged upon in class discussion.

(a) Our athlete turns once every three seconds. The head of the hammer travels through a distance $2\pi \times 1.5 = 3\pi$ m in this time, therefore

$$v = \frac{3\pi \text{ m}}{3 \text{ sec}} = \pi \text{ m/sec}.$$

The centripetal acceleration is

$$\frac{v^2}{R} = \frac{\pi^2}{1.5}$$

$$= 6.6 \text{ m/sec}^2.$$

(b) The force needed to maintain the centripetal acceleration is ma, = mv^2/R. We are not given m, but we do know the weight mg. Since mg = 196 nt, m = 196/9.8 = 20 kg. The force required is thus 20 x 6.6 = 132 nt.

(18 Supplemental) If the interests of the class are so inclined, this exercise can be taken as the starting point of a number of other problems. For example, how far will the hammer go, if it is released when its velocity makes an angle of 45° with the horizontal? [We saw from Exercise 14 that his is the angle which results in the maximum range for a given initial velocity.]

The initial vertical component of velocity is 3.14 sin 45° = 2.22 m/sec. The time that the hammer is in the air can be obtained by using the kinematic equ. (2), where d = 0 since the total vertical displacement when it hits the ground is zero. (Remember that the vertical component of the initial velocity is v_o sin 45° = 2.22 m/sec).

$$0 = v_{oy}t - \frac{1}{2}gt^2$$

$$t\,(v_{oy} - \frac{1}{2}gt) = 0$$

$$t = \frac{2v_{oy}}{g}$$

$$= 0.45 \text{ sec.}$$

The horizontal distance travelled in this time, at a horizontal velocity of 3.14 cos 45° = 2.22 m/sec., is 1 meter! A rather puny hammer thrower, undoubtedly! We have made the approximation that the difference in level between the starting point of the hammer and the ground is negligible, and in this case it isn't a very good approximation. But even allowing for this, it isn't a very realistic throw!

Since the world's record for hammer throwing (with a hammer whose mass is 7.3 kg) is 70 meters, the actual initial velcoity of a well-thrown hammer must be considerably larger than that given in this example: in fact, it must be increased in the ratio of the square root of the ranges in the two cases, i.e. $\sqrt{\frac{70m}{1\ m}}$, so it must be about 26 m/sec.

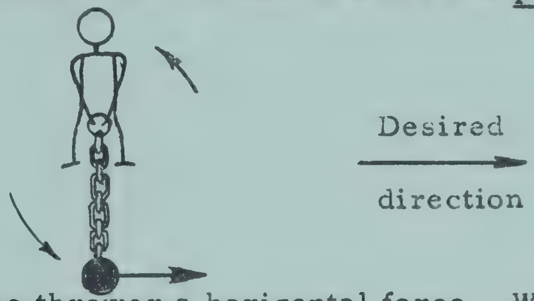
The force exerted by a champion hammer thrower would actually be

$$F = \frac{mv^2}{R} = \frac{7.3 \times 26^2}{1.5} \approx 3.3 \times 10^3 \text{ nt.}$$

In more familiar units, this force is about 700 lbs!

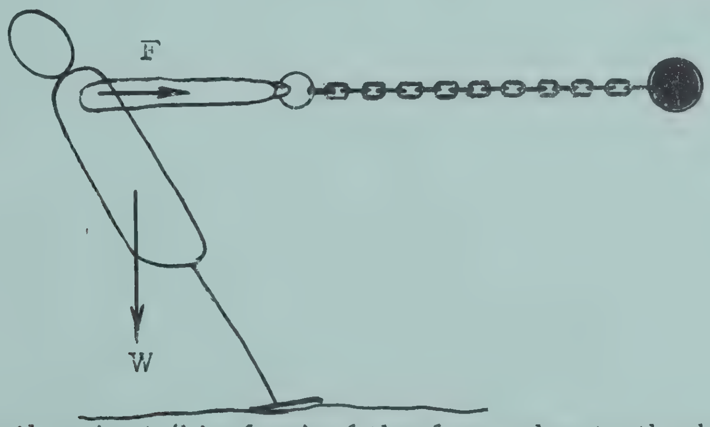
In what position does the thrower release the hammer?

The hammer must be released when the velocity is in the required direction, i.e. when its handle and the thrower's arms are perpendicular to the desired direction.



The hammer exerts on the thrower a horizontal force. Why doesn't he fall over? Where does the counter-acting horizontal force come from?

The answer is illustrated in the figure. The thrower leans back until the moment about the pivot (his feet) of the force due to the hammer is balanced by the moment of his own weight. Although at this stage the students have not been formally introduced to the concept of moments, the treatment of this point can be convincing although rather sketchy.

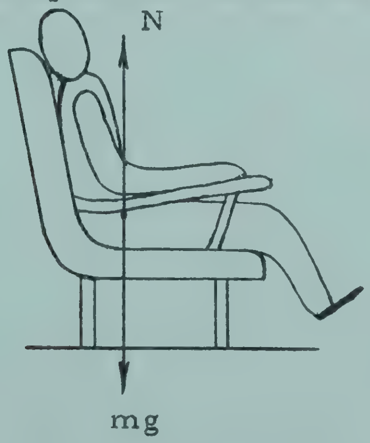


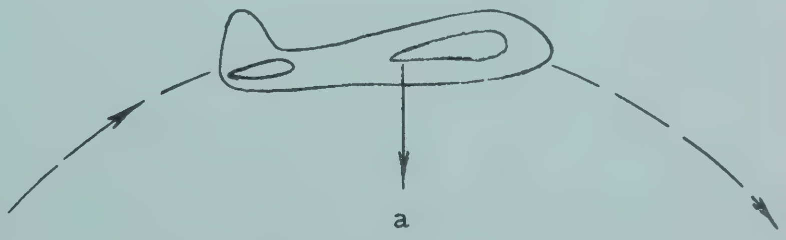
EXERCISE 19: Some people speak of a pilot coming out of a power dive in a jet airplane as weighing several times his own weight. We sometimes say that the same pilot, falling free before opening his parachute, is weightless. Do these statements make sense in the light of our definition of weight as the force of gravity on an object? What is meant by "several times his own weight" and "weightless" above?

Using weight in the sense that it has been defined in the text as the gravitational force on any body, then the weight of a person at or near the earth's surface is constant, and equal to mg, in all situations.

On the other hand, everyone in the class will probably have experienced sensations similar to the one described in the exercise: that with respect to the immediate surroundings, one's bodily sensations were just those which one would have if one's weight were different. An airplane in bumpy weather, a car accelerating rapidly or going over a sudden hump in the road, a roller-coaster, an elevator - even a garden swing, all provide such sensations. The sensation arises because one's interaction with the surroundings is altered - you are squashed down in your seat or feel lifted from it. The explanation can be given in two ways, using in both cases the fact that your seat and surroundings form a frame of reference which is accelerated with respect to the earth. In the case cited in the exercise, on a roller-coaster, a garden swing, the acceleration is the centripetal acceleration arising because the path of your seat is curved in space. In a car or elevator accelerating or decelerating, it is linear acceleration.

(a) Consider the motion in a frame of reference fixed on the earth. Then your seat, and you, if you stay attached to it, are being accelerated. Suppose for simplicity that it is in a vertical direction. The vertical forces on you are your weight, mg, and the force exerted on you by the seat. Call it N. It is this force whose effect tells you what your apparent weight is. The equation of motion says that $mg - N = ma$, i.e. $N = m(g - a)$.





Thus the force exerted on you by the seat can be zero, if a is as large as g, as it is for free fall, (If it becomes negative, you either fly off your seat, or are held there by the seat belt.) At the bottom of the loop, a is negative and the equation of motion is

$$N = m(g + |a|)$$

where $|a|$ means the magnitude of $\underline{a}$ (a positive quantity). Thus here your reaction with the seat can be considerably larger than the value mg which you normally experience.

(b) The alternative explanation of the phenomenon uses the fact that in a frame of reference accelerating with respect to the earth the "fictitious" inertial forces must be included as well as the weight mg. In the situation illustrated in the figure, we say that in the frame of reference at rest with respect to the airplane we are at rest, but the force on us is $\overrightarrow{mg} - \overrightarrow{ma}$ where $\overrightarrow{ma}$ is the fictitious force due to the acceleration. In this sense, our weight has changed, if we define it as the external force on bodies at rest in the frame of the airplane. The only thing to remember is that at other parts of the loop the vector $\vec{a}$ will not be vertical, so from this point of view the "weight" of an object in the plane changes direction both with respect to the earth and with respect to the plane, as we go through the loop.

To sum up, our interpretation of the situation depends somewhat on our definition of the word "weight", but the sensations felt are not illusory, and they can be quantitatively explained on the basis of the equations of motion.

EXERCISE 20: How fast must a plane fly in a loop-the-loop of radius 1.00 km if the pilot experiences no force from either the seat or the safety belt when he is at the top of the loop? In such circumstances the pilot is often said to be "weightless."

This exercise, and exercises 21 and 22 can be regarded as numerical examples of the previous exercise. They should be considered together.

At the top of the loop the centripetal acceleration is

$$a = \frac{v^2}{R}$$

vertically downward. The pilot is to experience no force from the seat or belt, i.e. $N = 0$. Thus

$$mg = ma$$

$$a = g$$

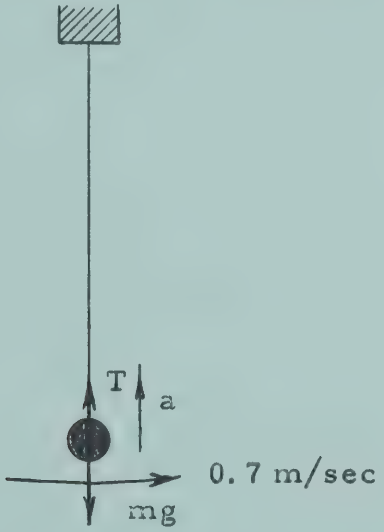
Thus $$v^2 = Rg$$

$$= 10^3 \times 9.8\ (\mathrm{m/sec})^2$$

$$v = 99\ \mathrm{m/sec.}$$

This speed is about 220 miles per hour.

EXERCISE 21: A pendulum with a mass of 0.4 kg swings through the bottom part of its arc with a velocity of 0.7 m/sec. The distance from the point of suspension to the bob is 1 m. What is the tension on the string at this lowest point?



The situation is very similar to that described in exercise 19. T, the tension in the string, takes the place of N. Here, at the bottom of the swing the centripetal acceleration is $a = v^2/R$ upwards. Thus, using the equation of motion or going to the frame of reference of the bob,

$$mg - T = -ma$$

$$T = m(g + a)$$

$$= m\left(g + \frac{v^2}{R}\right)$$

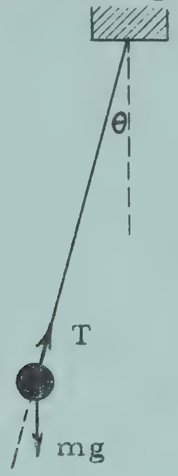
$$= 0.4\left[9.8 + \frac{(0.7)^2}{1.0}\right]$$

$$= 4.12\ \mathrm{nt.}$$

The tension is increased by a factor 1.05 from its value when the bob is at rest.

The following supplementary question can test the students' understanding of these concepts: What is the tension in the string when the bob is instantaneously at rest at the end of its swing? What is the acceleration (mainly its direction) at that point?

You will have to tell the students what the maximum angle of swing is. It can be shown (by considering the energy of the system) to be about 12°. (This is the angle θ in the diagram.)



Since the velocity is zero, there is no centripetal acceleration. Thus, by the equation of motion, there is no resultant force in the radial direction. This means that T is slightly less than mg, in fact

$$T = mg \cos \theta.$$

There is a resultant force in the tangential direction, however: it is $mg \sin \theta$. There is therefore a tangential acceleration $g \sin \theta$. [There obviously has to be, since the tangential velocity is changing.]

EXERCISE 22: A 5-meter string whose diameter is 2.00 mm just supports a hanging ball without breaking.

(a) If the ball is set to swinging, the string will break. Why?
(b) What diameter string of the same material should be used if the ball travels 7.00 m/sec at the bottom of its swing?

After exercises 19, 20 and 21 the answers to this question are easy.

(a) The answer to exercise 21 showed that the tension in the string when the bob is swinging through its lowest point is greater than mg, because of the centripetal acceleration. If the string can only withstand a tension of mg, it will break.

(b) The centripetal acceleration at the lowest point is

$$\begin{aligned} a &= \frac{v^2}{R} \\ &= \frac{(7.00)^2}{5} \\ &= 9.8 \text{ m/sec}^2. \end{aligned}$$

and it is upward. Using the equation of motion, the tension in the string T must then be given by

$$mg - T = -ma$$

$$T = m(g + a)$$

and since a is numerically equal to g, $T = 2mg$. This is just twice the tension required to support the ball in its rest position.

We may assume that the tensile strength of string is proportional to its cross-sectional area, i.e. to the square of its diameter. Since a string of diameter 2.00 mm will just support a tension of mg, the diameter necessary to support twice this tension is $\sqrt{2} \times 2.00 = 2.83$ mm.

EXERCISE 23: An electron (mass = 0.9×10^{-30} kg) under the action of a magnetic force moves in a circle of 2.0 cm radius at a speed of 3×10^6 m/sec. At what speed will a proton (mass = 1.6×10^{-27} kg) move in a circle of the same radius if it is acted upon by the same force?

Another exercise involving circular motion. It is stated in the exercise that the electron moves in a circle with constant speed. We deduce from this that the magnetic force on the electron is constant, and perpendicular to its velocity. The relation between this force, F, and the centripetal acceleration $a = v^2/R$ is given by the equation of motion:

$$F = m\frac{v^2}{R}$$

This is true for both the electron and the proton; the force and radius are the same in each case. If we call v_e and v_p the electron and protron velocities, and m_e and m_p their masses, then the equation of motion for each is

$$F = m_e \frac{v_e^2}{R} \qquad F = m_p \frac{v_p^2}{R}$$

We divide these equations to get

$$\frac{m_p v_p^2}{m_e v_e^2} = 1$$

$$\text{or} \qquad v_p^2 = v_e^2 \frac{m_e}{m_p}$$

$$\text{Thus} \qquad v_p = v_e \left(\frac{m_e}{m_p}\right)^{1/2}$$

$$= 3 \times 10^6 \times \left(\frac{0.9 \times 10^{-30}}{1.6 \times 10^{-27}}\right)^{1/2}$$

2.36

$$= 3 \times 10^6 \times (5.6 \times 10^{-4})^{1/2}$$

$$= 3 \times 10^6 \times \textcircled{1.8} \times 10^{-2}$$

7

$$= \textcircled{5.4} \times 10^4 \text{ m/sec.}$$

We did not need either R or the magnitude of F to solve the problem.

It may be of interest to the students to calculate the actual value of F.

$$F = \frac{m_e v_e^2}{R}$$

$$= \frac{0.9 \times 10^{-30} \times (3 \times 10^6)^2}{0.02} \text{ nt.}$$

$$= 4.05 \times 10^{-16} \text{ nt.}$$

[Note that R is given in cm, and we have converted it to m.] The force is extremely minute. Yet all of the students have seen a picture of this sort of motion, in Fig. 8 of Vol. I. There the path of the electron is a spiral, because its velocity decreases as it passes through the material which forms the visible track.

The force on a charged particle moving in a magnetic field is not discussed until Vol. IV, but there may be questions asked about it. In the situation assumed to hold in this exercise, and also holding in the "bubble chamber" photographed in Fig. 8, the magnetic field is perpendicular to the plane defined by the circular motion. The magnetic force is then constant in magnitude, and points towards the center of the circle. It is not profitable to discuss more complicated motion of electrons in magnetic fields at this stage.

EXERCISE 24: A body moving with a speed v is acted upon by a force that always acts perpendicular to the motion of the body but increases steadily in magnitude. Draw a sketch of the trajectory. Does the speed of the object increase or decrease?

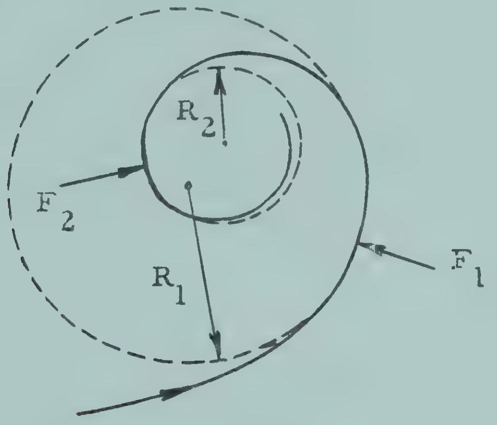
This is a qualitative exercise concerned with circular motion. The force acting on the body is always perpendicular to the velocity. Thus the acceleration produced by this force is perpendicular to the velocity at all points. The magnitude of the velocity therefore does not change, but its direction does change. We know that if the force were constant, the direction of v would change to make the body travel in a circle. If we consider what happens to the body in a short interval of time, we can say that during this time the perpendicular force was approximately constant, and equal to F_1, say, so the path during this time is approximately part of a circle, radius r_1, where

$$F_1 = \frac{mv^2}{r_1} .$$

At any time later, the same argument can be made about a segment of the path, except that now the force is greater, (F_2, say), so the path at this later time is approximately part of a circle of radius r_2, where

$$F_2 = \frac{mv^2}{r_2} .$$

Since F_2 is greater than F_1, r_2 will be less than r_1. In mathematical terminology, the radius of curvature of the path is decreasing with time. In the figure, we have sketched the path, and also some of the circles of which the path is part at any time.



constant speed v

The exact solution of this motion shows that the actual path of the particle, for a force increasing linearly with time, is the "Cornu spiral", which is well-known in optics. The exact solution is much too complicated to be undertaken with a high school class, so do not encourage students to do more than describe the qualitative features of the motion.

EXERCISE 25: (a) What is the period of a pendulum consisting of a mass of 2 kg suspended on a light cord 2.4 meters long if g = 9.8 newtons/kg? (b) If the period is 3 sec, what is g?

A straightforward exercise involving a simple pendulum. (a) The period T, for a simple pendulum is given by the equation

$$T = 2\sqrt{\frac{\ell}{g}}$$

$2\pi\sqrt{\ell/g}$

Here, ℓ = 2.4 m and g = 9.8 m/kg.

$$T = 2 \cdot 3.14 \left(\frac{2.4}{9.8}\right)^{1/2}$$

$$= 6.28 \cdot .495$$

$$= 3.1 \text{ sec.}$$

(b) If, keeping $\ell = 2.4$ m, but taking T = 3.0 sec instead of T = 3.1 sec, what would g have to be? The above relation is

$$(\frac{T}{2\pi})^2 = \frac{\ell}{g}$$

$$g = \ell(\frac{2\pi}{T})^2$$

$$= 2.4\ (\frac{6.28}{3.00})^2$$

$$= 2.4 \times 4.38$$

$$= 10.5 \text{ nt/kg.}$$

Comparing the results of parts (a) and (b) shows that this is a sensitive method for measuring g. It is very easy to set up a simple pendulum as a class demonstration. Accurate values of the period can be obtained by timing a large number of swings. You can verify that the period varies as $\sqrt{\ell}$, and also obtain a surprisingly accurate value for g.

The formula we have used was derived under the assumption that the angle of swing is ''small''. If the angle of swing is kept less than about 30° it will not be in appreciable error.

EXERCISE 26: A block of mass m rests on a horizontal platform. The platform is driven vertically in simple harmonic motion with an amplitude of 0.098 meters. When at the top of its path, the block just leaves the surface of the platform. (This means that at this point its acceleration is 9.8 m/sec^2 downward.)

(a) What is the period of the simple harmonic motion?
(b) When at the bottom point of its path, what is the acceleration of the block?
(c) What is the force exerted by the platform on the block at this bottom point?

An exercise in simple harmonic motion. Since the block follows the platform's motion, its motion must also be simple harmonic, which means that the equation for its motion must be of the form

$$F = ma = -kx$$

where x is measured from the middle of the motion.

(a) The period of the motion is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

How do we find k? If we know the value of F at a given value of x, we can calculate k. Information which is essentially just this is given in the exercise: we are told that at the top of its path (i.e. when x = R, the amplitude of the oscillation) the block just leaves the surface. So at this point the block is not in contact with the plane, and the only force acting on it is its weight mg. Thus

$$kR = mg$$

$$k = \frac{mg}{R}$$

The period T is given by $T = 2\pi\sqrt{\frac{mR}{mg}}$

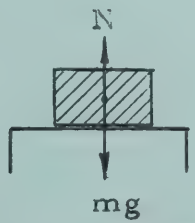
$$T = 2\pi\sqrt{\frac{R}{g}}$$

$$= 2 \times 3.14 \left(\frac{0.098}{9.80}\right)^{1/2}$$

$$= .628 \text{ sec.}$$

(b) At the top of the path, the force on the block is mg downwards, therefore at the bottom it must be mg upwards and the acceleration is therefore g upwards.

(c) The net force on the block is mg upwards. This is the resultant of its weight mg downwards, and the force exerted on it by the platform. The latter force must therefore be 2 mg upwards, In the form of an equation



$$mg - N = - mg$$

$$N = 2\ mg.$$

EXERCISE 27: A spring of force constant k = 196 newtons/m hangs vertically from a peg with its lower end at y = 0. Then a 2-kg mass is hung on the spring.

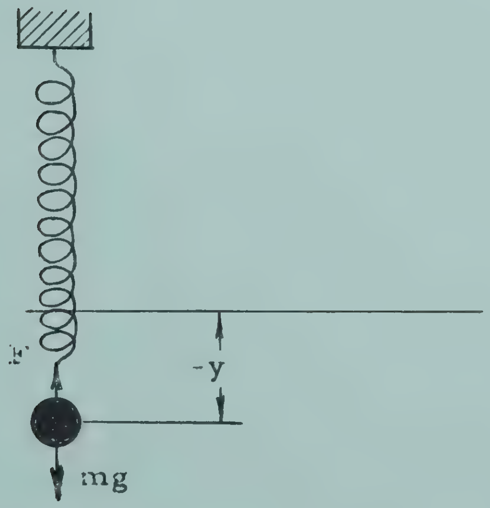
(a) Plot the net force on the mass as a function of the y coordinate of its position from y = - .25 meter to y = + .25 meter.

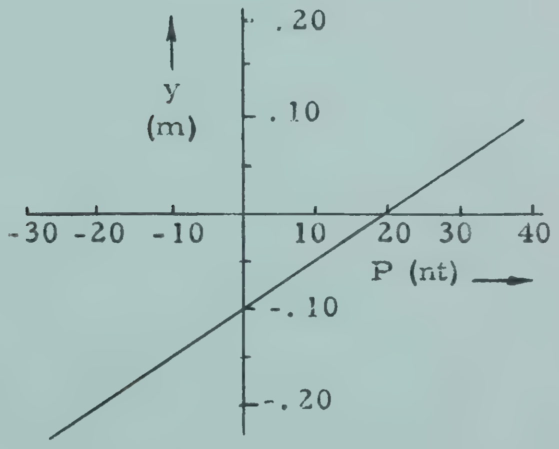
(b) At what position on the y axis should the mass be released from rest if you want it to stand still?

(c) Suppose the mass is released from rest at y = 0. Describe its subsequent motion. How far will it move before it again is momentarily at rest? How long a time does it take?

Another exercise involving simple harmonic motion. Note that we are dealing with a spring, which can undergo compression as well as extension. (An elastic string can only undergo extension.)

(a) When the spring is extended a distance y, the force exerted by it is $F = -ky$ (upwards), where y is positive for an upward displacement. The net force on the 2 kg mass is the resultant of F and the weight mg, which is downwards, i.e. it is a downward force $P = mg + ky$. Numerically, with y measured in meters, this is $P = (2 \times 9.8 + 196\,y)$ nt $= 19.6\,(1 + 10y)$ nt. The plot of P against y is a straight line, and is shown in the figure.





(b) The mass must be released at the position where there will be zero net force exerted on it. From the graph, or from the algebraic expression for P in terms of y, we see that this occurs where $y = -0.10$ m.

(c) If we define a new height coordinate z, measured from the rest position as obtained in part (b), i.e. $z = y + 0.10$ m, then in terms of this coordinate, the net force P is given by

$$P = -196\ z \text{ nt.}$$

The motion is therefore simple harmonic, with $z = 0$, i.e. $y = -0.10$ m., as the mid-point. If the mass is released from rest at $y = 0$, i.e. $z = +0.10$m, then this is the highest point reached in the motion. The amplitude R is 0.10 m. The mass will come to rest again at a point equidistant from the mid-point, and below it, i.e. .20 - 0.10 m, or $y = -0.20$ m. It has moved from $y = 0$ to $y = -0.20$ m, i.e. 0.20 m. It has gone through half of a complete cycle. The time for a complete cycle, the period T, is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2 \times 3.4 \left(\frac{2}{196}\right)^{1/2}$$

$$= 0.634 \text{ secs.}$$

The time taken to reach the lowest point is half of this, i.e. 0.317 sec.

EXERCISE 28: A mass of 1/3 kg moves on the x axis under the action of the force $F = -12x$. (F is in newtons when x is in meters.) When $x = 1$ meter, the mass is momentarily at rest as it reverses its motion.

(a) What is the period of the motion?

(b) How fast is the mass moving when $x = 0$?

(c) How fast when $x = 1/2$ meter, when $x = \frac{1}{\sqrt{2}}$ meters, and when $x = \frac{\sqrt{3}}{2}$ meters?

(d) Plot the speed vs. x.

Another exercise involving simple harmonic motion. This is a rather difficult exercise, and involves the equivalence of simple harmonic motion and circular motion.

(a) The equation of motion is

$$ma = -kx,$$

$$\frac{1}{3}a = -12x,$$

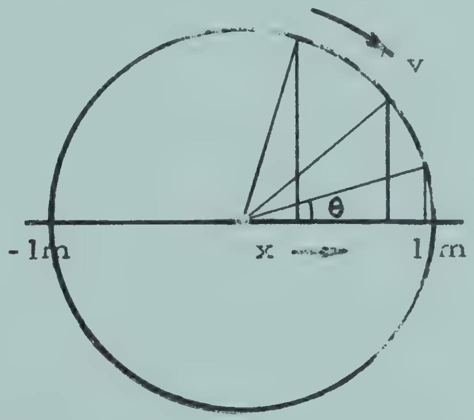
where a is in m/sec^2, and x in m. The period of the simple harmonic motion is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2 \times 3.14 \left(\frac{1}{3} \times \frac{1}{12}\right)^{1/2}$$

$$= 1.05 \text{ sec.}$$

(b) The amplitude R is given as 1 m. We can find the speed by considering the equivalent circular motion, that of a particle moving in a circle of radius m_1, with a period of 1.05 sec.



The speed of the body moving around the circle is

$$V = \frac{2\pi R}{T} = \frac{2 \times 3.14 \times 1}{1.05}$$

$$= 6.0 \text{ m/sec.}$$

When x = o the motion of the actual body is just equal to this, i.e. it is 6 m/sec.

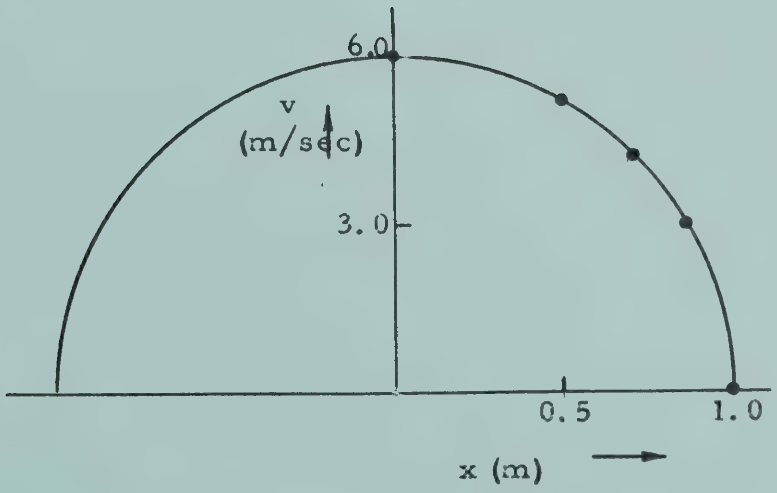
(c) The speed of the actual body is the projection on the x axis of the velocity of the body moving in the matching circle. If the radius vector of the latter makes an angle θ with the x axis, then the speed of the actual body v is

$$v = V \sin \theta.$$

The given values of x correspond to simple values of θ:

x(m)	θ	Sin θ	v(m/sec)
0	90°	1	6
$1/2 = .5$	60°	$\sqrt{3}/2 = .866$	5.20
$\frac{1}{\sqrt{2}} = .707$	45°	.707	4.25
$\sqrt{3}/2 = .866$	30°	.500	3.00
1	0	0	0

(d)



If the vertical scale is chosen so that 6 m/sec, the maximum velocity, has the same length as lm, the maximum distance, the points of (v, x) will lie on a circle with this radius. (Otherwise the curve will be an ellipse.) Because of the symmetry in the problem between x and -x, we can extend the curve to negative values of x without calculating any more points.

The curve is a circle because when x and v are expressed in terms of θ,

$$v = V \sin \theta$$

$$x = R \cos \theta$$

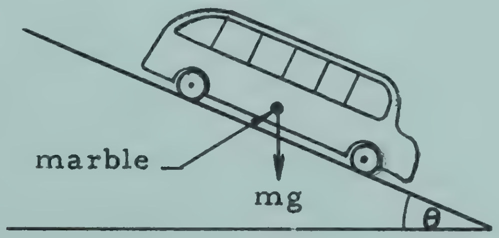
$$\left(\frac{x}{R}\right)^2 + (v/V)^2 = 1. \quad (\sin^2\theta + \cos^2\theta = 1)$$

This is the equation for a circle, when we choose the scales appropriately.

EXERCISE 29: While a bus is in motion along a straight line, a marble rolls from side to side across the floor. Its path is a straight line relative to the bus. Later it rolls again, and this time its path is a parabola which bends toward the front of the bus. Describe the motion of the bus in each case.

This is an exercise in the use of moving frames of reference. The bus is moving in a straight line, so we are concerned with acceleration or deceleration in the direction of motion. We are not given, and must not necessarily assume, that the straight line is horizontal.

We use a frame of reference fixed in the bus. The y axis is parallel to the length of the bus, and the x axis perpendicular to it. The problem is to detect, from observation of the motion of a marble on the floor of the bus, whether or not, and how much, the bus is accelerating or decelerating. (i) The marble rolls in the x direction i.e. it maintains zero velocity in the y direction. There is therefore no acceleration in the y direction relative to our frame of axes. What are the real forces acting on the marble? The only one is the component of the weight parallel to the floor, which is $+mg \sin \theta$. (If we count θ, the angle the road makes with the horizontal, to be positive when the bus goes downhill, and negative when uphill, then the sign of the force is given correctly in all cases.) How can a marble have zero acceleration under action of a real force? We must be in an accelerating frame of reference with respect to the ground, so



This needs to be clarified to say that the bus can accelerate downhill and keeps [illegible] a [illegible] in [illegible]

that there is a compensating "fictitious" force. If the acceleration is a, then this "fictitious" force is -ma. In our frame of reference, if we allow for all forces, real and fictitious, then the equation of motion still holds. The net force is thus

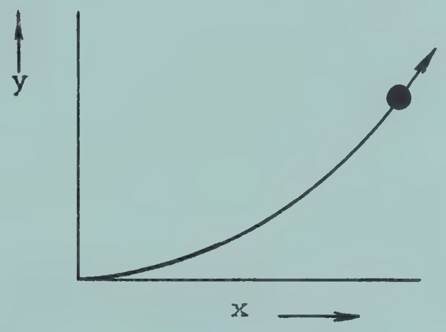
$$mg \sin \theta - ma$$

and this must be zero, since there is no acceleration of the marble in our frame i.e.

$$a = g \sin \theta.$$

A special case of this is $\theta = 0$. If the bus is travelling horizontally, then it must be maintaining a constant speed. Our arguments can be put rather more simply, for this special case. If we are given that the road is horizontal, then the weight of the marble has no component parallel to the floor. We must therefore be in an inertial frame of reference, i.e. there are no fictitious forces. Inertial frames are those travelling with constant velocity relative to the earth.

(ii) The path of the marble is a parabola which bends towards the front of the bus.



This means that in our frame of reference, the marble has a positive acceleration a_1 toward the front of the bus. The sum of real and fictitious forces on the marble must be included in the equation of motion:

$$mg \sin \theta - ma = ma_1$$

where a is the acceleration of the frame of reference (and the bus) with respect to the earth. It is therefore given by

$$a = g \sin \theta - a_1.$$

Once again, there is a whole range of solutions, depending on the slope of the road. If we are given that the road is horizontal, then

$$a = - a_1$$

and the motion of the bus is actually a deceleration of the same magnitude as the acceleration of the marble in the bus frame of reference. Another situation which would give an identical motion to the marble, however, is that in which $a = o$, and $g \sin \theta = a_1$. This is the case when the bus is going downhill at a constant speed, the slope being given by $\sin \theta = a_1/g$.

It is a very fundamental result that if we only make observations on objects in the bus frame of reference, and have no extra information about the direction of the gravitational field, we cannot distinguish between changes in the gravitational field, and variations in the acceleration of our frame of reference.

It may interest students to do experiments which are eventually the same as this, in their own cars. Instead of observing the motion of a marble on the floor of the car, they can use any form of levelling indicator: a spirit level, plumb line etc. They can verify experimentally the result that the levelling indicator will read roughly the same if they are going down a hill with uniform speed (or zero speed) as when they decelerate while on a horizontal road, provided that a in the second case is chosen correctly.

Supplement to Teacher's Guide: Acceleration in circular motion:

Traditionally one of the hardest subjects in elementary physics is the acceleration of a body moving at constant speed in a circle. Developing a clear understanding is important for several reasons:

(1) The blocks to understanding are of a fundamental nature and worth cleaning up in their own right

(2) Uniform circular motion is one of the two cases which can be treated relatively simply in which the vector nature of acceleration is important (The other case is that of a constant acceleration vector).

(3) The text uses circular motion to help verify the vector nature of $\vec{F} = m\vec{a}$. We must therefore be able to deduce $\vec{a}$ (given $\vec{r}$ and $\vec{v}$) independently of statements such as: "The acceleration points inwards along a radius because the force points inwards along a radius".

(4) Alternatively, for planetary motion where there is no "string" visible pulling inwards, we need knowledge of $\vec{a}$ to deduce the presence of the invisible gravitational force,

In the following pages we suggest some steps which should help to isolate the difficulties so that they are met one at a time and to bring them out into the open where they may be attacked.

The Objective

The object is to find the instantaneous acceleration of a body moving in uniform circular motion. From Vol. I, Ch 6, Section 8 we know that the average acceleration, $\vec{a}_{av}$, between times t_1 and t_2 (corresponding velocities $\vec{v}_1$ and $\vec{v}_2$) is

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}.$$

The instantaneous acceleration $\vec{a}$ at any time, t, between t_1 and t_2 is the limit of the average acceleration when the times t_1 and t_2 approach each other and approach t.

We suggest four major difficulties we have observed:

(1) Given two vectors $\vec{v}_1$ and $\vec{v}_2$, how do we take the difference? (this problem is especially hard when both vectors have the same length and we have no help from our incorrect natural desire to subtract the shorter from the longer).

(2) Which vector do we subtract from which? (Do we want $\vec{v}_2 - \vec{v}_1$ or $\vec{v}_1 - \vec{v}_2$? It does not matter as long as we divide by $t_2 - t_1$ or $t_1 - t_2$ respectively, noting that dividing a vector by a negative scalar reverses the direction of the resultant vector)

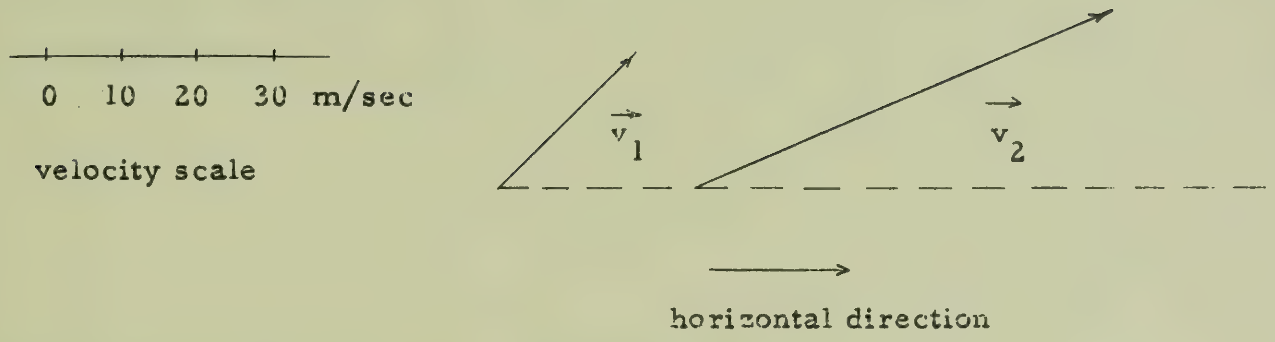
(3) The scale for drawing an acceleration vector may be chosen arbitrarily with respect to that of velocity. In particular, we can not draw any vector without specifying the scale we are using (such as 1 cm stands for 5m/sec velocity). General haziness about scales probably explains why many students can find $\vec{v}_2 - \vec{v}_1$, but do not know the next step to take to draw the acceleration.

(4) What happens when we take the limit of the average acceleration? Both the limiting magnitude and the limiting direction must be found. Students have difficulty seeing that although the vector $\vec{v}_2 - \vec{v}_1$ grows shorter when t_2 and t_1, get closer, the ratio $\frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ approaches a limiting value.

It is essential that students draw some velocity and acceleration vectors themselves. A relatively small amount of work with pencil, ruler, protractor, and compass will be very useful. A good analogy is that one learns algebra after arithmetic. Likewise, the algebraic expressions for vector acceleration are only grasped after some numerical (in this case graphical) examples.

You will undoubtedly think of many sample problems. We shall give one sequence which illustrates what one might do. It would be best if the problems were drawn on ditto, mimeograph, etc. and duplicated for the class.

Example 1:

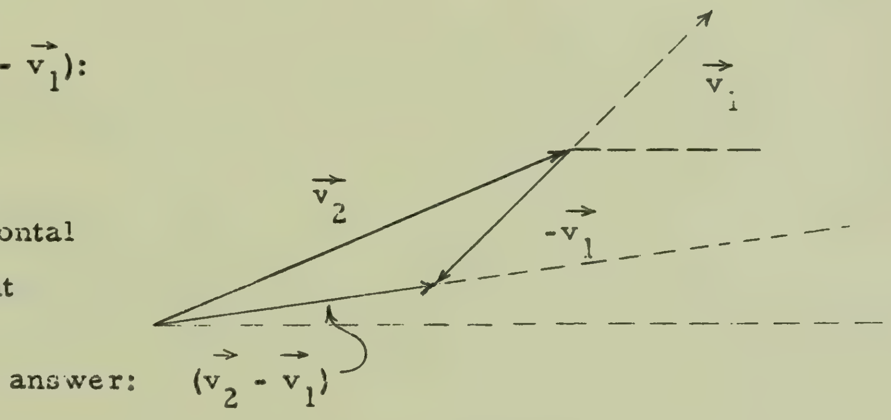


Problem: Find $\vec{v}_2 - \vec{v}_1$ graphically, find its magnitude in meters/sec, and give the direction in terms of an angle clockwise or counterclockwise from the horizontal.

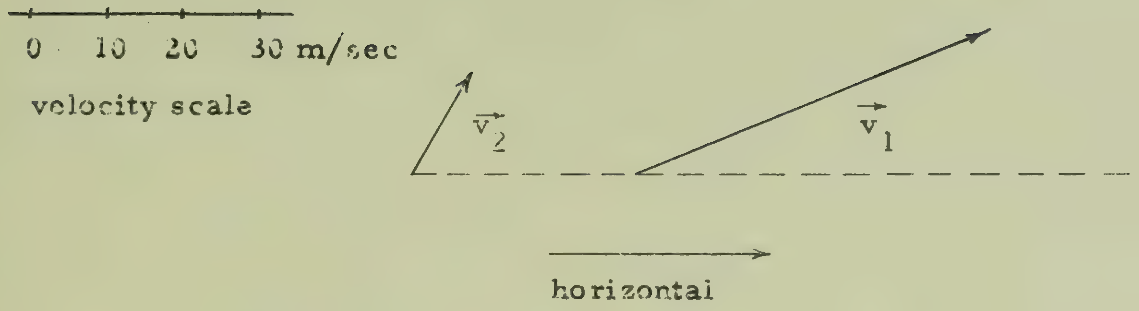
Solution:

magnitude of $(\vec{v}_2 - \vec{v}_1)$:
37 m/sec

direction:
8° above horizontal
and to the right

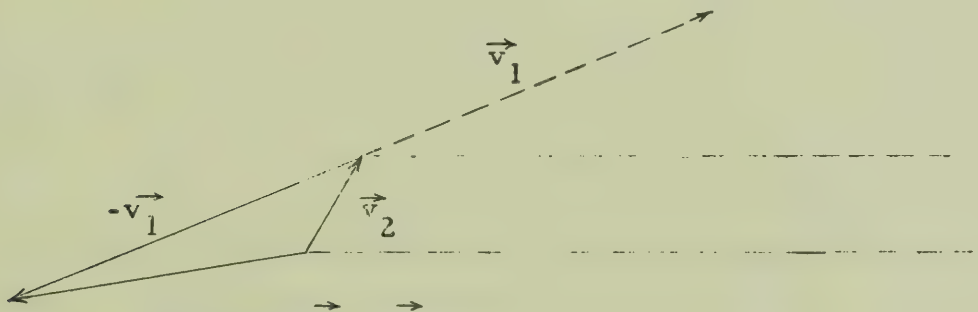


Example 2:



Find the same things as in # 1. (Here since $\vec{v}_2$ is shorter than $\vec{v}_1$, students may be tempted to subtract the wrong way.)

Solution:

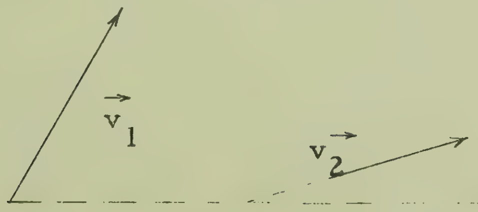


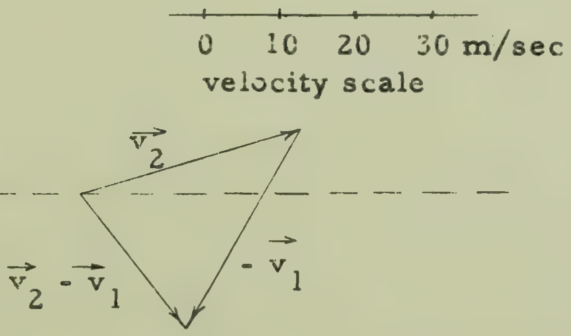
answer: $(\vec{v}_2 - \vec{v}_1)$

magnitude of $\vec{v}_2 - \vec{v}_1$ = 40 m/sec

direction: 8° below horizontal to left

Example 3: Same as #1 or #2 except two vectors of the same length.



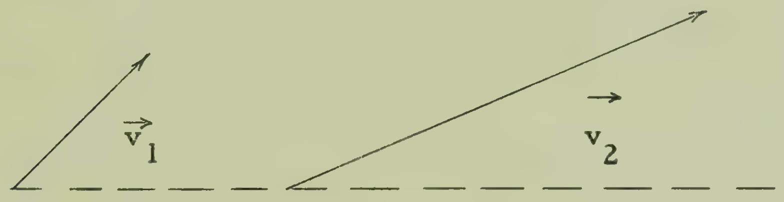


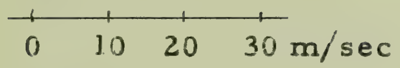
magnitude of
$\vec{v}_2 - \vec{v}_1$ = 22 m/sec
direction: 51° below horizontal to right

The examples so far merely stress the subtraction of vectors.

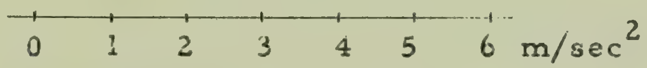
The next element we add is computation of acceleration.

Example 4: Given $\vec{v}_1$ and $\vec{v}_2$ the same as in #1, $t_1 = 2$ seconds, $t_2 = 6$ seconds.





velocity scale



acceleration scale

Find the magnitude of the average acceleration in the interval t_1 to t_2, draw a vector to scale showing its magnitude and direction.

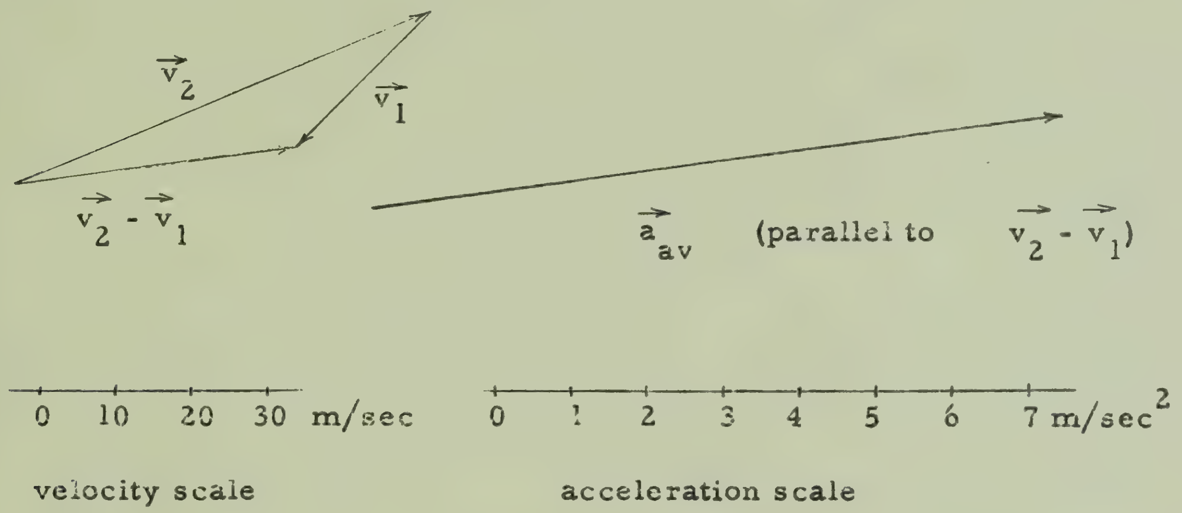
Solution: The average acceleration, $\vec{a}_{av}$, is given by $\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

Its direction is parallel to $\vec{v}_2 - \vec{v}_1$, and its magnitude is found by dividing the magnitude of the vector $(\vec{v}_2 - \vec{v}_1)$ by the time interval $t_2 - t_1$.

From #1 the magnitude of $(\vec{v}_2 - \vec{v}_1)$ is 37 m/sec

$$t_2 - t_1 = 6 - 2 = 4 \text{ seconds}$$

Therefore the magnitude of the acceleration is $\frac{37}{4} = 9.2 \text{ m/sec.}^2$ Choosing a scale of acceleration we plot the vector of proper length, parallel to $\vec{v}_2 - \vec{v}_1$.



It may be necessary to give several other examples to drive home the steps of

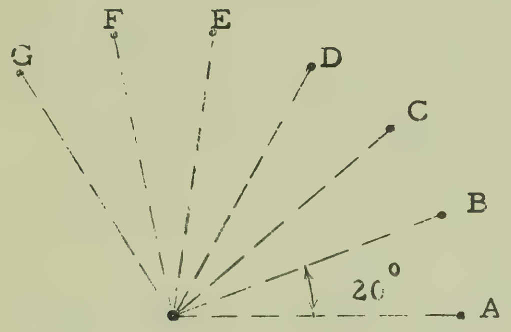
(i) Find $\vec{v}_2 - \vec{v}_1$

(ii) Measure the length of $(\vec{v}_2 - \vec{v}_1)$ and from the scale, determine the magnitude of $(\vec{v}_2 - \vec{v}_1)$ in m/sec.

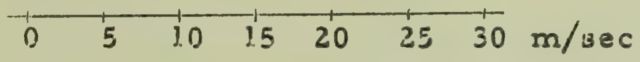
(iii) Divide the magnitude of $(\vec{v}_2 - \vec{v}_1)$ by the time interval $(t_2 - t_1)$ to get the magnitude of the average acceleration.

(iv) Using the acceleration scale to get the length, plot the vector $\vec{a}_{av}$ as parallel to $\vec{v}_2 - \vec{v}_1$.

We now turn to circular motion. We show a circular path with consecutive points A, B, C, D, E, F each separated by 20° of arc. We assume the particle is moving at 10 m/sec and that it takes 5 seconds to go from A to B, B to C, etc.

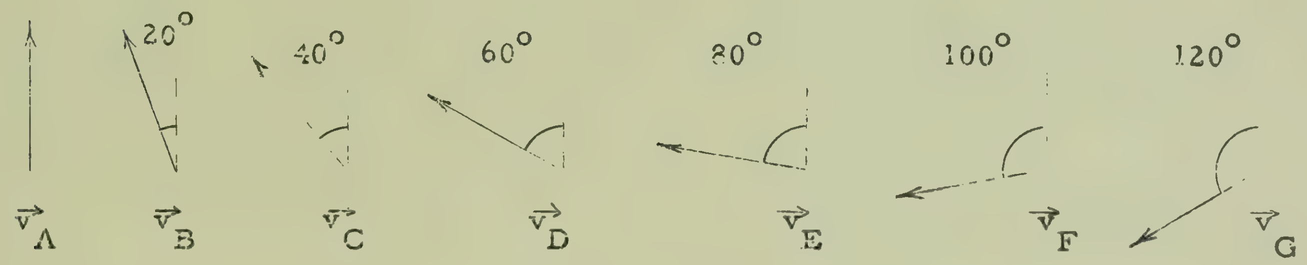


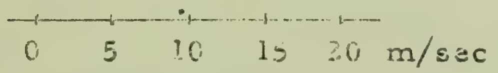
Problem: Plot to scale consecutively across the page $\vec{v}_A$, $\vec{v}_B$, $\vec{v}_C$, etc. where $\vec{v}_A$ is the instantaneous velocity of the particle at point A, etc. Specify the direction of each by a clearly specified angle.



velocity scale

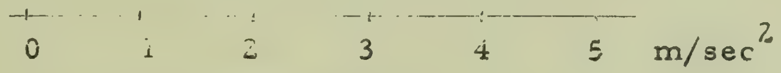
Solution: The velocity vectors are tangent to the path, point in the direction of motion, and have a length corresponding to 10 m/sec on our scale. Thus we have:





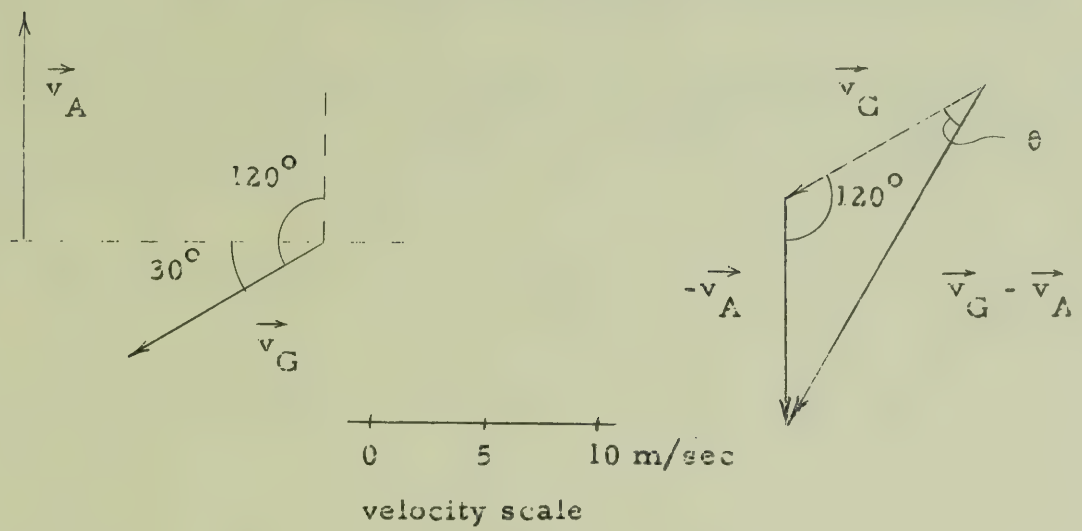
velocity scale

Problem: Compute the magnitude and direction of the average acceleration for the following intervals, and draw them on a clearly labeled figure using the scale indicated: Intervals AG, AE, AC, AB

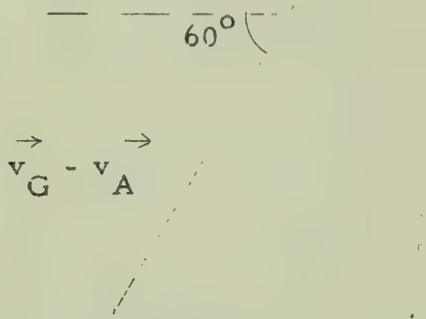


acceleration scale

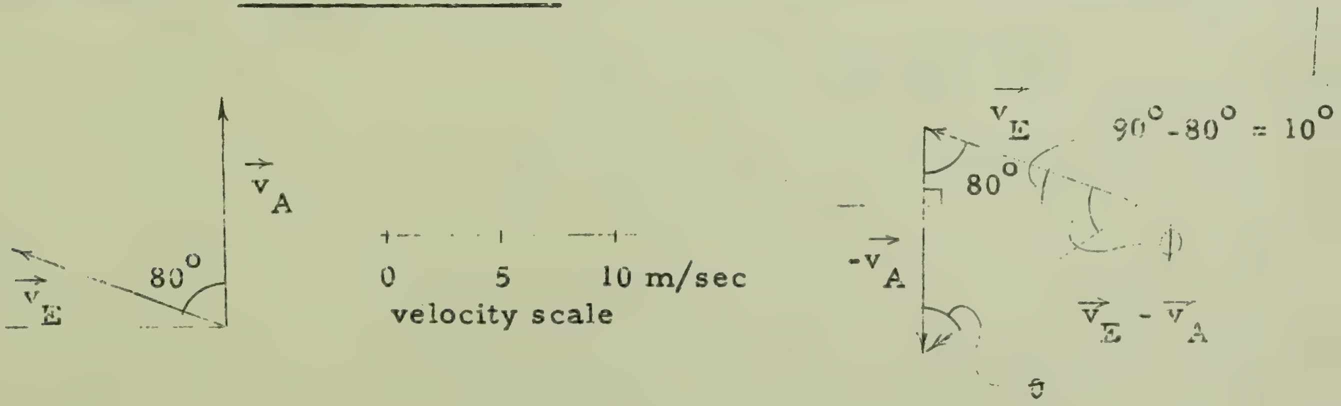
Solution: First we compute $\vec{a}_{av}$ for the interval AG



Measurements show the length of $\vec{v}_G - \vec{v}_A$ is 17.5 m/sec. By measurement or geometry (using the fact that we have an isosceles triangle) we find the angle θ is 30°. Using θ = 30° and since $\vec{v}_G$ points 30° below the horizontal, we see that $\vec{v}_G - \vec{v}_A$ is to the left at 60° below horizontal.

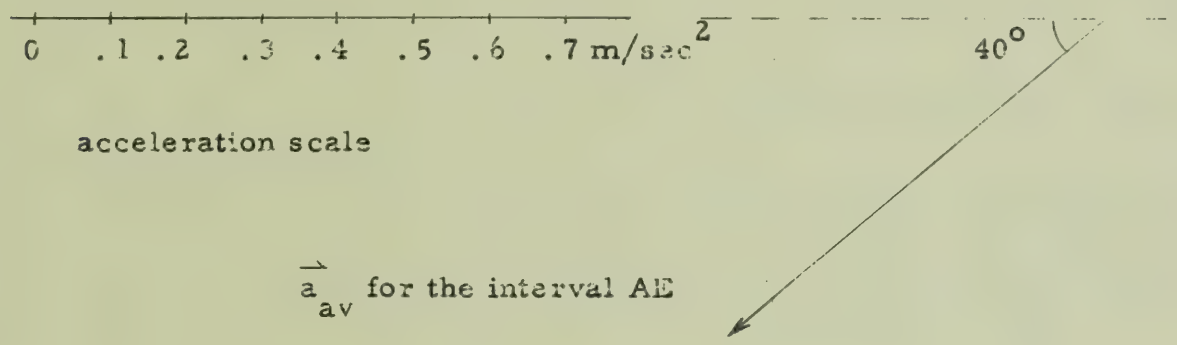


The time interval $t_G - t_A$ is 30 seconds. Therefore the average acceleration between A and G is 0.58 m/sec^2 in the direction 60° below the horizontal toward the left. For the interval AE

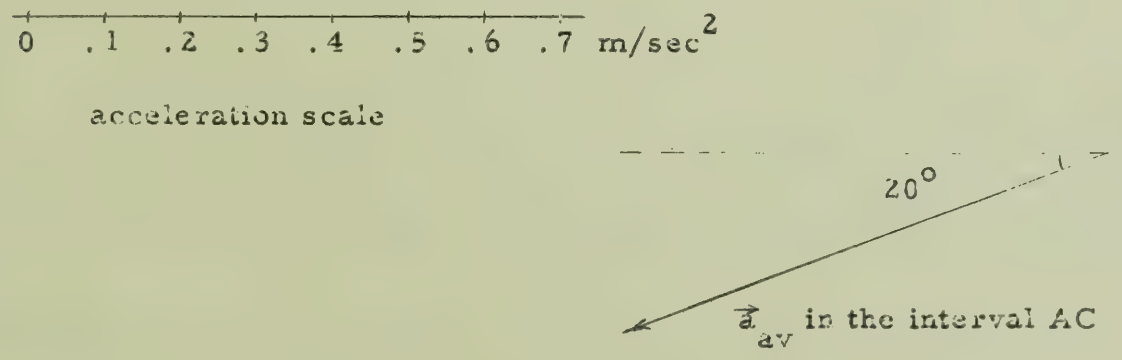


We can either measure the direction of $\vec{v}_E - \vec{v}_A$, or else use geometry. By the fact that we get an isosceles triangle, we see $\theta = \phi \therefore 2\phi = 180^\circ - 80^\circ = 100^\circ; \phi = 50^\circ$ so $\vec{v}_E - \vec{v}_A$ lies at 40° below the horizontal and to the left.

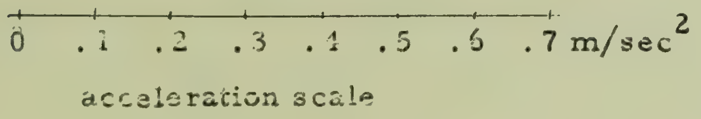
Measuring the length of $\vec{v}_E - \vec{v}_A$ shows its magnitude is 12 m/sec. The time interval is 20 seconds. Therefore the average acceleration has a magnitude of $\frac{12.9}{20} = 0.64 \text{ m/sec}^2$.

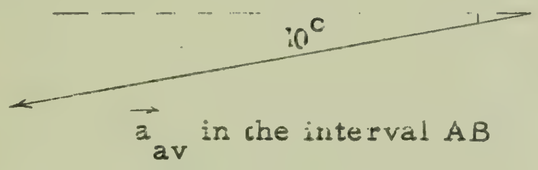

0 .1 .2 .3 .4 .5 .6 .7 m/sec²
acceleration scale
40°
$\vec{a}_{av}$ for the interval AE

For AC the answer is $\vec{a}_{av}$ has a magnitude of 0.68 m/sec² and a direction 20° below horizontal and to the left:


0 .1 .2 .3 .4 .5 .6 .7 m/sec²
acceleration scale
20°
$\vec{a}_{av}$ in the interval AC

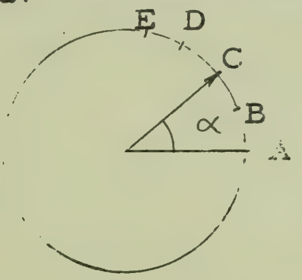
For AB $\vec{a}_{av}$ has a magnitude of 0.70 m/sec^2 and points at 10° below horizontal to the left.





Studying the results we note two things

(i) The magnitude of the acceleration vectors is nearly constant for points which are moderately close.
(ii) The direction of the acceleration vector points parallel but in the opposite sense to the radius vector from the center of the circle to the midpoint of the arc. Thus, in the interval AE, the midpoint is C, and the radius vector from the center of the circle to point C makes an angle α 40° to the horizontal, pointing up and to the right, while $\vec{a}_{av}$ is 40° below horizontal to the left.



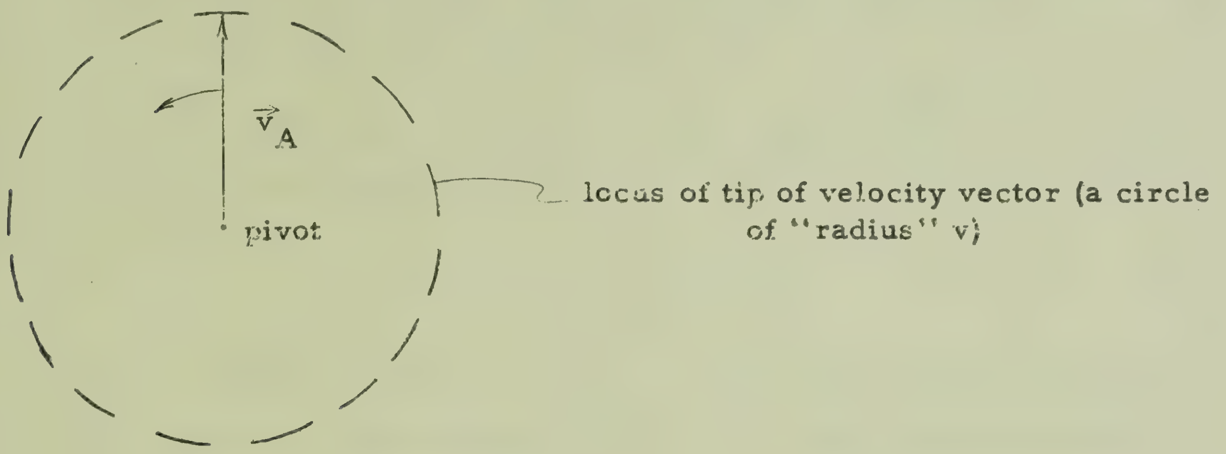
Your bright students may enjoy proving the second relationship.

Notice in particular that it shows as do our drawings that as our two points get closer together, the average acceleration over the interval gets closer and closer to perpendicular to the velocity at any point in the interval. Therefore, the instantaneous acceleration is perpendicular to the instantaneous velocity, and "antiparallel" to the radius vector to its position at that instant.

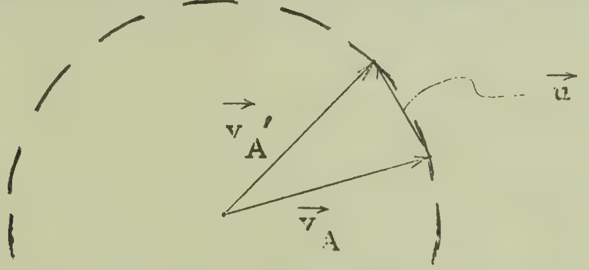
Derivation of v^2/r relation:

The previous problems lead naturally to a derivation of the magnitude of the acceleration. Our sequence of velocity vectors $\vec{v}_A$, $\vec{v}_B$, etc. suggests a

convenient way of describing the velocity by saying that it changes in time as though pivoted at its tail, turning at the same rate as the radius vector (since $\vec{v}$ is always perpendicular to $\vec{r}$)



Let T be the time for the particle to go around the circular path once. It is then also the time for the velocity vector to turn around once. We show the velocity at point A, and at a time, t, a slight bit later, when it is $\vec{v}_A{}'$.



In the figure we have labeled the vector from the tip of $\vec{v}_A$ to the tip of $\vec{v}_{A'}$ as $\vec{u}$. We see that $\vec{v}_A + \vec{u} = \vec{v}_{A'}$, or $\vec{u} = \vec{v}_A{}' - \vec{v}_A$. Therefore the average acceleration in the interval is

$$\vec{a}_{av} = \frac{\vec{v}_{A'} - \vec{v}_A}{t} = \frac{\vec{u}}{t}$$

We wish to compute the magnitude of the vector $\vec{u}$ to get the magnitude of the average acceleration.

Now u is a chord of the circle described by the tip of the velocity vector. We see that for a small enough angle between $\vec{v}_A$ and $\vec{v}_A{}'$ the chord has nearly the same length as the arc. Therefore we approximate the magnitude of $\vec{u}$ the arc length. Since the arc subtends a fraction $\frac{t}{T}$ of the whole circle, its length is $(2\pi v)\frac{t}{T}$.

Thus u, the magnitude of $\vec{u}$, is approximately $\frac{2\pi vt}{T}$. The magnitude of the average acceleration is

$$\frac{u}{t} \cong \frac{\frac{2\pi v t}{T}}{t} = \frac{2\pi v}{T}$$

Since $v = \frac{2\pi r}{T}$, the magnitude of the average acceleration over this short interval is approximately $\frac{2\pi v}{T} = \frac{v^2}{r}$. But as the interval gets smaller, the approximation of replacing the length of the chord with the length of the arc becomes better, so that we say in the limit of short time intervals, the magnitude of the average acceleration, becomes $\frac{v^2}{r}$. We call this the magnitude of the instantaneous acceleration.

PSSC Teacher's Guide Chapter 3 Vol 111

UNIVERSAL GRAVITATION AND THE SOLAR SYSTEM

PURPOSE OF THE ENTIRE CHAPTER: To introduce Newton's Law of Universal Gravitation in its historical context, and to present an account of man's continuing effort to understand the workings of the universe.

CONTENT OF ENTIRE CHAPTER:

a. Early attempts to describe celestial motion by geocentric models.
b. Copernicus' heliocentric model of the solar system.
c. Kepler's kinematic laws for the heliocentric model.
d. Newton's law of universal gravitation, $F = \frac{Gm_1m_2}{R^2}$

EMPHASIS: Compared with other chapters of Volume 3, the substantive content of chapter 3 is slight, at least as measured by the number of "formulas" which students may feel they must learn to use. Because of this, the chapter can be handled largely as a reading assignment with a bare minimum of class discussion and problem solving, without loss in continuity of the material of this volume.

However, an overly rapid survey of this chapter will miss some of the most exciting things in physics. A study of science cannot be divorced from a study of the history of science, nor can an appreciation of the contribution of great scientific figures be achieved without a knowledge of the facts of science. The professional scientists' enthusiasm for his work can be traced in part to his appreciation for the great contributions made by his predecessors in their periods in history. Just as they were able to contribute to the understanding of problems that disturbed them, so does the modern scientist, drawing on their successes and failures, hope to contribute to the clarification of problems of contemporary interest. Mechanics, viewed as a "modern" science, can be deadly dull. However, such problems were of vital interest to our ancestors, and all of modern physics derives from the work of men like Galileo and Newton. Mechanics, viewed from the scientific environment of its great innovators, can be vital and exciting. Hopefully, it is this flavor which your students can derive from the present chapter.

COMMENT: Do not belabor the details of the geocentric models. Some students will grasp the sphere-on-sphere and circle-on-circle motion readily. Others will be terribly confused. The only important point to make is that motion will appear different depending on the choice of the frame of reference - see, e.g., exercise 8 in Home, Desk and Lab.

Students may get the impression that gravitation is a completely settled issue. This is not true. There are a large number of basic questions about gravity which are still unresolved, even years after Einstein's theory of general relativity, such as the following:

1) Why does the gravitational field follow an inverse square law?
2) What is the speed of propagation of the gravitational field?
3) Does gravity theory hold for very large and very small distances?
4) Does the law of gravitation hold everywhere in the universe?
5) Is the gravitational "constant", G really constant, or does it change slowly with time over billions of years?
6) Is the gravitational force between two bodies independent of the distribution of matter in the rest of the universe?

We have no answers to these "simple" questions, or for many others of great fundamental interest. If gravity is a field of investigation which is currently somewhat static, it is not through exhaustion of physicists! It is extremely difficult to conceive and execute experiments to test details of gravity theory on a terrestrial scale, and the economy is not yet ready to support an intergalactic astrophysical year!

PSSC Teacher's Guide Chapter 3 Vol 111

Section 1 - Early Planetary Systems

PURPOSE: To present some of the heliocentric models for the solar system in their historical context.

EMPHASIS: This material can be handled almost entirely as a reading assignment. Do not belabor the details of the heliocentric models, or unduly emphasize the astronomical terminology here and in later sections.

COMMENT: It is impossible to separate a discussion of modern science from its historical roots. Our present day thinking is as much conditioned by the questions raised by the ancient Greeks as by any of the work this century. While the "answers" frequently proposed by the ancients may seem humorous in the light of our modern theory, no scientist can help but marvel at the enormous insight apparent in their questions, nor wonder where we would stand today if their original efforts had been steadily maintained instead of lapsing into disrepute for almost two thousand years.

The weakness in the approach of the Greeks to a study of astronomy, judged by modern standards, was their reliance on intuition rather than on direct observation in their search for reasonable models. This is attested by their insistence on the use of circular orbits for celestial motion, simply because the circle was considered "perfect". No one bothered to study the motion in sufficient detail, as did Brahe centuries later, to show that indeed motion was not circular anymore than anyone bothered to drop two stones of different mass and notice that the heavier did not hit the ground first as propounded by Aristotle. Because of this strong reliance on instinct, it is difficult to separate ancient science from its contemporary philosophy. Even today, basic philosophical questions are frequently as disturbing to scientists as direct observations for which they have no ready explanation.

The notion of a universe composed of bodies moving in "perfect circular" orbits certainly did not originate with Plato. These ideas go back at least as far as the Pythagorean school (580 to 500 B. C.) more than a century earlier. Indeed, at least two members of the Pythagorean school. Hicetas and Ecphantus, proposed a fair model of what we generally call the Copernican view of the solar system, with an earth which rotates daily on its axis and an immobile sphere of fixed stars. However, in this model, the sun and moon were assumed to move in circular orbits about the earth. Plato believed firmly that the earth stood still, but even in his day, Heraclides taught that the earth rotates on its axis once a day, and even suggested that Venus and Mercury moved in circular orbits about the sun while the latter revolves about the earth. The accepted "Copernican" model was actually probably first stated by Aristarchus of Samos (ca. 310 to 230 B. C.), who proportedly published the following hypothesis: "The

cleides

fixed stars and the sun remain unmoved, but the earth revolves about the sun in a circumference of a circle, the sun lying in the middle of the orbit." The basic suggestion for the Ptolemaic system undoubtedly arose from the work of the great ancient astronomer, Hipparchus, who died about 125 B. C. Hipparchus, unlike many of his predecessors and successors, was a great observer and made very careful measurements of celestial motion. He detected small variations in the plane of the ecliptic due to precession of the earth's axis, deriving a figure which agrees within one percent of its modern accepted value. This motion is so small and slow that it takes about 26,000 years for its complete cycle.

The heritage of Greek science was advanced by the Arabs, after their conquest of the Egyptian city of Alexandria which had become the principle center of Greek learning by the third century B. C. In fact, during the middle ages, the Ptolemaic system returned to Europe via translations from the Arabic, together with star catalogues complied by Arab astronomers.

PSSC Teacher's Guide Chapter 3 Vol 111

Section 2 - Copernicus' Planetary System

PURPOSE: To provide a brief description of the Copernican model of the solar system.

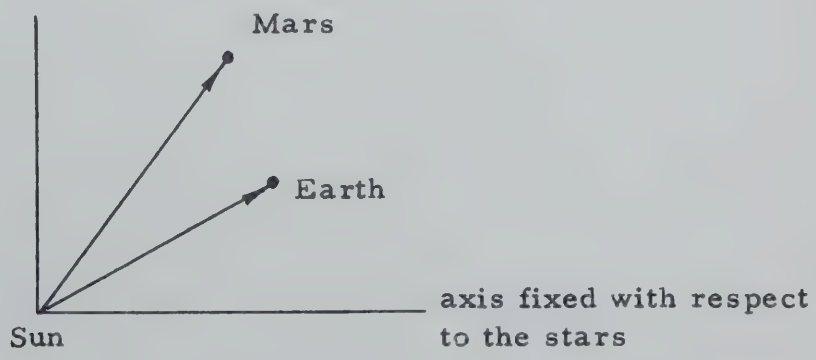
EMPHASIS: Treat briefly, as part of a longer reading assignment.

COMMENT: As noted in the discussion of the last section, the idea of a heliocentric solar system certainly did not originate with Copernicus. He himself was undoubtedly familiar with the model earlier proposed by Aristarchus. It is perhaps a wry commentary that despite the fact that he expounded ideas already some two thousand years old, his work was branded as heretical.

As discussed in Volume I, it is common knowledge that the familiar phenomenon of parallax should be noticeable when making observations from the moving system. Copernicus was concerned about this point, and the fact that he could not observe stellar parallax, particularly since, in his notion, the sphere of fixed stars was fairly near the earth. Copernicus, of course, had no idea of the enormous distances actually involved, and the fact that stellar parallax is far too small to measure by simple means. Actually, it was not until the year 1838 that stellar parallax was first noted by the German astronomer, F. W. Bessel. Remembering that the largest parallactic angle ever observed is 0.756 seconds of arc (for the nearest star, alpha Centauri), it is easy to see why the effect was so long unobserved. The correct notion of the almost infinite distance between the earth and the stars was probably first proposed by Giordano Bruno in the 15th century.

The description of planetary motion and the equivalence of the different descriptions (geocentric, heliocentric etc.) is greatly aided by the use of vectors. Figure 1 shows the radius vectors from the sun to the earth and from the sun to Mars. The axes are fixed with respect to the stars.

Figure I



It is a good approximation to say the planets move in circles about the sun at constant speed. Then in our diagram the two radius vectors $r_{\text{sun to earth}}$ and $r_{\text{sun to Mars}}$ simply turn about the sun with unchanging length. This is just the Copernican view.

To locate a planet such as Mars with respect to the earth we simply note $r_{\text{earth to Mars}} = r_{\text{earth to sun}} + r_{\text{sun to Mars}}$ as shown in Figure 2a and Figure 2b.

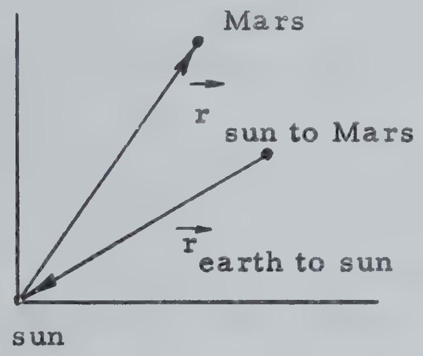

Mars
$\vec{r}_{\text{sun to Mars}}$
$\vec{r}_{\text{earth to sun}}$
sun

Figure 2a

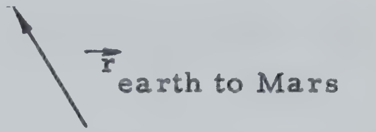


Figure 2b

We see the vector $r_{\text{earth to Mars}}$ changes as both planets move just as if the earth and Mars were joined by a rotatable joint at the sun. A model of such a mechanical linkage is easy to make with two sticks and a nail. Note that the vector from sun to earth must make one revolution relative to the reference axes a year, and that from sun to Mars one revolution in a Martian year (687 days). Students could be asked to describe the motion of the moon about the sun by such a method (using the vectors from sun to earth and earth to moon). As a more complicated example with three "rods", they could describe the motion of the earth's moon with respect to Mars.

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Sections 3, 4 - Projectile Motion

PURPOSE: To derive the equations of motion for a body moving with an arbitrary initial velocity in a uniform gravitational field.

CONTENT: (a) The (vertical) acceleration of a freely falling body due to gravity is experimentally found to be independent of the state of motion of the body, thus, the vertical motion of the body is independent of its horizontal motion.

(b) The horizontal motion of a freely falling body is independent of its vertical motion.

(c) We may solve for the motion of bodies projected with an arbitrary initial velocity $\vec{v}_o$, by considering separately the vertical (y) and horizontal (x) components of the motion. Denoting the vertical component of $\vec{v}_o$ as v_{oy}, and the horizontal compondent by v_{ox}, and taking the initial position as y = o, x = o, (y is positive for upward displacements)

$$y = v_{oy}t - \frac{1}{2} g t^2,$$

$$x = v_{ox}t,$$

in the absence of air resistance. From the second equation, $t = \frac{x}{v_{ox}}$

Substituting in the first equation,

$$y = \frac{V_{oy} X}{V_{ox}} - \frac{1}{2} \frac{g X^2}{(V_{ox})^2} ,$$

and the path of the projectile is a parabola.

APPROPRIATE LABORATORY: Experiment 490

EMPHASIS: Important material, treat fully. It is best to consider first cases with only an initial horizontal velcoity ($v_{oy} = 0$), before treating the general case. Stress the vector nature of forces, velocities and displacements.

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Section 4 - Tycho Brahe

PURPOSE: A brief account of the great observations of Brahe.

EMPHASIS: Treat as part of a reading assignment.

COMMENT: In evaluating the work of Copernicus, it is important to remember that, prior to Brahe, there actually existed no really precise data on the motions of celestial bodies. Brahe, who was no believer in the Copernican theory, was probably motivated as much by a determination to disprove Copernicus' model as by anything else. To him, one of his great achievements was the failure to detect stellar parallax, even with his highly accurate observations, thereby "disproving" the Copernican model.

At the same time, Brahe was not willing to accept the enormous complexities of the Ptolemaic model and developed a geocentric system of his own. In Tycho's system Mercury and Venus were assumed to revolve around the sun, while the sun and other planets revolved about the earth (similar to the model of Heraclides).

e

Section 5 - Kepler

PURPOSE: To introduce Kepler's laws of planetary motion in their historical context.

EMPHASIS: At a minimum students should "know" Kepler's three laws. Some class time should be spent on discussion and exercises here.

COMMENT: While Kepler certainly was seeking an explanation for the elliptical orbits which he described, the concept of a force was not well defined in his time (although he and Galileo were contemporaries), and Kepler had no success in explaining his own model.

* * *

It should be pointed out to students that Kepler's first and second laws are not equivalent. The first law describes the elliptical path followed by a planet, but says nothing about the speed of the planet at various points along the path. This information is furnished by the second law. The first two laws, while accounting for the regularities of motions of individual planets, do not relate the motions of different planets to one another. It is the third law of Kepler which stated the universality of the rules of planetary motion, and laid the groundwork for the later work of Newton.

Thus Kepler's laws are not merely a summary, in concise form, of the observations of Brahe, but essentially permit prediction of the future positions of the planets relative to one another.

Section 6 - Kinematic Description and the Dynamical Problem

PURPOSE: To emphasize that the difference between the geocentric and heliocentric models was, from the point of view of kinematics, merely a change in the frame of reference. However, when the dynamics of the motion is considered, the heliocentric system becomes highly preferable.

EMPHASIS: Treat briefly, but be sure students understand the effect of the choice of frame of reference on observed motion. Exercise 8 at the end of the chapter is highly appropriate for class discussion at this point.

COMMENT: There is a bit of philosophy expressed in the last paragraph of this section to the effect that the simplest model is probably correct from the dynamical point of view. While this moral is appropriate to the present discussion, it can hardly be classified as a generally accepted principle in physics.

* * *

A key sentence in the whole chapter is in the next to last paragraph in this section: "In the second place, we know that Newton's laws of motion hold only in certain reference systems." This point is so important and presents such a dangerous potential trap for the teacher that we shall amplify. You should not attempt to bring out the points below in any detail until the students have completed Section 8 in which the law of gravitation is given. Problem 21 is also pertinent.

Let us think of the earth and the sun, and let us assume we know the law of gravitation from the Cavendish experiment. If we say the earth goes about the sun with velocity v in a circle of radius R, we have from Newton's laws

$$\frac{mv^2}{R} = \frac{GmM}{R^2} \qquad \text{Eq. 1}$$

Where m is the mass of the earth and M the mass of the sun. If we attempted to say the sun goes about the earth, we would say

$$\frac{Mv^2}{R} = \frac{GmM}{R^2} \qquad \text{Eq. 2}$$

which is inconsistent with Eq. 1. We choose between these by experiments.

That is, Newton's laws do not hold with respect to all reference systems. It is a matter for experiment to decide whether any given reference system is one in which Newton's laws are obeyed.

* * *

An important philosophical point which we shall raise but not attempt to answer is the following: Why are some reference systems "Newtonian" (i.e. reference systems with respect to which Newton's laws hold) and others not. Thus, if we think of the solar system as being far away from all other matter, why should one set of coordinate axes fixed with respect to distant stars, by preferred over another which turns with respect to the stars. The opening paragraphs of Einstein's first paper on general relativity are concerned with this paradox.

Section 7 - Newton

PURPOSE: To present an account of Newton's deduction of the law of universal gravitation.

EMPHASIS: This section can be handled largely as a reading assignment, but the students will benefit by learning a bit about the life of Newton and some appreciation for the magnitude of his constrbutions to physics.

COMMENT: Newton was almost certainly led to his statement of the law of universal gravitation, as noted in the section, by consideration of Kepler's third law. It is easy to show that Kepler's second law, the law of equal areas, does not define an inverse square force uniquely. It can be shown in general that Kepler's second law would follow for any central force, i.e. a force directed along the line joining the centers of two bodies, no matter what the variation of the force with distance between the two bodies.

Section 8 - Universal Gravitation

PURPOSE: To introduce the law of universal gravitation, $F = G \times \frac{m_1 m_2}{R^2}$.

EMPHASIS: Students should know how to handle a wide variety of problems involving the law of universal gravitation. This is an important section, and should receive adequate class discussion. It is essential that students attempt quantitative exercises at this point, such as those given at the end of the chapter. A good chance for drill in scaling, ratio, and proportion, using the inverse square law.

COMMENT: Newton guessed that the gravitational force between two objects was proportional to the product of their inertial masses. The Cavendish experiment verifies the guess. Einstein, in this theory of general relativity, took this assumption of Newton to be a fundamental fact of nature, and asserted that it is inherently impossible to distinguish the effects of acceleration from those of a gravitational field (see supplement at the end of comments on Section 11 of this chapter).

What is generally referred to as Newton's third law, the law of action and reaction, appears naturally in the statement of the law of universal gravitation, since the force is unchanged by interchange of the masses, m_1 and m_2. Obviously then, the force on m_1 due to m_2 must be equal and opposite to that on m_2 due to m_1. While the law of action and reaction will not be discussed much in the present chapter, it is good to emphasize this symmetry of the law of universal gravitation and the implication of the third law. The third law, of course, is completely general, since it applies to all forces, gravitational and otherwise.

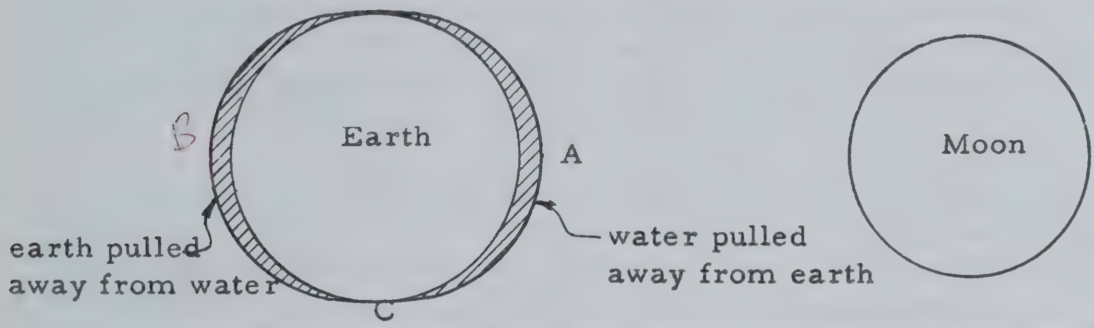
Note the wonderful quotation from Newton beginning "and the same year . . .". Students should appreciate how young he was (24) when he made these important discoveries. In physics, many of the most important discoveries have been made by men in their twenties or early thirties. Einstein discovered the special theory of relativity at age 26.

Section 9 - Some of Newton's Later Accomplishments

PURPOSE: To complete the story of Newton's contributions to our understanding of gravity.

EMPHASIS: Treat briefly, as a reading assignment. Interested students should be encouraged to do some outside reading about Newton.

COMMENT: Students living in the middle of the continent may not be familiar with tides. Although the details of tidal motion are complicated by local variations in the shore line and ocean floor, the main features of this motion are caused by gravitational attraction of the moon for the earth. This phenomenon is illustrated below. (Scale enormously exaggerated).



Now that students know that the force between the moon and the earth results in an acceleration, they may be able to better understand the tides. Essentially, the moon and earth freely fall toward one another. The acceleration of water at A toward the moon is greater than that of the earth (or water at C). The acceleration of the earth toward the moon is greater than that of water at B. Thus the moon's attraction produces two bulges in the ocean on opposite sides of the earth. Since the earth rotates once in 24 hours, there are two high tides per day at any ocean location, although they may not be of exactly equal height. While the influence of the sun must also be considered in calculating the exact size of the tides, its effect is considerably smaller than that of the moon.

Note that the force exerted by the sun upon a unit mass anywhere on the earth is about 100 times greater than the corresponding force exerted by the moon. Nevertheless, the tidal effect of the moon is about twice that of the sun. This is because the difference between the force of the moon on a unit mass on the earth's surface at A and on another at C, is more than twice as great as the difference in the sun's force at these points (this is because the fractional change in the distance from the moon to these points is almost 400 times greater than in the distance from the sun to these points.

resonance

The highest tides occur about twice a month at about the new moon and full moon, when the sun, earth, and moon are all in positions along the same line. Tides may vary considerably in magnitude from place to place along a coast, because of details of the local topography, the largest tides are most frequently seen in enclosed bays, which have a funneling effect. In the Bay of Fundy, for instance, the difference between high and low tide is sometimes over 50 feet.

Newton himself wondered whether the gravitational force due to a spherical mass could be calculated as he had assumed, by considering that all of the mass was essentially concentrated at a point at the center of the sphere. It was only after the invention of the calculus that he was able to prove this fact to his own satisfaction, as well as show that planets would follow elliptical orbits. The proof of this relationship, using only forces which are vector quantities, is fairly formidable and will not be given here. It is only true because of the inverse square distance dependence of the force due to a point particle. One can also show that bodies other than spherical cannot be replaced by a point mass in calculating their gravitational fields. The general gravitational interaction between bodies of arbitrary shape is extremely complicated.

To first approximation, the acceleration of gravity is constant over the surface of the earth. To second approximation, it varies from point to point on the surface because of variations in height of the terrain and the fact that the earth is not exactly spherical. To third approximation, the acceleration of gravity varies with the distribution of masses in the earth's crust at the particular location. This fact is of great importance to geologists, since hidden features of the earth's crust may be detected by accurate measurement of local variations in g. If g is measured over a large underground cave, which may be filled with oil, it will have a smaller value than if measured over a similar region of solid rock, and a still smaller value than if measured over a large deposit of lead or gold. The value of g also depends on the rotation of the earth, as discussed in exercise 21.

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Section 10 - Laboratory Tests of the Law of Universal Gravitation

PURPOSE: To emphasize that the gravitation law can be verified in the laboratory and to describe the method used by Cavendish to determine the gravitational constant G.

EMPHASIS: While students should have a feeling for the order of magnitude of G, and a qualitative appreciation for Cavendish's experiment, this section can be treated briefly.

Check with Hurley

OK

COMMENT: Cavendish's experiment enabled him to find the mean density of the earth, and determined that this was about 5 1/2 times that of water. Since the surface rocks have a density of only 2 to 3 times that of water, the interior of the earth must obviously be made of a material of higher density than the surface crust on which we live. The molten basalt, or lava, which underlies much of the earth's crust is itself far too light to account for the measured density of the earth. A good deal of the earth's mass apparently resides in a relatively small heavy "core" at the center of the earth, of density about ten times that of water. This core is presumably composed of metals like iron and nickel rather than rocks. The core, as determined by seismic measurements, is essentially spherical in shape and about 2,000 miles in radius. Actually, the core itself is apparently not uniform in density but shows a distinct rise in density, increasing as the center of the earth is approached, starting at about 1,000 miles from the center. At the center of the earth the density is about 16 times that of water.

Section 11- A small Discrepancy

PURPOSE: To point out that Newton's Law of Universal Gravitation is not exact.

EMPHASIS: Qualitative material - can be treated largely as a reading assignment.

COMMENT: As noted in the comment at the beginning of this chapter, gravity is not a closed story as far as physics is concerned. There are many aspects of the gravitational field which are poorly understood, if understood at all. Students should not be left with the impression that we know all there is to know about this phenomenon just because we happen to have a law of force which correctly describes the observed behavior of large objects in a gravitational field. In just the same way as the dynamical description of a force forms an important step beyond the kinematic description of the motion, so also does a microscopic description of the cause of the force bring us closer to an understanding of the phenomenon.

Einstein's theory of general relativity does not in itself tell us anything about the cause of the gravitational force. This theory, which is described briefly in the following supplement, is essentially an extension of Newton's law which enables a correct quantitative description of all observed gravitational effects. You may wish to use some of the material in the supplement to tell your students a little bit about general relativity, which because of its mathematical complexity, is usually avoided like the proverbial plague in all elementary courses in physics.

Supplement on General Relativity

According to the basic ideas of the dynamics of Galileo and Newton, the laws of motion are only valid in a so-called "inertial frame", that is, a system which is at rest or moving with constant velocity. These two possible states are also clearly undistinguishable, since forces are only measurable when a body is accelerated. Everyone knows that Newtonian dynamics begins to break down for bodies moving with speeds near that of light, but Einstein's theory of special relativity has satisfactorily extended the basic notions of classical dynamics to the range of all possible speeds.

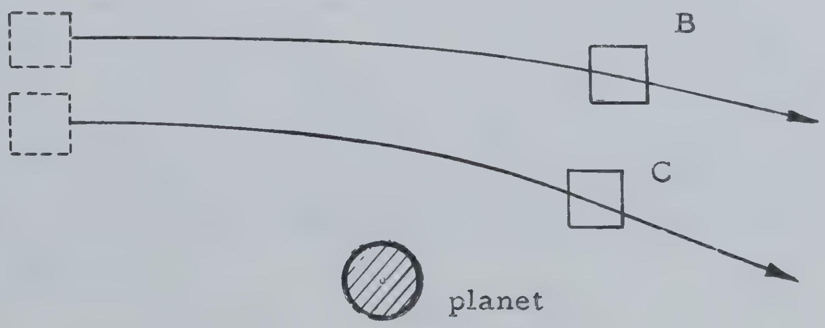
The existence of gravity, however, complicates matters considerably. We know, of course, that the surface of the earth is not an inertial frame in the Newtonian sense. If we accelerate a body upward, it does not continue to move up but reverses direction and falls back to the floor. We generally describe the situation by saying that we are in an inertial frame, but that there is a force--namely the "force of gravity"--also acting on the body. Indeed, it is evidently quite impossible to isolate any body from the gravitational forces exerted by the mass of the rest of the universe. Thus, the definition of an inertial frame in terms of a state of "rest" or "uniform motion" is not particularly meaningful in a universe which contains mass, since all bodies in the universe are accelerated with respect to one another.

Einstein, in his theory of general relativity, has attempted to restate the laws of physics in a manner consistent with this fact, namely as they would appear to an observer subject to acceleration. This thoery is, as is well known, of enormous mathematical complexity, and it is hardly to be recommended as an exercise for high school students. However, the basic concepts of the theory are simple enough to see by considering a few simple examples.

Suppose we were drifting along in space in a rocket ship in the popular "weightless" condition which every student of the funny papers knows about. In this state we could perform dynamical experiments on ourselves and the surrounding objects floating in our room and quickly decide that we were in a true inertial frame, since the laws of motion would be obeyed exactly. If we jumped up from the floor, we would most certainly hit the ceiling, no matter how gently we jumped. Now imagine, during the course of these experiments, that this "weightless" interlude suddenly comes to an end. We would find ourselves standing on the floor, feeling our own weight. All of the surrounding objects which a moment earlier had been floating around the room would suddenly fall to the floor. What could we conclude had happened?

Without looking out the window of the rocket ship (this, incidentally, wouldn't really help), we could conclude that one of two things must have happened. Either our rocket ship has suddenly accelerated upward (relative to the floor) with an acceleration of about g, or we have just come to rest (with the floor "down") on the surface of the earth or some similar planet. In either event, it would be obvious that the laws of physics had suddenly changed and we were no longer in an "inertial frame". We would be hard put to devise an experiment to tell whether we had in fact been accelerated or had entered a gravitational field, since either occurence would have had a similar effect on our observations. This is precisely what we mean when we state that the "inertial mass" of a body and its "gravitational mass" are the same. It is this "principle of equivalence" which lies at the heart of general relativity--that is, that the effects of an acceleration are in principle undistinguishable from the effects of a gravitational field. But what about our inertial frame? We had a perfectly good way of measuring this earlier, simply by observing the validity of the laws of motion. Evidently, during this time interval, when our rocket ship was an "inertial frame", we must have been in uniform motion in the absence of a gravitational field (impossible in our universe), or in "free fall" in a gravitational field. Thus, the true inertial frame is not one which is in uniform motion in our space, but rather one which is in free fall.

General relativity, then, is simply the description of nature as viewed by an observer in a true inertial frame--patiently doing experiments in a freely falling box. To this observer, the laws of dynamics would be exact within his box, but he would quickly conclude that the space outside his box had some very peduliar properties. To illustrate, imagine that we are in free fall in such a box, moving at a fairly high speed through space. Through our window we observe another person moving along parallel to us at the same speed and also doing dynamical experiments. We could compare experiments by flashing messages back and forth and agree that our results were identical and both of us were in inertial frames. If we then arrive in the vicinity of a large planet (in such a manner that we don't hit it--in that case the experiment is obviously concluded!) both boxes would be deflected by the gravitational field, as shown below.



Our box, labeled B in the figure, would be deflected somewhat and the other box, labeled C, since it is closer to the planet, would be deflected rather more. Since both we and the observer in C are in free fall, neither of us would feel or be able directly to measure any effects due to the acceleration, within the confines of our own boxes. We are both still in inertial frames as far as our internal measurements can tell us. However, since we are now obviously drifting apart, we would have to conclude that the distance between us suddenly started to increase without either of us apparently having accelerated. Instead of describing what has happened in terms of some "force" which we could not detect, we might rather describe the occurence in terms of a distortion in space and time which we could observe.

This is the essence of the theory of general relativity. The gravitational field, according to this viewpoint, causes a "warping" of space and time in the vicinity of a mass. Gravity then enters not as a "force" but as a property of space and time.

This whole procedure would be rather fruitless if it did not permit us to extend the laws of physics and to predict certain observable events. In fact, general relativity predicts certain facts which are not correctly explained either by Newtonian dynamics or special relativity. One of these involves the curvature of light in a gravitational field. In general relativity, the "shortest distance between two points" is not a straight line but is a "geodesic", the path which light would follow in traveling between the two points, bent by the gravitational field. According to special relativity, light has energy, and since mass and energy are equivalent, light will be deflected by a gravitational field. However, the amount of deflection is not given correctly by this simple argument, because the force of gravity on a moving object depends on its speed. We can actually observe the bending of light traveling from distant stars which passes very close to the sun during a solar eclipse. The effect is very small, about two seconds of arc, but it is accurately measurable, and agrees precisely with the predictions of general relativity. General relativity also predicts that the time scale should be affected by gravity in such a way that a perfect clock, when it enters the gravitational field of a massive body, would appear to start running slower to an observer far from the clock, but would appear to be keeping perfect time to an observer moving with the clock. This change in time scale can be verified by observation of the "red shift" in the spectra of light from extremely massive stars relative to that from stars of smaller mass. Spectra from the massive stars are found to be shifted slightly toward longer wavelengths corresponding to a decreased frequency and hence a longer time scale. This effect is due entirely to the mass of the stars, quite independently of the relative motion of the stars and the observer.

The theory of general relativity is of considerable importance in physics, but is more the working tool of the cosmologist in his study of the universe. The cosmologist is concerned with such questions as "Is the universe finite?"--that is, would a geodesic directed outward from the known universe go on forever, or traveling always out, return to its starting point? Much of the effort of modern astronomers is directed toward answering this question. At present, we do not yet know whether the space we know is finite or infinite, but it is certainly "warped", in the relativistic sense.

For Home, Desk, and Lab

GENERAL: Much of the material of this chapter is largely of historical interest, and will not require intensive drill. However, students should at a minimum be expected to "know" the Law of Universal Gravitation (section 8), and be able to solve numerical problems using this law. The exercises for this chapter contain a large number of examples of the use of this law, and some of them should be assigned. The following table may be useful.

Section	Easy	Medium	Hard	Class Discussion	Laboratory
1			1, 2		
5	6	5, 7		5, 7	Exp. 510
6	3, 8	4		4, 8	
8	10, 14*, 16, 17, 18	11, 12, 13, 15	24, 25	10, 11, 12, 16, 25	
general	19, 23	9, 20, 21, 22		19, 21	

*Exercise 14 should be preceded by Exercise 13.

For Home, Desk, and Lab

EXERCISE 1: Observe the position of one of the planets with respect to the fixed stars once or twice a week for a month or two. Begin by plotting a map of the stars surrounding the planet. Then mark the new position of the planet on this map at each observation.

A home exercise for ambitious students. Students who try this would be well advised to construct a crude sextant to obtain reasonably quantitative results. Otherwise, it would be best to start with a scaled "star map" of the region of the sky under study.

EXERCISE 2: The four largest moons of Jupiter can be seen through binoculars. Plot their positions, relative to the planet, every clear night for about three weeks. See if you can determine their periods. The moon closest to Jupiter moves so fast that its motion relative to Jupiter can be easily detected by observations just a few hours apart.

- Indicate nomi
di
stelle
maggiori
Notare
maggiori
etc.

Another home chore for amateur astronomers equipped with reasonably good binoculars (inferior ones may have too much spherical and chromatic disturtion to permit resolving the moons). The binoculars must be mounted on a rigid support for this observation. Remind students that Galileo made this measurement with optical equipment far more inadequate than a pair of commercial binoculars!

Jupiter actually has eleven satellites. The four largest of these, Io (I in following table), Europa (II), Ganymede (III) and Callisto (IV) were discovered by Galileo in 1610. The smallest of the four, Europa, is about the size of our moon, while the two largest, Ganymede and Callisto, are larger than the planet Mercury. Pertinent information about the satellites is given in the following table:

The Satellites of Jupiter

Designation	Discovery	Distance from Jupiter (Mi)	Diameter (Mi)	Period of Rev.
I	Galileo, 1610	261,000	2318	1 d 18 h 28 mi.
II	" "	416,600	1967	3 d 13 h 14 mi.
III	" "	664,400	3200	7 d 3 h 42 mi.
IV	" "	1,168,800	3218	16 d 16 h 32 mi.
V	Barnard, 1892	112,600	100 ?	0 d 11 h 57 mi.
VI	Perrine, 1904	7,114,700	80 ?	250 d 17 h
VII	" 1905	7,288,600	25 ?	260 d 1 h
VIII	Melutte, 1908	14,600,000	15 ?	739 d
IX	Nicholson, 1914	14,900,000	15 ?	745 d
X	" , 1938	7,300,000	15 ?	270 d
XI	" "	14,000,000	15 ?	735 d

The seven small satellites are far too small to be observed with any but the largest astronomical telescopes (and even then, under very favorable conditions). The outer satellites, in fact, are so small that they could not even be seen from Jupiter without a telescope!

EXERCISE 3: What is the rate of rotation of the sphere of the fixed stars as seen by someone on the moon? What is that of the earth about the moon as seen from the moon? What is the rate of rotation of the sun as seen from the moon? Remember that we always see the same side of the moon.

Question relating to frames of reference. (Note that the period of rotation of the moon, 27.3 days, is the same as its period of revolution about the earth - thus we always see the same side of the moon.)

The sphere of fixed stars appears to rotate at a rate given by the period of rotation of the body on which the observer is situated. Thus, for an observer on the moon, the fixed stars rotate once every 27.3 days, the earth would not appear to rotate, and the sun would appear to rotate every 27.3 days.

around the moon

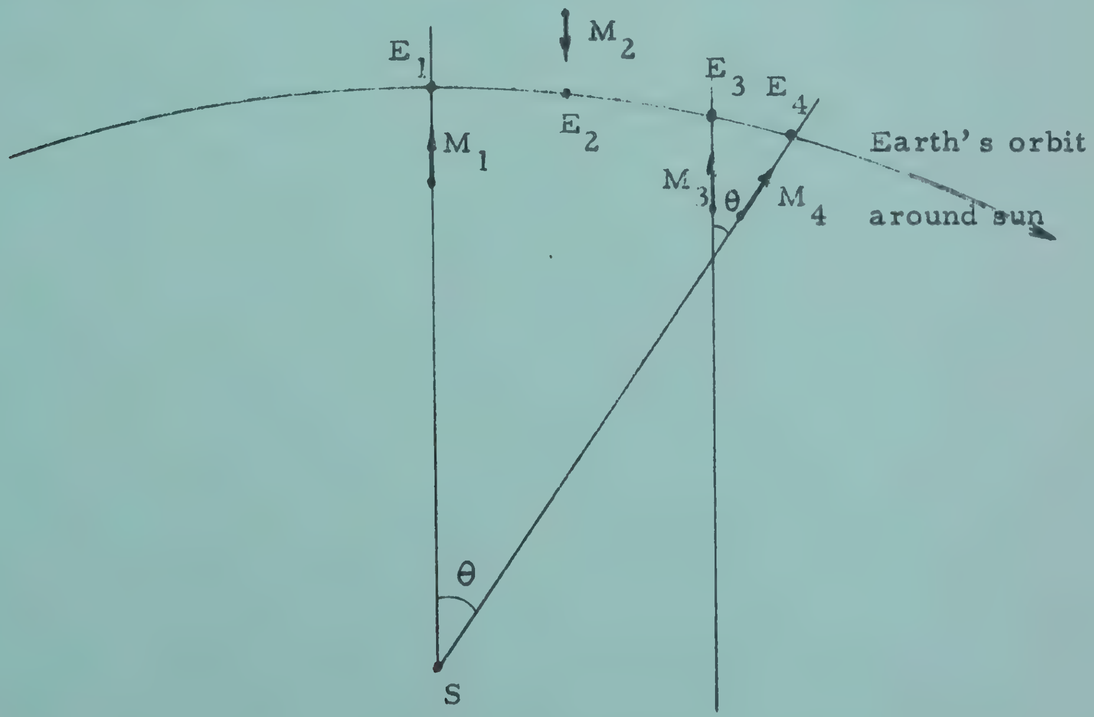
Problem 3 solution for problem as stated on page 24 of Chap 3 T. G.

This is a tricky question relating to frames of reference. The rotation of the earth about its axis, that of the earth in its orbit, that of the moon in its orbit about the earth, and that of the moon about its own axis are all in the same direction. Furthermore, the rate of rotation of the moon about its orbit and its rate about its own axis are identical. The period of each of these motions is 27.3 days.

Imagine an arrow, planted extending through the moon, with its head pointing toward the earth. The head will always remain pointing toward the earth. As the moon spins about its axis, the angle between this arrow and an imaginary diameter of the fixed star sphere will change. The arrow rotates 360° with respect to such a line in 27.3 days. Therefore, for an observer on the moon (to whom the arrow appears fixed) the diameter of the fixed star sphere (and thus the sphere itself) appears to rotate with a 27.3 day period.

The earth, however, remains fixed in the direction that the arrow points. Thus it neither rises nor sets for an observer on the moon. Its apparent rate of rotation is zero.

Now, the apparent rate of rotation of the sun is a more difficult problem. In Figure ____ are shown various configurations of the earth-moon system.



All rotations are clockwise. Position 1 is a new moon position for an earth observer. Position 3 is one sidereal period later. The moon arrows are parallel. The moon has rotated 360° with respect to the fixed star sphere. (Position 2 is an intermediate position after 180° of rotation. Note that at position 1, the sun is at its zenith for an observer on the moon situated at position of the tail of the moon arrow. However, at position 3, it is not yet again high noon for this observer. Noon does not come until position 4 is reached.

Now the angle along the earth's orbit, between positions 1 and 4, is θ. This is also the angle between M_3 and M_4. Thus one full apparent sun rotation requires a time $T_{\text{sidereal}} + T_\theta$

That is:

$$T_{\text{sun}} = T_{\text{sidereal}} + T_\theta$$

But $T_{\text{sidereal}} = 27.3$ days, and θ, the angle that the earth moves along its orbit in a time, T_{sun}, is given by:

$$\theta = W_{\text{earth}} T_{\text{sun}}$$

where w_{earth} is the angular velocity of the rotation of the earth in its orbit.

Then T_θ can be calculated:

$$T_\theta = \frac{\theta}{w_{\text{moon}}} = \frac{w_{\text{earth}} T_{\text{sun}}}{w_{\text{moon}}} = \frac{27.3}{365} T_{\text{sun}}$$

Thus

$$T_{\text{sun}} = T_{\text{sidereal}} + T_\theta = 27.3 + \frac{27.3}{365} T_{\text{sun}}$$

and $$T_{\text{sun}} = \frac{27.3 \times 365}{365 - 27.3} = \underline{29.5 \text{ days}}$$

It is interesting to note that positions 1 and 4 are both new moon phases for an earth observer, and therefore 29.5 days is also the period of our lunar month.

EXERCISE 4: Pretend that the earth's orbit is circular and the moon moves in a circle around the earth. The radius of the earth's orbit is 1.5×10^{11} meters and that of the moon's orbit is 4×10^{8} meters. By what fraction of itself does the moon's distance from the sun change? How often does it get to its nearest approach to the sun?

(a) How far has it moved around the sun between successive "approaches" to the sun?

(b) Does the moon show retrograde motion as seen by an observer at the sun?

This exercise reviews the kinematics of rotating systems, emphasizing the rotation of frames of reference.

Since the plane of the moon's orbit about the earth coincides with the plane of the ecliptic (the plane of the earth's orbit about the sun), the problem is completely straightforward. At its maximum distance, the moon is at $(1.5 \cdot 10^{11}\text{m} + 4 \cdot 10^{8}\text{ m})$ from the sun, and at its minimum distance $(1.5 \cdot 10^{11}\text{m} - 4 \cdot 10^{8}\text{ m})$ from the sun. Thus, the fractional change in the moon's distance from the sun is $\dfrac{8 \cdot 10^{8}\text{ m}}{1.5 \cdot 10^{11}\text{m}} = 5.3 \cdot 10^{-3}$. The moon is closest to the sun once every 27.3 days (its period of revolution).

(a) During 27.3 days, the moon (and earth) have moved $\frac{27.3}{365}$ x circumference of the earth's orbit, or $\frac{27.3}{365}\ 2\pi \cdot 1.5 \cdot 10^{11}\text{m} = 7.1 \cdot 10^{10}\text{m}$.

(b) The moon does not show retrograde (reverse) motion as seen by an observer on the sun, since during the 13.7 day (27.3 x 1/2) period that the moon is traveling "backward" a distance $8 \cdot 10^{8}$ m, it (and the earth) are traveling "forward" a distance of about $13.5 \cdot 10^{10}$m (1/2 of $7.1 \cdot 10^{10}$m). ✓ Therefore, from the sun, the moon would not show retrograde motion relative to the fixed stars (it would, of course, show retrograde motion relative to the earth), but would appear to be moving at a slightly varying speed.

EXERCISE 5: How fast, in m^2/sec, does the radius from sun to earth sweep out area? The radius from earth to moon?

This is a straightforward problem related to Kepler's second law. Using the data given in exercise 4:

Area swept out by radius from earth to sun per second $= \pi r^2 f$, where f is the frequency of revolution in cycles per second. Since the earth revolves once in 365 days ($3.2 \cdot 10^7$ sec), $f = \frac{1}{3 \cdot 2 \cdot 10^7 \text{sec}}$.

Thus, area swept out by earth to sun radius per second =

$$\pi \cdot (1.5 \cdot 10^{11} \text{m})^2 . \quad \frac{1}{3.2 \cdot 10^7 \text{sec}} = 2.2 \cdot 10^{15} \text{ m}^2/\text{sec.}$$

For the radius from the earth to the moon, since the period is 27.3 days ($2.4 \cdot 10^6$sec.), thus

area swept out by earth to moon radius per second

$$= \pi (4 \cdot 10^8 \text{m})^2 \cdot \frac{1}{2.4 \cdot 10^6 \text{sec}} \boxed{= 8.6 \cdot 10^{10} \text{m}^2/\text{sec.}} - 2.1 \times 10^{11}$$

EXERCISE 6: In addition to planets, the solar system contains other "wanderers" called comets. Many comets reappear at regular intervals, apparently becoming brighter and larger in a few weeks as they approach the earth and then becoming dimmer and smaller until they are invisible for a period of years. What kind of an orbit do you think they have?

This easy "thought question", is merely to remind students that an elliptical orbit, even a highly eccentric one like that of a comet, represents a periodic motion.

EXERCISE 7: Halley's comet has a period of 75 years. Its smallest distance from the sun is 8.9×10^{10} meters. How far from the sun does it get? (see the footnote on page 3-7). Sketch its orbit and the earth's orbit together. (It was Newton who told Halley how to compute the orbit of a comet. Halley found and calculated the orbit and period of the comet that bears his name in the course of a general analysis he made of comets' orbits.)

A quantitative exercise involving the orbit of a comet, and Kepler's third law.

5.7

From Table I, we take $R^3/T^2 = 3.5 \cdot 10^{18}\ m^3/sec^2$. Since the period of Halley's comet is 75 years (=75 x 365 x 24 x 3600 sec = $2.4 \cdot 10^9$ sec) OK $T^2 = 5.8 \cdot 10^{18}\ sec^2$, and $R = \left(3.5 \cdot 10^{18} \frac{m^3}{sec^2} \cdot 5.8 \cdot 10^{18} sec^2\right)^{1/3} = 2.7 \cdot 10^{12}\ m$.

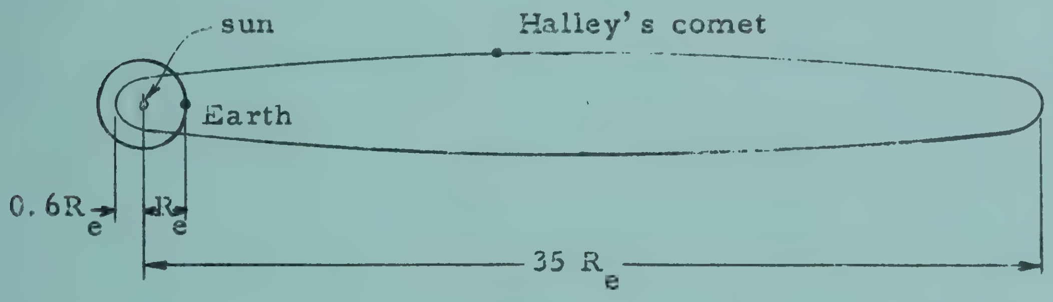
Since, from the footnote, R is defined as $\frac{1}{2}$ the sum of the shortest and longest radii, the maximum distance of the comet from the sun is

$$\frac{R_{max} + 8.9 \cdot 10^{10}}{2} = 2.7 \cdot 10^{12}\ m$$

$$R_{max} = 5.3 \cdot 10^{12}\ m$$

OK

(The distance of closest approach, $8.9 \cdot 10^{10}$ m is negligible compared to the long diameter). If this orbit is drawn to scale together with that of the earth, it is evidently highly elongated, as shown below.



Can't we draw a better ellipse than this? and to scale?

EXERCISE 8: Three fireflies X, Y, and Z are on a moving bicycle at night. X is at the very center of one of the axles, which turns with the wheel. Y is on the rim of the wheel. Z is on the frame of the bike outside the circumference of the rim.

(a) Describe the motions that X and Y have from Z's point of view.
(b) Describe the motions of Y and Z as viewed by X.
(c) Describe the motions of X and Z as viewed by Y.
(d) Describe the motions of all three as seen by an observer standing near the bicycle.

A moderately difficult exercise best used for class discussion and intended to show the different reference frames - an illustration of the principles involved in relating heliocentric and geocentric descriptions of the universe.

The basic difficulty encountered in this problem may be illustrated by an example. Suppose we ask "what is the motion of a man sitting on a bench as viewed by a man looking out the window of a moving car." Two answers could be given. A perfectly sensible answer is that the man is not moving at all. Even though the car moves, we can see from the car that the man on the bench is sitting still, hence is "at rest". A second answer is that in a reference system fixed with respect to the car, the man on the bench is moving backwards at the speed of the car. Certainly the first answer is the more natural in terms of our experience. In no sense does the motion of the car fool us. We intuitively choose to describe the position of the man relative to the ground.

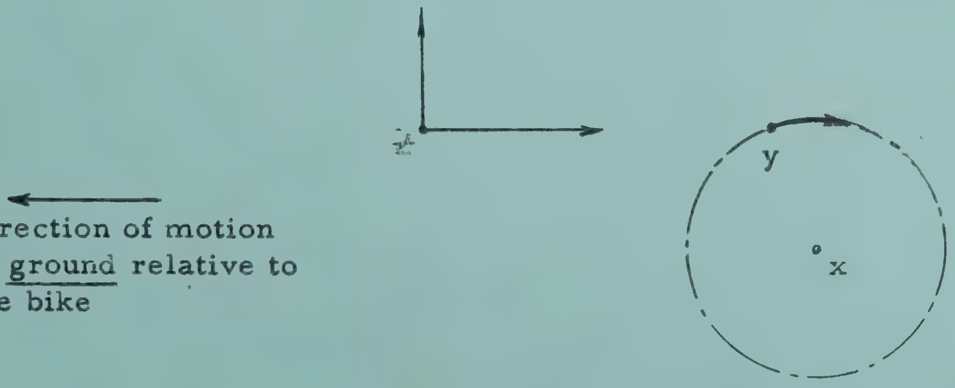
The lesson is simple: to describe motion of an object, we pick a reference system and describe the motion relative to the reference system. In our example there are two possible choices (1) a system at rest on the ground and (2) a system at rest with respect to the car. In giving our answer we must state clearly which reference system we are using.

The man in the car can tell us what motion the bench sitter has with respect to either system. For that matter, so could the man on the bench!

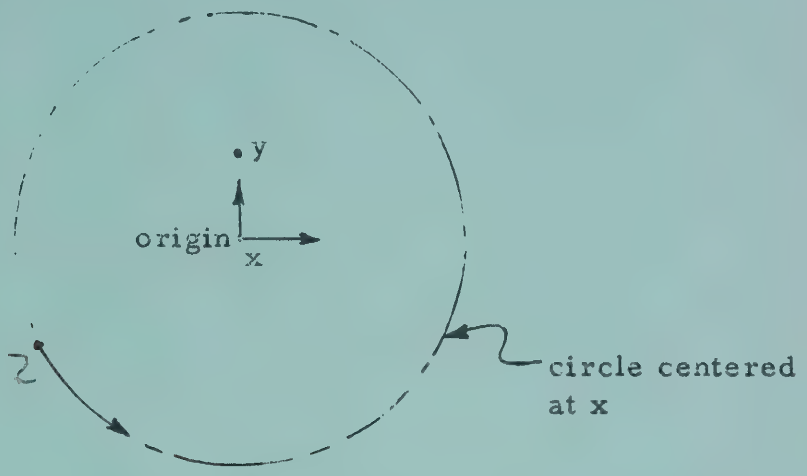
In Exercise 8 we are presented with four reference systems. We can most easily think of them in terms of the orientation and origin of some coordinate axes. We interpret the words "as viewed by" as specifying coordinate axes as follows:

(a) We assume axes with origin at Z, on the frame, and of fixed orientation relative to the frame. Then X is at rest, and Y describes a circle.

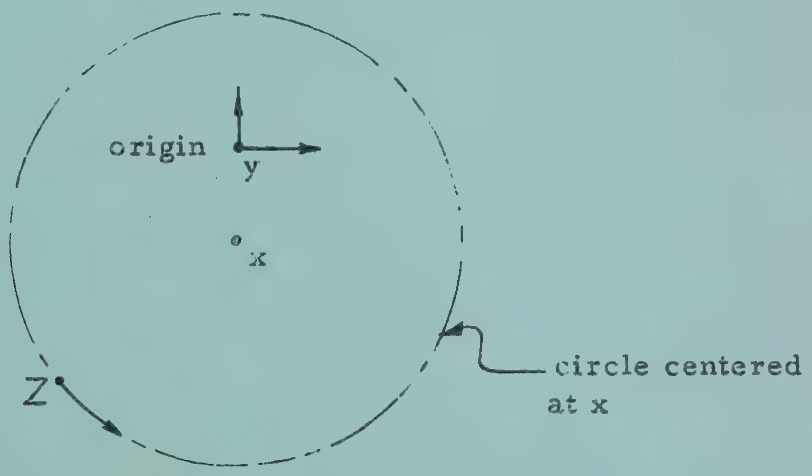
from the point of view of Z, y is in simple harmonic motion



(b) The axes are centered on the turning axle and are "rigidly attached" to it. Hence they rotate with the wheel. Clearly Y, on the rim, is at rest. X, on the frame, is at a fixed distance from the origin. Hence, as the axes turn relative to him he may be said to turn in the opposite sense relative to the axes, generating a circle.

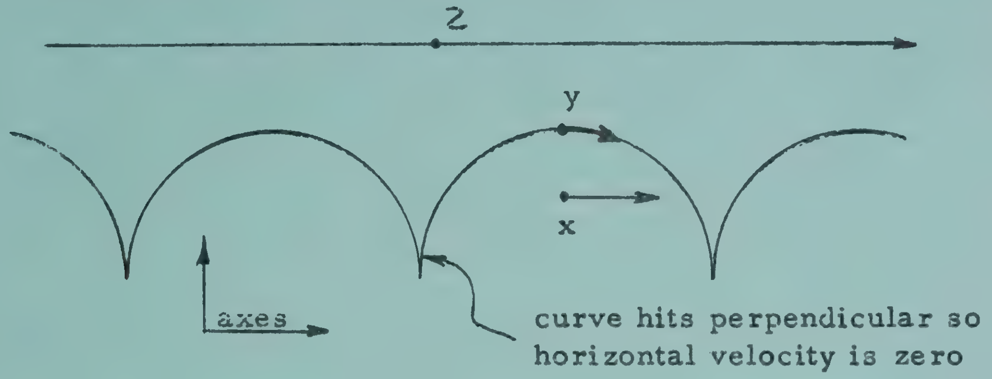


(c) We assume the origin of the axes at Y and that they are rigidly attached to the wheel (i.e. rotating with the wheel.)



X is at rest, but Z moves in a circle centered at X. Note that the only difference between (b) and (c) is that the origin of the axes is shifted, but the relative positions of X, Y, and Z are the same.

(d) The axes are fixed on the ground. Then X on the axle and Z on the frame move with uniform speed in a straight horizontal line. Y on the wheel goes in a cycloid. Since the wheel does not slide on the ground, but only rolls, the horizontal velocity of Y is zero at the instant Y's part of the wheel is in contact with the ground. Thus we get:



Note that the simplest method of describing the motion of Y is to say that Y moves in a circle relative to the axes of part (a), and that the axes of part (a) move at a constant speed horizontally with no turning relative to the ground.

Students may have fun teasing their teachers with consideration of whether the intensity of the firefly's light changes as he turns relative to an observer, the way an airport beacon does. They may also bring in three dimensional effects (if Z is far out on the handle bars he "sees" an ellipse for the motion of the firefly on the rim).

EXERCISE 9: Two skaters of equal mass on a circular rink go around the rink in the same time. One skater is twice as far from the center of the rink as the other. Compare:

(a) their angular velocities;
(b) their actual velocities;
(c) the centripetal forces acting on them.
(d) What exerts the centripetal force on them?

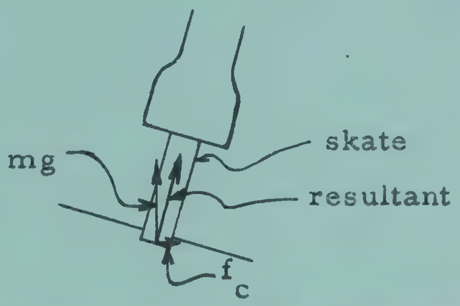
This exercise is a review of rotational motion.

(a) Their angular velocities (i.e., the rate at which they go around the rink) are identical; $w_1 = w_2$

(b) If $r_1 = 2\ r_2$, then since $V = rw$, $V_1 = 2V_2$.

(c) Since the centripetal force $fc = \frac{mv^2}{r} = mrw^2$, and $r_1 = 2r_2$, $fc_1 = 2fc_2$.

(d) The centripetal force is exerted on the skaters by the frictional force exerted on the skates by the floor of the rink, if the rink is level and hard (as in a roller-skating rink with a level surface). If the rink is soft (e.g. an ice rink, where the skate pushes a short distance into the ice), or if the floor of the rink is inclined ("banked" on the turns), the centripetal force is exerted by the reaction force of the floor on the skates - (due to the effective angle of the floor).



EXERCISE 10: At what height above the earth's surface will a rocket have half the force of gravity on it that it would have at sea level?

An elementary exercise involving the fact that gravity follows an inverse square law.

Since the gravitational force varies as $\frac{1}{R^2}$, the force will be smaller by a factor 2 when $\frac{1}{R^2}$ is smaller by a factor 2, than R_o, the radius of the earth. (i.e., when $R = \sqrt{2}\, R_o$). If $R_o = 4000$ mi. $\sqrt{2}\, R_o = 5700$ mi. The height above the earth's surface at which the force of gravity would be reduced by one-half is thus $R - R_o = 1700$ mi.

EXERCISE 11: A satellite circles the earth once every 98 minutes at a mean altitude of 500 km. Calculate the mass of the earth. The masses of planets are actually calculated from satellite motions, and one reason for establishing artificial earth satellites is to get a better value for the mass of the earth.

An exercise involving the law of universal gravitation. (It should be pointed out that one cannot calculate the mass of the earth from the data given without a prior knowledge of the gravitational constant

$$G \left(= 0.667 \cdot 10^{-10} \frac{m^3}{kg\ sec^2}\right).$$

Kepler's third law states merely that R^3/T^2 is a constant for all satellite motion, but does not specify the nature of the constant.

Using the law of universal gravitation, we may equate the centripetal force with the gravitational force on the satellite (if the orbit is stable). Thus, if we designate the mass of the satellite by m_s, and that of the earth by m_e,

$$m_s R w^2 = \frac{G m_s m_e}{R^2},$$

$$\text{so } m_e = \frac{R^3 w^2}{G}.$$

Substituting: $R = 6.37 \cdot 10^6 \text{m} + 5.00 \cdot 10^5 \text{m} = 6.87 \cdot 10^6 \text{m}$

$$w = 2\pi \cdot \frac{1}{98.60} \text{sec}^{-1} = 1.07 \cdot 10^{-3} \text{sec}^{-1},$$

$$m_e = \frac{(6.87 \cdot 10^6 \text{m})^3 (1.07 \cdot 10^{-3} \text{sec}^{-1})^2}{0.667 \cdot 10^{-10} \dfrac{\text{m}^3}{\text{kg sec}^2}}$$

$$= 5.5 \cdot 10^{24} \text{kg}.$$

Does the student know w?

$$\frac{m_s V^2}{R} = \frac{G m_s m_e}{R^2}$$

$$m_e = \frac{m_s V^2 R^2}{G m_s R} = \frac{V^2 R}{G}$$

$$V^2 = \frac{4\pi^2 R^2}{T^2}$$

$$m_e = \frac{4\pi^2 R^3}{G T^2}$$

$$m_e = 4\pi^2 \ldots$$

$$m_e = 5.5 \times 10^{24}$$

EXERCISE 12: (a) At what height will a satellite stay over a place on the equator of the earth?
(b) How fast is it accelerating toward the earth?
(c) Using the inverse-square law and g at the surface of the earth, find the gravitational field at this height.

This is a straightforward exercise involving the law of universal gravitation, good for class discussion.

(a) The satellite will obviously be stationary with respect to a point on the earth only if its period, T, is the same as that of the earth, namely 24 hours or $8.64 \cdot 10^4$ sec.

If the satellite is in a stable orbit, the centripetal force, $m_s \frac{v^2}{r}$ must equal the gravitational force $\frac{Gm_s m_e}{r^2}$, where r is the distance from the satellite to the center of the earth. Thus,

$$m_s \frac{v^2}{r} = G\, m_s m_e / r^2,$$

but $2\pi r = vT$, so $v^2 = 4\pi^2 r^2 / T^2$

substituting, $$r = \left(\frac{Gm_e T^2}{4\pi^2}\right)^{\frac{1}{3}} = \left(\frac{0.667 \cdot 10^{-10} \frac{m^3}{kg\ sec^2} \cdot 6.0 \cdot 10^{24} kg \cdot (8.64 \cdot 10^4 sec)^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$= 4.2 \cdot 10^7 m$$

Since the radius of the earth is $6.4 \cdot 10^6$m, the satellite is at a height of $4.2 \cdot 10^7 m - 6.4 \cdot 10^6 m = 3.6 \cdot 10^7 m$, or about 22,000 miles.

(b) The satellite is accelerating toward the earth with an acceleration:

$$a = F \text{ (of gravity)}/m_s$$

$$= \frac{G\, m_e m_s}{r^2} / m_s$$

$$= \frac{G\, m_e}{r^2} = \frac{0.667 \cdot 10^{-10} \frac{m}{kg\ sec^2} \cdot 6.0 \cdot 10^{24} kg}{(4.2 \cdot 10^7\ m)^2}$$

$$= 2.3 \cdot 10^{-1} m/sec^2.$$

(c) This result can also be obtained by noting that a, at a distance r, must be $\left(\frac{r}{ro}\right)^2$ smaller than a at a distance ro. Since a = 9.8 m/sec^2 at the surface of the earth ($ro = 6.4 \cdot 10^6$), at a distance $r = 4.2 \cdot 10^7$m,

$$a = 9.8 \text{ m/sec}^2 \left(\frac{6.4 \cdot 10^6}{4.2 \cdot 10^7}\right)^2 = 2.3 \cdot 10^{-1} \text{ m/sec}^2.$$

EXERCISE 13: (a) If T is the period of a satellite moving around just above the surface of a planet whose average density is p, show that pT^2 is a universal constant. (b) What is the value of the constant?

An "analytical" exercise involving the law of universal gravitation.

(a) As in exercise 12, we may write

$$\frac{m_s V^2}{r} = \frac{m_s}{r} \frac{(4\pi^2 r^2)}{T^2} = \frac{G m_p m_s}{r^2}$$

If $r \approx r_o$, the radius of the planet, we may write

$$\frac{4\pi^2 r_o m_s}{T^2} = \frac{G m_p m_s}{r_o^2}$$

But $m_p = \frac{4}{3}\pi r_o^3 p$. Thus $\frac{4\pi^2 r_o m_s}{T^2} = \frac{G m_s \cdot \frac{4}{3}\pi r_o^3 p}{r_o^2}$.

Cancelling m_s and r_o, we solve for pT^2,

$$pT^2 = \frac{3\pi}{G}$$, a universal constant.

(b) Evaluating, $pT^2 = \dfrac{3\pi}{0.667 \cdot 10^{-10} \dfrac{m^3}{kg\ sec^2}} = 1.41 \cdot 10^{11} \dfrac{kg\ sec^2}{m^3}$.

EXERCISE 14: Compute the periods of satellites moving around
(a) the earth and
(b) the sun, close to their surfaces.

Another exercise involving satellite motion and the law of universal gravitation. This problem should follow exercise 13.

From the previous exercise, we know that pT^2 is a universal constant for a satellite moving close to the surface of any body. Hence,

$$p_s T_s^{\,2} = p_e T_e^{\,2} = 1.41 \cdot 10^{11} \frac{\text{kg sec}^2}{\text{m}^3}$$

Since $p_e \approx 6 \cdot 10^3 \text{kg/m}^3$

$$T_e = \left(\frac{1.41 \cdot 10^{11} \frac{\text{kg sec}^2}{\text{m}^3}}{6 \cdot 10^3 \text{ kg/m}^3} \right)^{1/2} = 4.8 \cdot 10^3 \text{ sec} = 80 \text{ min.}$$

Taking $p_s \approx 1.4 \cdot 10^3 \text{ kg/m}^3$

$$T_s = \left(\frac{1.41 \cdot 10^{11} \frac{\text{kg sec}^2}{\text{m}^3}}{1.4 \cdot 10^3 \text{kg/m}^3} \right)^{\frac{1}{2}} \approx 10^4 \text{ sec} \approx 167 \text{ min.}$$

EXERCISE 15: Find the weight of a 100-kg man on planets with the following masses and radii:

m	R
$2.4 \cdot 10^{23}$kg	$2.5 \cdot 10^{6}$m
$6.0 \cdot 10^{24}$kg	$6.4 \cdot 10^{6}$m
$1.9 \cdot 10^{27}$kg	$7.0 \cdot 10^{\textcircled{6}}$m 7

Can you name any of these planets?

Another exercise involving the law of universal gravitation. Since the weight is simply the force of gravity on the 100 kg man, and

$$W = F = \frac{Gm_p m}{r_p^2},$$

We can construct the following table:

Planet	m_p (kg)	r_p (m)	W (newtons)
I	$2.4 \cdot 10^{23}$	$2.5 \cdot 10^{6}$	260
II	$6.0 \cdot 10^{24}$	$6.4 \cdot 10^{6}$	980
III	$1.9 \cdot 10^{27}$	$7.0 \cdot 10^{7}$	2600

Planet II is clearly the earth or its twin sister. Planet I, with a gravity about 1/4 that of the earth might be Mars, while planet III, with a radius ten times that of the earth, might be Jupiter.

EXERCISE 16: A 70 kg boy stands 1 meter away from a 60-kg girl. Calculate the force of attraction (gravitational) between them.

On the surface this is a simple "formula" problem involving the law of universal gravitation. It's real worth depends on what you do with it to give the students a feeling for the magnitude of the gravitational force in terms of things such as the weight of one cubic centimeter of water (approx. 10^{-2}nts) or the fraction of the weight of either student (F/wt of girl = 4.8×10^{-10}).

$$\text{Solution:} \quad F = \frac{Gm_1 m_2}{r^2}$$

$$m_1 = 60 \text{ kg}, \; m_2 = 70 \text{ kg}, \; r = 1 \text{ m}, \; G = .667 \times 10^{-10} \frac{\text{nt. m}^2}{\text{kg}^2}$$

substituting: $F = 2.8 \times 10^{-7}$ nts

Obviously, it must be something other than gravity which attracts boys and girls!

EXERCISE 17: Find the gravitational attraction of the two atoms in a hydrogen molecule.

This simple exercise, involving the law of universal gravitation, is intended to show the smallness of gravitational forces on an atomic scale.

$$F = \frac{G m_1 m_2}{R^2}$$

$$m_1 = m_2 = 1.67 \times 10^{-27} \text{kg}$$

$$R \approx 2 \text{ A}^{\circ} = 2 \times 10^{-10} \text{m}$$

$$F = \frac{.667 \times 10^{-10} \cdot (1.67 \times 10^{-27})^2}{(2 \times 10^{-10})^2} \approx 4 \times 10^{-45} \text{nt.}$$

By comparison, the electric force between a proton and an electron in a hydrogen atom is about 10^{-8} nt, about 10^{37} times larger! Gravitational forces are completely negligible on an atomic scale.

EXERCISE 18: If a planet of twice the diamter of the earth has a mass six times as great, how does the gravitational field at its surface compare with the gravitational field at the surface of the earth.

A problem to emphasize the mass and distance dependence of the gravitation law, and to provide practice in more abstract reasoning (i.e. an algebraic rather than numerical problem). Gives more practice in ideas of scaling (Vol. I).

Since the planet exerts a force as though all of its mass were at a point concentrated at the center, the force F_1 on the mass m by the first planet of mass M and radius R is

$$F_1 = \frac{GmM}{R^2}$$

The force, F_2, by the second planet on the mass M is found by replacing M by 6 M and R by 2 R.

Therefore, $$F_2 = \frac{Gm\,6M}{(2R)^2} = \frac{6}{4}\ \frac{GmM}{R^2} = 1.5\ F_1$$

The gravitational force at the surface of the second planet is 1.5 greater than that at the first.

You might ask the students how the densities of the two planets compare (more practice in scaling). Since density = $\frac{\text{mass}}{\text{volume}}$ and volume is proportional to R^3 the density of second planet is $6/2^3$ or 3/4 that of the first.

EXERCISE 19: The earth is acted upon by the gravitational attraction of the sun. Why doesn't the earth fall into the sun?

A problem intended primarily for class discussion to help clarify the students thoughts on the causes of the circular orbits. A good starting point is the quotation from Newton in the text "Nor could the moon without some such force be retained in its orbit. If this force was too small, it would not sufficiently turn the moon out of the rectilinear course; if it was too great it would turn it too much and draw the moon from its orbit toward the earth."

The class should understand that the earth is falling freely towards the sun. If it started from rest, it would fall on a straight line into the sun. The actual situation may be appreciated by thinking of the problem of bombing a target from a plane. The bomb is not released directly over the target otherwise it would overshoot the mark. The faster the plane, the greater the overshoot. The case of an "orbit" is reached when the overshoot causes the object to miss the earth completely.

In the same manner, the earth does not fall into the sun simply because it is in a stable orbit, that is it has a sufficiently tangential velocity so that it "misses" the sun as it falls toward the sun. (Note that we (or anyone else!) cannot say how the earth happened to get the correct initial velocity to keep it in its present orbit. Refer to later material on conservation of angular momentum.)

EXERCISE 20: (a) If the earth were not present but the moon remained for a moment at its present distance from the sun, moving with the same speed as it now does, what would be the subsequent path of the moon?

(b) Calculate the ratio of the force of attraction exerted by the sun on the moon to the force exerted by the earth on the moon.

(c) Why does the sun not capture the moon, taking it away from the earth?

A very difficult problem for the students to work through with any degree of rigorous reasoning. The major point to bring out is that both earth and moon are attracted to the sun. The earth and moon revolve about their common mass center which in turn goes about the sun.

(a) We must first recognize that for a planet to go in a circular orbit of a given radius, the speed must be proper, but it is independent of the mass of the planet. (Since the gravitational force acting on the planet is proportional to the mass of the planet). Secondly we must recognize that the speed of the earth in its orbit about the sun (a circle of radius 94, 000, 000 miles traversed in one year) is much greater than that of the moon about the earth (circle of radius 240, 000 miles in 1/12 year). Thus at any instant the moon moves about the sun at almost the same speed as the earth. Under the conditions of part (a) the moon would be going at nearly the correct speed to travel the same orbit as the earth now travels. (see Exercise 4).

(b) One can say either

$$\frac{F_{\text{sun on moon}}}{F_{\text{earth on moon}}} = \frac{M_{\text{sun}}}{M_{\text{earth}}} \times \frac{R^2_{\text{moon - earth}}}{R^2_{\text{moon - sun}}}$$

where $R_{\text{moon-earth}}$ is the distance from moon to earth. Or else we can use the results of (a) together with $F = MV^2/R$.

Thus, the moon-earth force must be given by

$$F_{\text{moon-earth}} = \frac{MV_{\text{moon}}^2}{R_{\text{moon-earth}}}$$

where V_{moon} is the speed of the moon about the earth (actually about the moon-earth mass center) and M = mass of the moon.

Likewise, we know from (a) that the moon would go in a circle about the sun if it had the earth's velocity, V_{earth}, about the sun, and were at the earth's

distance $R_{\text{earth-sun}}$

$$\text{Thus } F_{\text{moon-sun}} = \frac{MV^2_{\text{earth}}}{R_{\text{earth-sun}}}$$

$$\frac{F_{\text{moon-sun}}}{F_{\text{moon-earth}}} = \frac{V^2_{\text{earth}}}{V^2_{\text{moon}}} \times \frac{R_{\text{moon-earth}}}{R_{\text{earth-sun}}}$$

$$\text{But } V_{\text{earth}} = \frac{2\pi R_{\text{earth-sun}}}{T_{\text{earth}}}$$

$$\frac{F_{\text{moon-sun}}}{F_{\text{moon-earth}}} = \frac{R^2_{\text{earth-sun}}\ T^2_{\text{moon}}}{T^2_{\text{earth}}\ R^2_{\text{moon-earth}}}\ \frac{R_{\text{moon-earth}}}{R_{\text{earth-sun}}}$$

$$= \frac{R_{\text{earth-sun}}\ T^2_{\text{moon}}}{R_{\text{moon-earth}}\ T^2_{\text{earth}}}$$

?

1/13

$$= \frac{94,000,000}{240,000}\ \frac{(1/12)^2}{1}$$

$$= 2.7$$

(The period of the moon as it revolves around the earth is taken as 1/12 of the period of the earth around the sun).

(c) The sun exerts forces on both the moon and the earth. If there were no attraction between moon and earth, they could both continue to orbit around the sun at the same radius and at the same speed. The sun would neither "capture" one or the other. When we add the attraction between the earth and the moon, we must have them going about each other to prevent their falling together. Thus we see their is no capture even without an attraction between moon and earth. This question is sometimes put in a different form, "when the moon is toward the sun from the earth, why doesn't the sun capture the moon?" A good answer is, "It already has."

EXERCISE 21: Assume the earth to be perfectly round with a radius of 6,400 km.

(a) How much less does a man with a mass of 100 kg apparently weigh at the equator than at the poles because of the rotation of the earth?

(b) How fast would the earth have to spin in order that he would exert no force on a scale at the equator?

(c) How many times larger is the speed of rotation in (b) than the actual speed?

A moderately hard problem aimed at understanding the consequences of circular motion. We are asked about the apparent weight of a man, that is to say the reading of a scale on which he weighs himself. When the man stands on a scale at the pole, his mass center is not accelerated, and he is therefore in static equilibrium. The force of the scale upwards, F_s, just balances the force of gravity, F_g, downwards, giving

$$F_g = F_s$$

When he stands on the same scale at the equator, he is riding around with the earth at a velocity V in a circle of radius, R. The earth's radius. Therefore his motion is accelerated and the pull of gravity towards the center (still the same F_g as at the pole) must exceed the new push of the scale, F_s^1, away from the center of the earth to provide the centripetal acceleration. Using M for the mass of the man, Newton's second law becomes

$$\frac{Mv^2}{R} = F_g - F_s^1$$

$$\therefore F_s^1 = F_g - \frac{Mv^2}{R} = F_s - \frac{Mv^2}{R}$$

Using T for the number of seconds in one day (= 24 x 60^2 seconds)

$$V = \frac{2\pi R}{T} \quad \text{and}$$

$$F_s^1 - F_s = \frac{Mv^2}{R} = \frac{M\,4\pi^2\,R^2}{R\,T^2} = \frac{4\pi^2 M R}{T^2}$$

Solve prob b & c

b $F = \frac{mv^2}{r}$

Using R = 6400 km = 6.4×10^6 m, M = 100 kg.

$$F_s^1 - F_s = \frac{4\pi^2 \times 100 \times 6.4 \times 10^6}{(24)^2 \times (60)^4}$$

= 3.4 nts. lighter apparent weight at the equator.

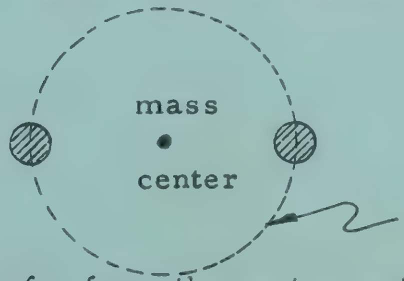
Caution: Students should not confuse this apparent change in weight with the actual change in weight resulting from the inverse square dependence of the gravitation force, and the fact that the earth is not perfectly spherical. The effect in this problem is related only to the rotation of a spherical earth about an axis (however, the fact that the earth is an oblate spheroid is also due to its rotation.)

EXERCISE 22: The two stars of a double star, far away from any other large masses, revolve in circular orbits, always remaining the same distance apart. Sketch their orbits if

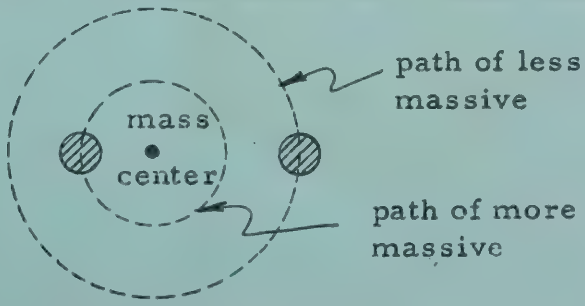
(a) they have equal masses,
(b) one has twice the mass of the other.
(c) What is the ratio of the radii of the two orbits in each case?

A moderately hard exercise to emphasize that when a pair of bodies revolve about each other, both actually move. Thus although we often say the moon goes about the earth once a month, actually they both revolve about their common mass center once a month.

(a) If the stars have equal masses, their mass center is half-way between, and they revolve about it. Their paths are circles, about the mass center. Since they are equally distant from the mass center, both circles have the same radius.



(b) The more massive is twice as far from the center as the less massive.



(c) In (a) both circles have the same radius. In (b) the circle of the less massive has twice as large a radius as that of the more massive.

If students do not know what is meant by the "center of mass", they will be unable to follow the quantitative description of the orbits, but should be able to appreciate qualitatively what is going on.

EXERCISE 23: Imagine that the moon does not revolve about the earth and that the moon is held at its present distance from the earth by a huge hydraulic jack. If the jack is suddenly removed, the earth and moon will collide. Now imagine that the same jack supports a small stone at the same position. The stone and the earth will also collide when the jack is removed. Which collision will take place sooner after the jack is removed? Why?

A variant on Galileo's experiment of dropping weights. Since the force on the moon or stone is proportional to its mass, the two objects experience the same acceleration towards the earth when at the same distance from the earth. Therefore, they would accelerate "in step" and take the same time. (Strictly speaking, since the earth also would move, the moon would hit a bit sooner because the earth moves towards it more than towards the stone.)

If you try this exercise, simplify the problem by letting the earth and moon be represented by point masses - otherwise, things are unnecessarily complicated by the finite ratio of the bodies, and students may worry about what is meant by the point of support of the moon, and the point of collision.

EXERCISE 24: Assume that the gravitational attraction exerted by m_1 on m_3 in Fig. 00 is not influenced by the presence of m_2, and the attraction of m_2 on m_3 is not changed by the presence of m_1. Using the universal law of gravitation, show that the force of attraction exerted on m_3 by the single "body" made of m_1 and m_2 together is proportional to its mass.

Students may have difficulty in seeing what ideas are assumed and what to be proven in this exercise.

Call $F_{1 \text{ on } 3}$ the force of mass 1 on mass 3. Then our assumption that we can add the forces shows that the force of 1 plus 2 on 3, $F_{1 + 2 \text{ on } 3}$, is given by

$$F_{1 + 2 \text{ on } 3} = F_{1 \text{ on } 3} + F_{2 \text{ on } 3} = G \frac{m_1 m_3}{r^2} + G \frac{m_2 m_3}{r^2}$$

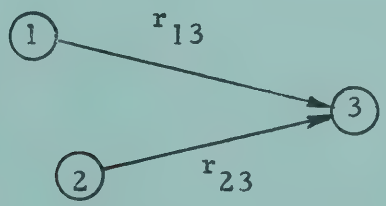
$$= G \frac{(m_1 + m_2)}{r^2} m_3$$

which is just the force of a single mass of magnitude $(m_1 + m_2)$ on m_3.

If you do this exercise, be sure that students do not get the mistaken idea that the gravitational force is a scalar! Gravity, like any force, is a vector, and the magnitude of the resultant force $F_{1 + 2 \text{ on } 3}$ is equal to the sum of the magnitude of $F_{1 \text{ on } 3}$ and $F_{2 \text{ on } 3}$ here not simply because the distance $r_{2 \text{ to } 3}$ and $r_{1 \text{ to } 3}$ are equal, but because they are identical in magnitude and direction that is m_1 and m_2 are superimposed.

In general

$$|F_{1 + 2 \text{ on } 3}| \neq |F_{1 \text{ on } 3}| + |F_{2 \text{ on } 3}|$$



Chapter 4 - Momentum and the Conservation of Momentum

The purpose of the chapter is to introduce the student to the idea of momentum and to show that it is a useful quantity in dealing with the motion of systems of objects. This is because the momentum of an isolated system must always remain constant.

Section 1 introduces the idea of momentum by considering impulse, $F\Delta t$, in connection with $m\Delta v$, momentum. By itself, the concept of impulse is not particularly important. It is used only as an analytical tool for introducing momentum and to give students a connection between momentum and the more familiar idea of forces.

The teacher should keep clearly in mind that, as presented here, the conservation of momentum is shown as an experimental fact. It does not rest on the idea of impulse, and it is not proved mathematically.

Content By Sections

1. $\vec{F}\Delta t$ is the impulse, a vector.
2. $M\Delta\vec{v}$ is momentum. Both impulse and momentum have the units Kg m/sec or newton-seconds.
3. $(M\Delta\vec{v})_1 = -(M\Delta\vec{v})_2$ when two objects interact. Concept of an isolated system.
4. Experiments show that for an isolated two-body system, the total momentum never changes.
5. Application of momentum conservation to rockets.
6. Determination and characteristics of center of mass.
7. Extension of momentum conservation to systems of more than two objects.
8. Newton's Third Law: If B exerts $\vec{F}_1$ on A, A must exert $-\vec{F}_1$ on B.

EMPHASES For an understanding of momentum and momentum conservation, Sections 1 through 4, and a general familiarity with Section 8, are the most important. Section 5 on rockets provides an interesting application of momentum but is not essential. The purpose of Sections 6 and 7 on center of mass and the generality of momentum conservation is to contribute to the idea of momentum conservation as a general principle. If the basic ideas of momentum and its conservation have been acquired these sections can be gone through lightly.

Section 1 - Impulse

PURPOSE To introduce impulse.

CONTENT

1. The product $\vec{F}\Delta t$ is defined as the impulse.
2. Impulse is a vector.
3. It is often possible to measure the product $\vec{F}\Delta t$ even though neither $\vec{F}$ nor Δt can be measured separately.

EMPHASIS The students should know the definition of impulse but it is not necessary to spend much time on this section. It will be natural to continue to discuss impulse as momentum is considered in Section 2 and following sections.

COMMENT Some students will accept the vector nature of impulse without the detailed development given in the text. The best way to present this to your class depends on how much time you devoted to vectors in Volume 1. For example, if your students know that the product of a scalar and a vector is another vector, the fact that $\vec{F}\Delta t$ is a vector is obvious. Force always has direction. Including the time the force acts does not change its directional nature. (Incidentally, the equation $F\Delta\vec{t} = m\Delta\vec{v}$ would be meaningless if $F\Delta\vec{t}$ were not a vector and $m\Delta\vec{v}$ were a vector. If we did not specify the direction of the force, we would not know the direction of the resulting change in velocity. Other students prefer the approach in which $\vec{F}_1\Delta t + \vec{F}_2\Delta t$ is rewritten as $(\vec{F}_1 + \vec{F}_2)\Delta t$. This is completely analoguous to the treatment in the text where $m\Delta\vec{v}_1 + m\Delta\vec{v}_2$ is rewritten as $m(\Delta\vec{v}_1 + \Delta\vec{v}_2)$.

* * *

If you emphasized the graphical interpretation of distance and speed when you covered Chapter 5 in Volume 1, your students may get additional insight into the additive effect of many impulses if you review the graphical material briefly. The graphical presentation of impulse as the area under an F vs t graph need not be stressed merely to enable the students to do calculations involving impulse.

In many practical cases where impulse is important, neither the force nor the time duration are known. In fact, the importance of the concept of impulse is that, although we may know neither $\vec{F}$ nor Δt, we do have a real experimental handle on the product, namely the change in momentum of an object. (Note that we measure $m\Delta v$ and infer $\vec{F}\Delta t$ only if we want it.)

Hence, the graphical presentation of impulse is not of prime importance. Furthermore, some students may not understand how to picture graphically the total impulse produced by a force which changes direction. (It is probably not worth the time to introduce the idea of graphing separately the three components, x, y, and z, of the forces.)

Section 2 - Momentum

PURPOSE To define momentum.

CONTENT Momentum is defined as the product $m\vec{v}$. Momentum is a vector; it has units of kg m/sec (or, equivalently, newton-seconds).

EMPHASIS This is very important material which should be stressed with exercises (e.g., 1-7) so that the students know how to calculate momenta and the changes in momenta produced by impulses.

COMMENT The main objective is to be sure that the students know that momentum is $m\vec{v}$ and that any two objects which have the same momentum, no matter how it was acquired, have a common property. The common property they have is that both could be stopped by the same impulse, $\vec{F}\Delta t$. Students should see that it is the product $m \cdot \vec{v}$ that is important. There is an infinite number of combinations of masses and velocities that will produce any given momentum. All the combinations resulting in the same momentum have a common property. They can all be stopped by the same impulse, $\vec{F}\Delta t$.

Although two objects might have equal velocities, there is no simple relation between their velocities and the forces needed to stop them; the force would depend on their masses.

(Although it should not be taken up now, later chapters will show that kinetic energy is another dynamic property; two objects with equal kinetic energies can be stopped by the same amount of work $\vec{F} \cdot \Delta\vec{s}$.)

* * *

Classroom work should emphasize both the magnitude and the vector nature of momentum. For example, a 2 ton car moving 30 mph north does not have the same momentum as a 1 ton car moving 60 mph south.

You may wish to introduce changes in momentum even at this stage. Thus, a 1 1/2 ton car going north at 40 mph suddenly turns and goes west at 40 mph . What is its change in momentum? If the tires squeal a bit, that's fine. It emphasizes that forces must act to change the momentum of an object.

* * *

Some students may not understand fully the distinction between "velocity. . . a kinematic quantity--and. . . momentum. . . a dynamic [quantity]" (next to last sentence on page 4-3). One way of looking at this distinction is that if we know only an object's velocity we have simply a description of what it is doing---where it is headed and how fast. But we do not know anything about what was required to give the object this velocity. Neither do we know what would be required to change this velocity. However, if we know the object's momentum, we can go beyond a description of the object's immediate behavior. We can infer something about what produced this velocity and we can determine what will be required to change this velocity. This distinction does not need to be labored at this point.

Section 3 - Changes in Momentum When Two Bodies Interact

PURPOSE To present experiments basic to momentum conservation.

CONTENT Experiments show that whenever two objects interact with each other (and when there are no other forces on either of the two), a momentum change in one object is accompanied by an oppositely directed, equal momentum change in the other.

EMPHASIS This is very important material which should be treated fully; stress particularly the isolation of the two interacting objects.

COMMENT In digging into changes of momentum in even the simple cases used in this section, students will, either consciously or unconsciously, have to isolate the system they are examining from external forces (such as friction, air resistance, etc.). Eventually, in order to apply conservation of momentum to practical occurrences, they will have to learn what is meant by the isolation of two objects, (what to include in the system, and what to exclude) and how to recognize isolation (or approximate isolation) when it occurs in a practical situation.

Once a student realizes what is meant by isolation he can learn easily that one object cannot "gain" momentum unless its partner "loses" the same amount. This fact cannot be deduced; it is based solely on experiments. If students know what isolation means, they can illustrate conservation of momentum with many everyday examples. Of course, if they do not understand isolation they can think of many cases where momentum seems to disappear. For a discussion of isolation see COMMENTS ON ISOLATION at the end of this section.

Go over the various examples given in this section of the text and be sure the students recognize the isolation and what might happen if the various two body systems were not isolated.

An ice surface is really not completely frictionless as anyone knows who has walked across an icy patch. How would basketball shoes affect the illustration of the man and boy who push each other?

Be sure that the students realize that in the gun explosion, the gases cannot excape around the bullet. When the bullet leaves the barrel and the gases do escape, they usually carry a negligible

portion of the momentum. (On the other hand, since the escaping gas does carry some momentum the recoiling gun has slightly more momentum in one direction than the bullet does in the other direction).

* * *

Note on Figure 7. It might be worthwhile to measure and analyze Figure 7 in class even though the analysis of this figure is given in the text. The students can measure the motion of each disc during ten time intervals (or 2 seconds). The large disc traveled 20.0 cm and the small disc traveled 41.2 cm; of course the students might make errors of several percent. There are two qualitative features that need clarification for some students.

(1) The small disc, by chance, happens to go one-half diameter during each flash interval.

(2) The first distance interval is less than the others because the "explosion" occurred about midway between flashes.

Supplementary Questions you can ask students about Figure 7:

(a) Assuming that the explosion occurred very quickly, estimate when the explosion occurred compared to the flash. Students who look at the small disc will easily see that the explosion occurred about halfway between flashes. However, some students who look only at the dark spaces between the successive positions of the large disc may be confused because the dark space between the first two positions seems so small. Remind these students that the large disc moved not only through this dark space but also through a distance equal to the thickness of its white rim.

(b) How large is the small disc? Get this value from the velocity as given in the text rather than from direct measurement.

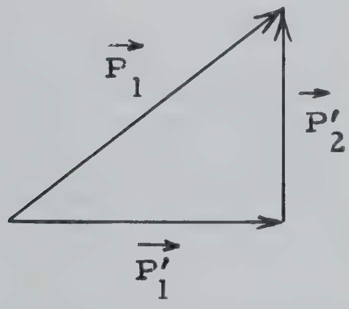
Making use of the happenstance that the small disc travels a diameter in 2 flashes, the students can estimate that since it travels 20.6 cm/sec or 20.6 cm/5 flashes, it travels $\frac{2}{5} \times 20.6 = \frac{41.2}{5} = 8.15$ cm in two flashes. Therefore, the diameter is 8.15 cm.

* * *

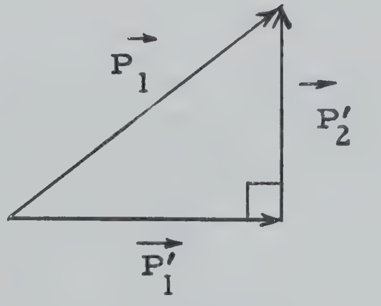
In the collision of the billiard balls in Figure 9, the balls are suspended as pendula with long threads. However, even if rolling

balls had been used, the spinning of the balls would not have affected the conservation of momentum during the collision. It would affect the conservation of kinetic energy of translation.

Some students may be particularly interested in the fact that when an off-center collision occurs between a moving (non-spinning) billiard ball and a stationary ball of equal mass, the two moving balls, after the collision, always make an angle of 90°. Many students will note this when they perform Experiment 540, A Collision in Two Dimensions. This fact depends on the perfect conservation of energy (toward which students are led by the final paragraph of Exp 540. See Teacher's Guide comments on Exp 540) and can be easily proved to students after they have studied energy. If $\vec{P}_1$ is the momentum of the moving ball before the collision, and $\vec{P'}_1$ and $\vec{P'}_2$ are the momenta after the collision, their conservation of momentum gives $\vec{P}_1 = \vec{P'}_1 + \vec{P'}_2$. Thus the three vector momenta must form a triangle as shown at (a).



(a)



(b)

The energy before the collision is $\frac{1}{2}mv_1^2 = \frac{P_1^2}{2m}$, and if energy is conserved during the collision,

$$\frac{P_2^2}{2m} = \frac{P'^2_1}{2m} + \frac{P'^2_2}{2m} .$$

But this just say

$$P_1^2 = P'^2_1 + P'^2_2 ,$$

which is nothing more nor less than the Pythagorean theorem for the triangle shown at (a). The triangle must then be a right triangle as at (b) and the two balls go off at 90°.

The Figures 9 and 10, the angle between P'_1 and P'_2 is only 85.5° rather than 90°. This is mostly due to the fact that energy is not completely conserved in the collision. Momentum should be completely conserved and the fact that $\overrightarrow{\Delta P}_1$ and $\overrightarrow{\Delta P'}$ are not on a straight line in Figure 10b by about 1° is experimental error due to distortion in the photograph and friction in the system (an external force).

The angle should be less than 90° if energy is lost in the collision. This can be seen by considering the case where the most energy is lost, namely when the balls stick together. In this case, the angle is 0°, certainly less than 90°.

When the collision is head-on, the incident ball stops dead and the struck ball goes straight ahead with the speed of the incident ball. In this case, the angle between P'_1 and P'_2 is not defined.

* * *

COMMENTS ON ISOLATION. In dealing with momentum problems, the isolation of a system (what to include in the system, and what to exclude) is not always simple. The only case in which true isolation occurs is in an interaction somewhere in outer space. Such cases are usually highly artificial, but easy to understand.

A second kind of isolation is illustrated by smooth objects on a frozen lake (assuming the ice is frictionless), or billiard balls on a table (again neglecting friction). Here external forces are not small. Gravity pulls down, while the ice or the table pushes up. However, because the surface is rigid, the forces *automatically adjust* themselves to be equal and opposite, and the *net* force up or down on the object is zero. In such cases, then, there is a kind of pseudo-isolation with large external forces acting, but adding up to zero.

A third common case of isolation involves consideration of the *time* during which forces act. Consider the question of changes of momentum between two billiard balls colliding on a billiard table. No one doubts that a billiard ball set rolling on the table will eventually come to rest because of friction. Hence, two billiard balls on a table do not form a really isolated system. But consider

just the instant of collision. The table, an object external to the system, exerts frictional forces on the balls during the collision. These forces may not even be small. The important thing is that they are negligibly small compared to the forces exerted between the balls during the collision. During the collision, the forces between the balls are very large and, though they are exerted during a short time, cause an appreciable impulse or change of momentum. During the same short collision time, the much smaller frictional forces (external) do not cause an appreciable impulse. Therefore the momentum of the two-ball system may be said to be conserved during the collision even though it is not conserved if longer times are considered.

Another illustration of this case is an exploding rocket. Here, gravity acts on the rocket as an external force. However the explosion takes place in such a short time that the impulse imparted by gravity to the rocket during the explosion is negligible and momentum conservation may be used to describe the situation immediately before and after the explosion.

Section 4 - The Law of Conservation of Momentum

PURPOSE To introduce the law of conservation of momentum for two interacting objects.

CONTENT Experiments show that for an isolated system composed of only two interacting objects, the total momentum never changes.

EMPHASIS This is very important material. It should be stressed.

COMMENT Spend as much time as is necessary to give the students a deep understanding of momentum conservation for systems of only two objects. The later sections of this chapter are merely applications or extensions of this principle to more complex situations. Highest priority should be given to establishing the principle, even in the special simple case. The more complex situation in later sections are less important.

In order to be sure that the students understand momentum conservation, return in the class discussion to each of the examples given in the text in section 3. Ask individual students to tell exactly what the isolated "system" is in each case and to restate the law of conservation of momentum as it applies specifically to each system. For example the boy and the man form an isolated system because they are standing on smooth ice. Since they were stationary before the pushing started, the original momentum was zero. Hence, the total momentum (vector sum) must always be zero. If the man acquires momentum in one direction, the boy must acquire an equal and opposite momentum.

* * *

Some students may not interpret properly the historical comments about momentum given in the last three paragraphs of section 4. Be sure they understand that, as presented in our text, momentum conservation is based completely on experiment and could not be proved from anything that has been established earlier (i.e., it does not follow from the law of inertia and Newton's (2nd) law of motion). The text refers to Wallis giving theoretical arguments for momentum conservation. These arguments were based on Newton's third law

as discussed in Section 8 of this chapter. The text, in presenting momentum conservation before Newton's third law, is following the most logical, modern course. We commonly observe momentum conservation in quite novel situations in which we know nothing about the forces. From momentum conservation we can infer the third law. Historically, however, the third law seemed more fundamental for cases in which forces were contact forces. You may want to return to these historical comments after section 8 has been treated. At this point you will want to see that students are not left with the view that clever mathematics or clever "theory" could be used to prove momentum conservation. They should be secure in their knowledge that momentum conservation is based on experiment; the only way it could be proved would be if one assumed something specific about the interaction forces (i.e., Newton's 3rd law).

Section 5 - Rockets

PURPOSE To apply momentum conservation as a guide to the problems of rocket propulsion.

CONTENT A rocket ship gains forward momentum equal to the momentum carried backward by the exhaust gas.

EMPHASIS If you deal with this section, it should be treated briefly as an example of momentum conservation. Avoid a detailed discussion of rocket technology unless you have both the class time to spare and a familiarity with the latest rocket news. Your abler students can dig into it outside of class.

COMMENT The estimates given in the text for the ratio of fuel mass to final rocket mass are rather conservative. Some modern fuels have somewhat higher exhaust speeds than the 2 km/sec quoted in the text. 2.5 km/sec to 2.7 km/sec are typical for conventional rockets; higher exhaust speeds exist. Scientists are working on chemical fuels and preheated gaseous fuels which may raise these values to 4 km/sec to 6 km/sec within 5 years. When this fuel efficiency is achieved, manned earth satallites will be rather easy to produce. A further increase of exhause speed to about 100 km/sec would make space travel feasible. Such exhaust speeds will require non-chemical techniques such as exhausting a stream of ions which had first been accelerated electrically. (Presumably some advanced, compact power source such as a nuclear reactor would have to be carried as the electrical power source).

* * *

In the absence of atmospheric drag and the pull of gravity, the equation relating the attained speed, v, the exhaust speed, v_e, the original total mass of fuel and rocket, m_o, and the remaining mass, m_a, is:

$$\frac{m_a}{m_o} = e^{-v/v_e}$$

or $$2.30 \log_{10} \left(\frac{m_a}{m_o}\right) = -\frac{v}{v_e}$$

or $$\log_{10} \frac{m_o}{m_a} = 0.434 \frac{v}{v_e}$$

Note that this formula assumes that all the mass which left the rocket $(m_o - m_a)$ moved with the speed v_e to the rear. Note also that v_e is measured with respect to the rocket.

A lower limit of the exhaust speed attained in the 80 ton Atlas rocket which put a 4 ton load in orbit (on Dec 18, 1959) can be obtained by setting v = 8 km/sec. Then

$$\log_{10} 20 = \frac{0.434(8)}{v_e}$$

or

$$v_e = \frac{0.434(8)}{1.3} = 2.67 \text{ km/sec}$$

Since there is some drag, and since some of the original mass is not fuel, and since the force of gravity must be overcome, the Atlas fuel-rocket system must produce an exhaust speed of more than 2.67 km/sec.

* * *

If you read about rocket fuels in the newspapers, you will probably come across the term "specific impulse". For example, a rocket and its fuel might give a specific impulse of 250 pound seconds per pound. (This unit is pounds force times seconds per pound mass). In MKS units this would be (9.8) (250) = 2.45×10^3 newton seconds/kilogram. This is exactly what the text refers to as an exhaust speed of 2.45 km/sec since

$$\frac{1 \text{ newton second}}{\text{kilogram}} = \frac{1 \text{ newton second}}{\text{kg}} \times \frac{1 \text{ kg} \frac{\text{m}}{\text{sec}^2}}{1 \text{ newton}} = 1 \text{ m/sec}$$

The unit is expressed in units of newton seconds per kilogram to show explicitly the impulse (in newton seconds) derived from burning and exhausting a kilogram of fuel. The average force depends on the rate at which fuel is burned; it would be 4.9×10^4 newtons if the fuel mentioned above were consumed at the rate of 20 kg/sec. (The specific impulse times the mass of fuel burned per second gives the force). In order to raise a large rocket (say 100 tons or 10^5 kg), a fuel consumption rate of about 400

kg/sec might be used. This together with a system of specific impulse $3 \times 10^3 \; \frac{\text{newton seconds}}{\text{kg}}$ would give a thrust of 12×10^5 newtons which would be enough to overcome the force of gravity, 9.8×10^5 newtons. After 100 seconds the mass of fuel ejected would be 410^4 kg or 40 % of the original mass. As the fuel is burned, the acceleration increases because the mass is decreasing. In order to avoid the mechanical problems inherent in excessive acceleration, a new slower (probably more efficient) rocket engine often takes over after the mass has decreased enough (multistage rockets). Of course, when this switch occurs, it is most efficient to reduce the mass further by ejecting the old rocket engine together with the storage tank that had held the already ejected fuel.

* * *

The equation relating m_a, m_o, V and v_e can be derived using integral calculus. Consider yourself to be riding along next to a rocket of mass m. It ejects a mass fragment, dm, backwards (to your left) with a speed v_e as seen by you. Then the rocket must increase its forward speed by an amount, dv, so that momentum is conserved. Thus:

$$m\,dv = - v_e\,dm$$

This is not written as a vector equation. It simply equates, in magnitude, the momentum which goes off to the right with that which goes off to the left. The minus sign is used because dm represents a decrement in the mass m of the rocket (i.e. it is a negative number). Then:

$$\frac{1}{v_e}\,dv = - \frac{dm}{m}$$

where v_e is a constant, m is not.

Integration of this equation gives:

$$\frac{v}{v_e} = - \ln_e m + c$$

c = constant of integration.

To evaluate c, note that initially, when the rocket was fired v = o

and $m = m_o$, therefore:

$$o = -\ln_e m_o + c \quad \text{and} \quad c = \ln_e m_o$$

Thus: $$\frac{v}{v_e} = -\ln_e m + \ln_e m_o = \ln \frac{m_o}{m}$$

Here, m is the mass remaining when the speed is v. When all the fuel has been burned, we can write m_a for m and V for v, getting

$$\frac{V}{v_e} = \ln \frac{m_o}{m_a}$$

Section 6 - Center of Mass

PURPOSE To introduce the center of mass of a system composed of two objects.

CONTENT The center of mass is at a point on the line joining two masses m_1 and m_2 such that the distances x_1 and x_2 from the center of mass to the respective objects are related by the equation, $m_1x_1 = m_2x_2$.

If a system of two objects is isolated, the velocity of the center of mass, $\vec{v}_{cm}$, does not change. The total momentum of the system is $(m_1 + m_2)\vec{v}_{cm}$.

EMPHASIS This section can be covered quickly at this stage. You can further develop the center of mass concept as you go along when it is particularly useful in the analysis of specific problems.

COMMENT If you have the time, discuss Problem 20 with the class, particularly drawing the diagram suggested in the TG solution to that problem.

* * *

Note that until the center of mass was introduced, the only objects to which Newton's laws could be applied properly were point particles. Any large object has many internal forces acting and these must be considered together with any external force in order to predict the object's motion.

The center of mass has the special property that, independent of internal forces, it obeys Newton's law of motion. This fact explains the phraseology in the text (e. g., last sentence in this section) such as the "center of mass moves as though all the mass is there" or "it always moves as though all the mass was there". You need not discuss this point with the class unless specific questions are raised.

* * *

If an external force is applied to an otherwise isolated object, the center of mass accelerates in the direction of the force in accordance with Newton's law of motion. If the line of action of the force goes through the center of mass, the entire object moves along with the center of mass without twisting. On the other hand if the line of action of the force does not go through the center of mass, the object will start to rotate about its center of mass while the center of mass moves according to Newton's law. Note that no matter which part of the object is subjected to the force, it is the center of mass which moves according to Newton's law. Note further that if the object is rotating when the force ceases to act, this rotation can continue. The center of mass can, in accordance with the law of inertia, move only in a straight line at constant speed. However, the other parts of the object can (and in general do) rotate about the center of mass because the internal forces produce the required acceleration.

* * *

Students can get a good deal from a careful study of Fig 21. If the wrench is 20 cm long and the flash interval is 1/30 second, what is the speed of the center of mass? The students should be able to find that the wrench moves a distance equal to its length (or very slightly over that) in two flashes). Hence $v = \frac{.20}{.067} = 3\text{m/sec}$.

Which way is the wrench moving? Very careful measurements are needed to find that the wrench enters the picture approximately perpendicular to its motion and leaves parallel to its motion.

Is the picture distorted by the camera? Yes, the wrench size seems different at the two ends of the picture.

Ask the students for the average speed of different points of the wrench. This may be a tricky question for some.

What is the speed of rotation of the wrench about its center of mass? It rotates about once in 14 flashes (about $\frac{1}{2}$ sec).

* * *

An interesting home exercise of class discussion can be developed from the collision of the two equal mass billiard balls of Figure 9 of the text.

Make a tracing of the positions of the center of the two balls at various times. This might look like Figure (a). Join the positions of the two balls after the collision as shown at (b) and mark the positions of the midpoint of the line, i. e. the center of mass. Do this for the two balls before the collision as at (c). This may be a little messy, but can be done. Point out that the center of mass moves with uniform velocity.

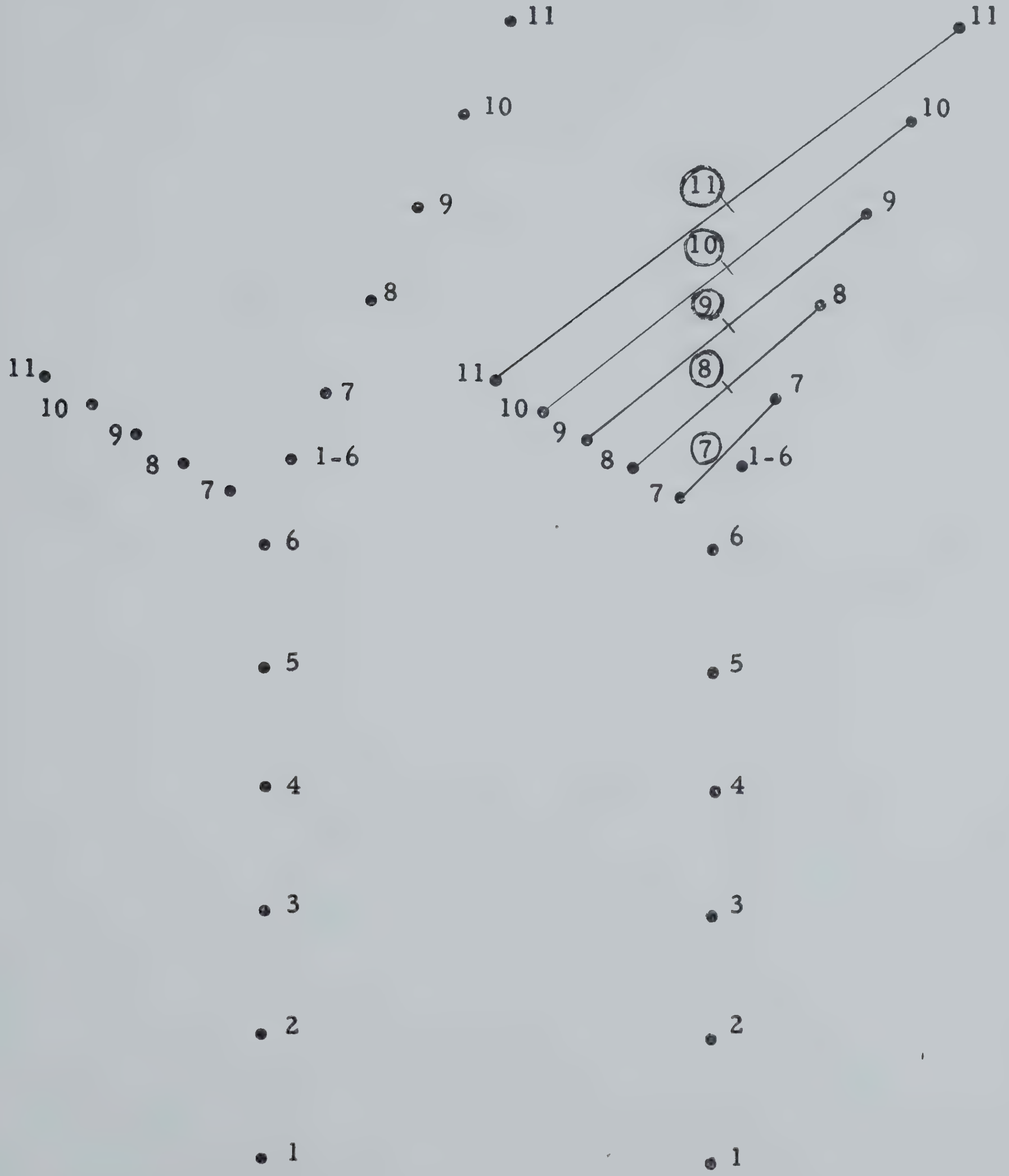


Figure (a)

Figure (b)

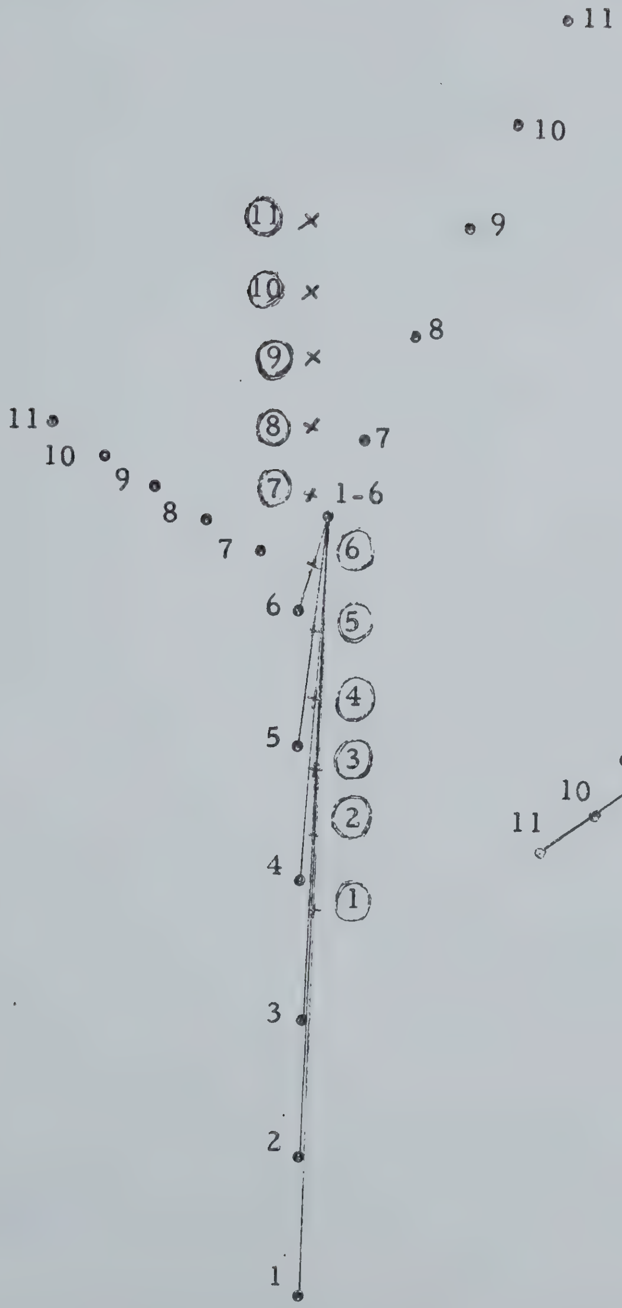


Figure (c)

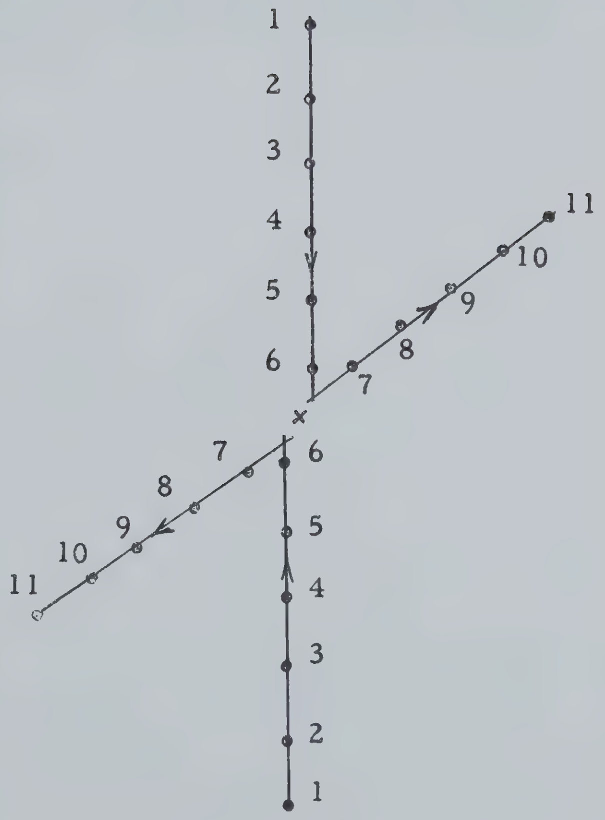


Figure (d)

An extension of this might be to suppose we were riding on the center of mass and see what the collision would look like. Take a second sheet of tracing paper and mark a spot, C. M. for the center of mass. Place it over the center of mass in the previous tracing and mark the position of the balls at this time. Do the same for each successive position of the center of mass. A final picture will looks like (d). The things to notice are that the balls seem to come in from opposite directions with equal velocities (actually equal momenta, but here we are dealing with equal masses) and to go out in a different direction with the same velocities as they came in. Actually the velocities as they leave are less than they were as they came in since energy was not conserved.

Another exercise might be developed from one of the collisions of unequal-mass billiard balls. Join the two balls with a straight line as before, and see if the students can tell where the center of mass is by assuming it must move in a straight line and with equal spacings before and after the collision. Again the second tracing, keeping the position of the center of mass fixed, may be made. The collision can be seen to be very simple when observed in this coordinate system.

Section 7 - The Conservation of Momentum in General

PURPOSE To extend momentum conservation to systems composed of many objects

CONTENT Momentum is conserved in any isolated system independent of the number of objects in the system.

EMPHASIS It is important for the students to know that momentum conservation is general. However, most of them will probably be convinced from the earlier material. Detailed understanding of the argument is not essential. This section can be handled as a reading assignment with a brief discussion of the conclusions if you choose.

COMMENT Since momentum conservation for a two body system is introduced as a fact based on experiment, there is only modest additional gain in extending momentum conservation to include any isolated system. Inasmuch as momentum conservation depends on experiment it cannot really be proved. However, once it has been established for a two body system, we need only a principle of additivity to extend it to an n-body system. Since the parameters involved, force and momentum, are vector quantities which have been established to be additive, we can conclude that the generalization will be valid. Even if this plausibility argument does not satisfy every student, you should be sure that they recognize that momentum conservation has been established experimentally for all types of systems.

* * *

Some students may wonder about what happens to the center of mass if light is emitted. (For example, one student wanted to know if momentum was conserved if mass 2a and 2b in figure 22 exchanged light signals.) You can avoid this point entirely. However, if the question is raised, you might handle it as follows.

A photon of light which carries an energy, E, can be shown by electrodynamics to be associated with a momentum E/c. (It is this momentum which gives rise to the pressure of light which you may have seen demonstrated in a PSSC film.) Now imagine a closed box, having a mass M, at rest in your coordinate system. (Its center of mass is, then, also at rest.) Suppose that a photon

of energy E is emitted from one end of the box and absorbed at the other. No external force has acted on the box and its contents, so its center of mass must have remained fixed. Look at Figure (a).

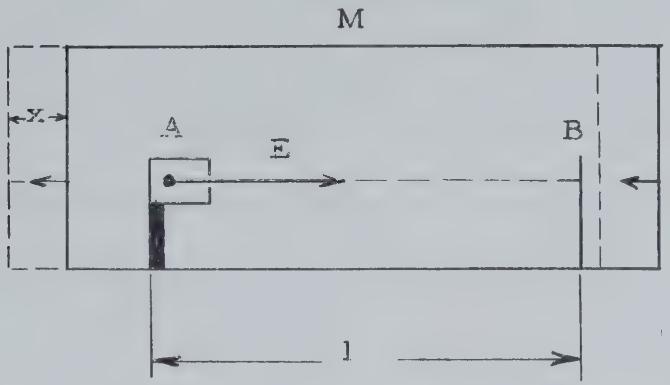


Figure (a)

The photon has momentum E/c, therefore when it leaves A the box must recoil with momentum E/c = Mv. Thus the speed with which the box goes to the left is:

$$v = \frac{E}{Mc} \qquad \text{(c is the speed of light)}$$

When the light is absorbed at B the photon gives the box an impulse which is equal in magnitude but opposite in direction to the one that was imparted to the box when it was emitted from A. Thus everything is again brought to rest. But the box has moved to the left in the process. How far?

The photon was in flight for a time

$$t = \frac{l}{c}$$

During this time the box was recoiling with the above speed, v. Therefore the distance of the displacement of the box must have been:

$$x = vt = \frac{E}{Mc}\ \frac{l}{c} = \frac{El}{Mc^2}$$

Thus the mass, M, of the box has moved a distance, x to the left. Since no external force has acted on the system, the center of mass must have remained fixed. Then when the photon traveled

a distance l from left to right, a mass m_{photon} must have been transferred with it. To find a value for this,

$$m_{photon} l = Mx = M\frac{El}{Mc^2}$$

$$m_{photon} C^2 = E$$

corresponding to Einstein's mass equivalence of energy.

Section 8 - Forces of Interaction

PURPOSE To introduce the ideas of the equal and opposite forces between interacting objects.

CONTENT If object A exerts a force on object B, object B exerts an oppositely directed force of equal magnitude on A.

EMPHASIS The equality of the two interaction forces is an important concept which the students have used and will continue to use. There is no need to stress the distinction between Newtonian and non-Newtonian forces at this time.

COMMENT Do not bother to try to give a rigorous definition that will distinguish Newtonian from non-Newtonian forces. As physicists analyze matter and interactions in more detail, a time lag is found to exist in the action of any type of force. Most physicists do not believe that any signal can travel faster than light; hence many of the familiar force fields propagate very much like the light emitted by a super nova as discussed in the text.

* * *

It is not necessary to analyze the super nova example in any detail. If you attempt to be precise, you may find some students asking questions which are almost semantic.

What is the definition of a force?

Does the light emitted by a super nova, comprise a material part of the super nova? If not, how can the center of mass of the super nova (at rest before the light was emitted) be moving after the light is emitted? Is something wrong with momentum conservation?

If the light is part of the super nova, why is it so strange that we give the super nova a counter shove?

What is the nature of the force with which the super nova shoves the light? Since light must always travel with the speed of light, how can you speak of the light as being shoved?

If your students have come to consider momentum conservation only in cases where the force acts for a very short time, there may be some confusion about the equal and opposite forces of this section. They should now understand that when two bodies interact, $\Delta P_1 = - \Delta P_2$. Then if $\Delta P_1 = F_{2-1} \Delta t$, and $\Delta P_2 = F_{1-2} \Delta t$, it follows that $F_{2-1} = - F_{1-2}$. Students who think only of the complete change can only see that the *average value* of F_{2-1} equals minus the *average value* of F_{1-2}. In order to appreciate this section, they must realize that $\Delta P_1 = -\Delta P_2$ for *any interval of time* they may wish to consider. Then if the particular Δt is chosen small enough so that the forces do not change appreciably during the time interval, the two forces are equal and opposite at that arbitrary time. They are therefore equal and opposite at all times. The Δt used in this proof is not "the time taken by a collision or explosion". It is any small interval of time we choose.

The words "equal and opposite forces" may seem contradictory--- in fact, they *should* seem contradictory if your students have been taught vectors throughly. Two vectors are equal only if they have the *same* direction. The words just occur so often in physics that "equal in magnitude and opposite in direction" has been contracted to "equal and opposite":

* * *

It might be helpful, when discussing the isolation of a system, to introduce the following well-known paradox:.

A horse is harnessed to a wagon and tries to pull it down the street. However, as hard as he pulls on the wagon, the wagon pulls back just as hard on him. Then how can he ever get going?

The paradox vanishes as soon as you remember that you must isolate the system in which you are interested. If you wish to understand the motion of the wagon, isolate the wagon and consider the external forces which act on it. The horse pulls it forward and the friction of its wheels on the ground hold it back. When the horse's pull exceeds the force of friction, the wagon accelerates forward.

If you are interested in the motion of the horse, isolate it. The external forces which act are the pull of the wagon, backwards, and the push of the ground forward on the horse's feet (in reaction to their backward push on the ground). When the latter exceeds the former the horse accelerates forward.

For Home, Desk, and Lab.

Table of Problems

Section	Easy	Medium	Hard	Class Discussion	Experiments
1	1, 3, 11	2*, 4*	22*	4, 22*	520
2	5, 6	7*		5	
3 and 4	10*, 16*, 21	8*, 9, 12, 13*, 15	14, 17	9*, 14	530 540
5		18*	22*	18, 22*	[redacted]
6	19, 21, 22*		20*	20*	
7			23, 24*	24*	
8					

Although problem 22 can be solved with only the material of section 1, it is probably best to defer it until after section 5.

Asterisks indicate important problems.

EXERCISE 1: When an impulse of 2 newton-sec is applied to a 6 kg object, what happens? If the mass is 3 kg, what happens?

When an impulse is applied to any object, that object changes its momentum by an amount equal to the impulse. If the 6 kg object started from rest it attained a momentum of 2 newton-seconds after the impulse was applied.

$$6\text{ kg} \times v = 2 \text{ newton second} = 2\ \frac{\text{Kgm}}{\text{sec}} \qquad \frac{\text{Kg-m}}{\text{sec}}$$

$$6 \times \text{velocity} = 2$$

$$\text{velocity} = \frac{1}{3}\ \text{meter per second}$$

Note that the units kilogram-meter per second are the same as newton-seconds

In the second case, if the mass were $\frac{1}{2}$ as great the speed would have to be twice as great to have the same final momentum. The velocity of the 3 kg mass would be 2/3 meter per second.

If we do not assume any particular initial condition we cannot give a definite answer to the problem. This might be worth pointing out, but for a simple problem like this, hardly worth exploiting.

EXERCISE 2: A constant force applied to a 2 kg object at rest moves it 4 m in 2 sec. What impulse was applied to the object?

There are two different ways in which this problem may be [illegible] solved. The first involves finding the force and multiplying it by the time to get the impulse; the second involves finding the change in momentum directly and realizing this equals the impulse.

The object moves with constant acceleration for 2 seconds since

$$d = \frac{1}{2} at^2$$

$$4 = \frac{1}{2} a\, 2^2;\ a = 2 \text{ meters per sec}^2.$$

Then since F = ma, F = 2 x 2 = 4 newtons. The impulse is then ✓

4 newtons x 2 seconds = 8 newton~~s~~-sec.

In the second method we note that its average velocity during the period is 4 meters ÷ 2 seconds or 2 meters per sec. Its final velocity is twice this, or 4 meters per sec. [Alternatively we could write $v = at$, $d = \frac{1}{2} at^2$, and eliminate a, obtaining $v = \frac{2d}{t}$ as above.] But if the object has a velocity of 4 m/sec and a mass of 2 kg, its momentum is 8 kg m/sec. The impulse therefore must have been 8 kg m/sec or newton-seconds as before.

EXERCISE 3: A skier on level ground at the bottom of a hill has a mass of 80 kg and is moving at a speed of 10 m/sec. Through some miscalculation, he finds himself brought to rest in a snowbank during a Δt of 1.5 sec.

(a) What impulse did the snow apply to the skier?
(b) What was the average force exerted by the snowbank to produce this change of speed?

a) Since the skier had a momentum of 80 x 10 = 800 kg - m/sec and was completely stopped by the snow his change of momentum was 800 kg - m/sec and this was therefore the impulse applied to him. This is the same as 800 newton seconds.

b) $F\Delta t$ = impulse. Then 1.5F = 800 or the average force was 533 newtons. Note that the force may have been much larger at times and smaller at other times but that its average was 533 newtons.

EXERCISE 4: A body of mass 10 kg moves at a constant velocity of 10 m/sec. At an instant of time, which we call t = 0, a constant force pushes on the body, and acts for 4 sec. After the constant force stops acting, the velocity of the body is measured and is found to be 2 m/sec in a direction opposite to the original motion. Calculate the impulse of the force and the magnitude and direction of the force.

The initial momentum of the body was 10 x 10 = 100 kg-m/sec. The final momentum was 2 x 10 = 20 kg-m/sec in the opposite direction. We might say -20 kg-m/sec. The total change in momentum was then 100 - (-20) = 120 kg-m/sec. This is also, of course, the total impulse. Since the force acted for 4 seconds $F\Delta t = 4F = 120$. F = 30 newtons. Its direction was opposite to the initial direction of motion.

EXERCISE 5: What is the momentum of a 70 kg man moving north at 100 km/hour (60 miles/hour)? What impulse will stop him?

The man's momentum is mv = 70 kg x 100 x 1000 meter x $(3600 \text{ sec})^{-1} = 1.9 \cdot 10^3$ kg-m/sec north. The purpose of this exercise is obviously to emphasize the north or vector nature of momentum and impulse. The impulse necessary to stop him is therefore $1.9 \cdot 10^3$ kg-m/sec or newton-seconds directed south.

EXERCISE 6: A 0.5 kg ball is thrown straight up at 3 meters/sec.

(a) What is its initial momentum?
(b) At the top of its rise, what is its momentum?
(c) What impulse stopped it and how long did that impulse take?
(d) If the ball were 1.0 kg, what would be different in (a), (b), and (c)?

This exercise may be used to emphasize the fact that impulse, as well as being applicable does not when forces act for a very short time, may equally well apply in cases involving other types of forces. The problem also can be used to emphasize the role which mass plays in impulse and momentum.

a) momentum = mv = .5 x 3 = 1.5 kg-m/sec

b) Since it stops momentarily at the top, its momentum is zero.

c) The change in momentum was 1.5 kg-m/sec and therefore the impulse which stopped it was 1.5 newton-sec directed down.

$$F\Delta t = 1.5 \text{ and } F = 0.5 \times 9.8 \text{ newton}$$

$$\therefore \Delta t = \frac{1.5}{0.5 \times 9.8} = 0.31 \text{ sec}$$

d) In (a) the momentum would double. The answer to (b) would still be zero. In c, the impulse to stop it would double, but so would the force; hence the time would remain the same.

EXERCISE 7: A body is made of two masses m_1 and m_2, 2 kg and $\frac{1}{2}$kg respectively, tied together by a light thread (Fig. 23a). It moves on a frictionless table starting from rest. We set it in motion by applying an impulse of 10 newton-secs to the body, but unfortunately the thread connecting m_1 and m_2 breaks during the course of the impulse (Fig. 23b). As a result m_2 goes off with great speed while m_1 at the end is only moving at a speed of 0.20 meters per second.

(a) How big an impulse did m_1 receive?
(b) How big an impulse did m_2 receive?
(c) How fast is m_2 moving at the end of the impulse?
(d) How big an impulse was delivered before the string broke?

The final momentum of m_1 is 2-kg x 0.2 m/sec or 0.4 kg-m/sec. The total momentum of the two bodies is 10 kg-m/sec since that was the total impulse applied. The final momentum of m_2 is then 9.6 kg-m/sec.

Therefore,

a) m_1 received an impulse of 0.4 kg-m/sec, and since the total impulse was 10 newton-sec,

b) m_2 received an impulse of 9.6 kg-m/sec

c) $m_2 v_2 = 9.6$; $v_2 =$ 19.2 meters/sec.

d) This part can only be answered by making some assumptions about the thread. One of the assumptions that leads to a simple solution is that the thread was taut to begin with and is inextensible so that the two masses move with the same velocities. Since m_2 gained no more velocity after the thread broke, its final velocity is indicative of the velocity of both m_1 and m_2 at the time the thread broke. The total momentum at this time was, then,

$$(2 + \frac{1}{2}) \times 0.2 = \underline{0.5 \text{ kg-m/sec}} \text{ or newton-sec}$$

and this was the total impulse delivered up to that time. It is highly unlikely that, for a real impulsive type of force, this is a good assumption. For example if the thread were at all slack, m_1 could have been struck a quick blow giving it a velocity of 20 m/sec. The thread would then straighten out and break, slowing m_1 and giving m_2 a pull. In this case the whole impulse would have been delivered before the thread broke.

EXERCISE 8: Fig. 24 shows a flash photograph of an "explosion" of two billiard balls. The larger one is of mass 201 grams and the smaller of mass 85 grams. The pictures were taken at intervals of one-thirtieth of a second. Your measurements on the photograph should give you a simple check on the conservation of momentum. If you find that the momenta of the two balls are not equal in magnitude, how would you explain this?

The momentum before the "explosion" the momentum should again be zero. This can be so only if the two balls have momenta of equal magnitude and opposite in direction. Suppose that between any two flashes the larger ball goes a distance D. If t is the time between flashes it is moving with a velocity D/t and with a momentum 201 D/t. Likewise the

smaller moves with a velocity d/t, and momentum 85 d/t. It follows that d/D = 201/85 = 2.36. Therefore if the distance travelled by the smaller ball is 2.36 times the distance travelled by the larger ball between flashes conservation of momentum is demonstrated in this instance, provided (and many students will probably forget this) that the two balls are also shown to be moving in exactly opposite directions.

If the two momenta do not turn out to be of equal magnitude or opposite in direction there can be several explanations.

a) During the "explosion" some interaction with other bodies (air, table) took place.

b) The table was not frictionless.

c) The measurements made were not accurate or not treated correctly afterwards.

d) There was distortion in the photographing and printing of the picture. This is often not negligible.

e) And probably several others which we have forgotten! But remember, it need only check as well as the original measurements were made! (Note that spinning of balls would have no effect.)

EXERCISE 9: A man is in the middle of a pond on perfectly frictionless ice. Can you think of some way in which he might get himself to shore?

This problem, which is quite a standard one, can serve nicely as the basis for reviewing isolated systems as well as conservation of momentum.

If we suppose for the moment that there is no air, then there is no way for the whole man to get to shore. He can only go off in one direction if he throws something in the other. Fortunately it can be very small, since he will move steadily once he gets going. If there is air friction he can make a propeller with his hands. If there is no air friction, but still air he can make a jet with his breath---and turn his head to breath in.

It must be added, in the spirit of a high school student, that probably all the man has to do is wait because the problem didn't say he was at rest on the ice!

EXERCISE 10: A 20 kg cart is moving at a speed of 2 meters/sec. A boy whose mass is 60 kg jumps off the cart. What is the change in velocity of the cart if he jumps off the cart so that when he hits the ground he is

(a) moving at the same velocity as the cart?

(b) moving with no velocity relative to the ground?

(c) moving with twice the velocity of the cart?

This is a standard problem on the conservation of momentum.

Initially cart (20 kg) and boy (60 kg) are moving with a velocity of 2 meters/sec and hence the total momentum of the system is (20 + 60) x 2 = 160 kg-m/sec.

a) In this case the boy has a final momentum of 60 x 2 = 120 kg-m/sec which leaves 160-120 = 40 kg-m/sec for the cart. Since it has a mass of 20 kg it has a velocity of 40 ÷ 20 = 2 m/sec. There is no change of the velocity of the cart. We could have seen this in the first place.

b) If the boy ends up with no velocity and hence no momentum the cart must have all the original 160 kg m/sec. It then is going with a velocity = 160/20 = 8.0 meters/sec. Therefore the change is 6 M/sec. Physically this comes about because the boy must jump backwards off the cart so as to arrive on the ground with no momentum. In doing this he pushes the cart forwards.

c) If the boy moves with twice the velocity of the cart he is moving with a velocity of 4 meters/sec. His momentum is then 60 x 4 = 240 meters/sec. If the final momentum of the cart is P_f

$$P_f + 240 = 160; P_f = -80 \text{ m/sec.}$$

It has a velocity of -80/20 = -4 or 4 meters/sec in the direction opposite to that in which it was originally moving. The change is therefore -6 M/sec.

EXERCISE 11: The same boy gets on the same cart as in problem 10 and moves along at the same initial speed again. Then he jumps off the cart again, applying a steady force to the cart for 0.2 sec and giving the cart a forward impulse of 20 newton-sec.

(a) What force did he apply?

(b) What was the final velocity of the cart?

(c) With what horizontal velocity did he hit the ground?

a) The impulse is the product of the force and the time or 20 = F x 0.2 or the force was 100 newtons.

b) The initial momentum of the cart was mv = 20 x 2 = 40 kg-m/sec. An impulse of 20 newton-sec was applied giving a final momentum of 60 kg-m/sec corresponding to a velocity of $\frac{60}{20}$ = 3 m/sec. Of course this can also be done using F = Ma rather than momentum.

c) The momentum of the boy, originally was 60 x 2 = 120 m/sec. The boy applied an impulse of 20 newt-sec to the cart and by Newton's Third Law had an opposite impulse of 20 newton-sec applied on him. His final momentum was then 120-20 = 100 kg-m/sec. His final velocity, then, was 100/60 = 1.7 m/sec.

EXERCISE 12: Two heavy frictionless carts are held together at rest by a loop of string (Fig. 25a). A light spring is compressed between them. When the string is burned, the spring expands from 2 cm to 3 cm length, and the carts move apart. After moving for a while, they collide with bumpers fixed to the table. Both hit at the same instant, but cart A moved 0.45 meters while cart B moved 0.87 meters (Fig. 25b).

(a) What was the ratio of the velocity of A to that of B after the interaction?

(b) What was the ratio of their masses?

(c) What was the ratio of the impulses applied to each cart?

(d) What was the ratio of the acceleration of the carts?

This problem is a simple one on momentum conservation which also can be used to introduce the ideas of Newton's Third Law, which does not come until section 8 of text.

a) Cart A moved 0.45 meters during a time t and Cart B moved 0.87 meters in the same time. Their velocities were

$$v_a = \frac{0.45}{t}, \quad v_b = \frac{0.87}{t} \text{ and } \frac{v_b}{v_a} = \frac{0.87}{0.45} = 1.9 \text{ or}$$

$$\frac{v_a}{v_b} = 0.52$$

b) Since the two momenta must be equal in magnitude

$$m_a v_a = m_b v_b \text{ or } \frac{m_b}{m_a} = \frac{v_a}{v_b} = \frac{1}{1.9} = 0.52 \text{ or } \frac{M_a}{M_b} = 1.9$$

c) The impulses were equal and opposite since momentum is conserved or since no net external $\vec{F}$ acted on the system. Therefore the impulses were equal but opposite.

$$\text{Since average } a = \frac{v}{t} \quad \frac{a_a}{a_b} = \frac{V_a}{V_b} = \frac{1}{1.9} = 0.52$$

EXERCISE 13: If a proton (mass 1.67×10^{-27} kg) with a speed of 1×10^{7} m/sec collides with a helium nucleus it bounces back with speed of 6×10^{6} m/sec. The helium nucleus moves forward with a velocity of 4×10^{6} m/sec after the bombardment.

(a) Can you compute the mass of the helium nucleus? If so, what is it?

(b) Can you compute the force that acted during the collision? If so, what is it?

This is a conservation of momentum problem with a nuclear physics flavor.

a) Let m_p be the mass of the proton and v_p its velocity before the collision. Since we assume the helium nucleus to be at rest, the total momentum before the collision is $m_p v_p$. After the collision it is $m_{he} v_{he} - m_p v'_p$ where v'_p is the final velocity of the proton, the minus sign occuring because velocity v'_p is opposite in direction to all the other velocities.

Then

$$m_p v_p = m_{he} v_{he} - m_p v'_p$$

$$m_p \; 10 \times 10^6 = m_{he} \; 4 \times 10^6 - m_p \; 6 \times 10^6$$

$$m_{he} = \frac{16}{4} m_p = 4 \, m_p = 6.68 \times 10^{-27} \text{ kg.}$$

A quick check shows that these velocities also satisfy the requirement of conservation of energy, but that is a later story.

b) The force cannot be computed unless you know how long the collision lasted. Actually from other types of experiments we know that the forces are exerted over distances of about 2.0×10^{-15} m. It took about $2.0 \times 10^{-15} \div 1 \times 10^{7} = 2.0 \times 10^{-22}$ sec for the collision; hence the forces are about

$$F = \frac{mv}{t} = \frac{1.67 \times 10^{-27} \times 1 \times 10^{-7}}{2 \times 10^{-22}} = 83 \text{ newtons}$$

--which is only an order-of-magnitude estimate. The average velocity might be smaller; the distance might be smaller since the proton and

helium might barely touch. Even if v is 10 times smaller, the force is of the order of 10 newtons which is fantastically large considering that this is the force between two such small particles.

EXERCISE 14: In Fig. 26 the ball came in at the top of the picture and the little ball at the bottom. As you see, a collision took place in the middle.

(a) Plot the change in the velocity of the big ball and the change in the velocity of the small ball to the same scale. Are these changes opposite in direction?

(b) Are they equal in magnitude? If the magnitudes differ, by what factor should they be different?

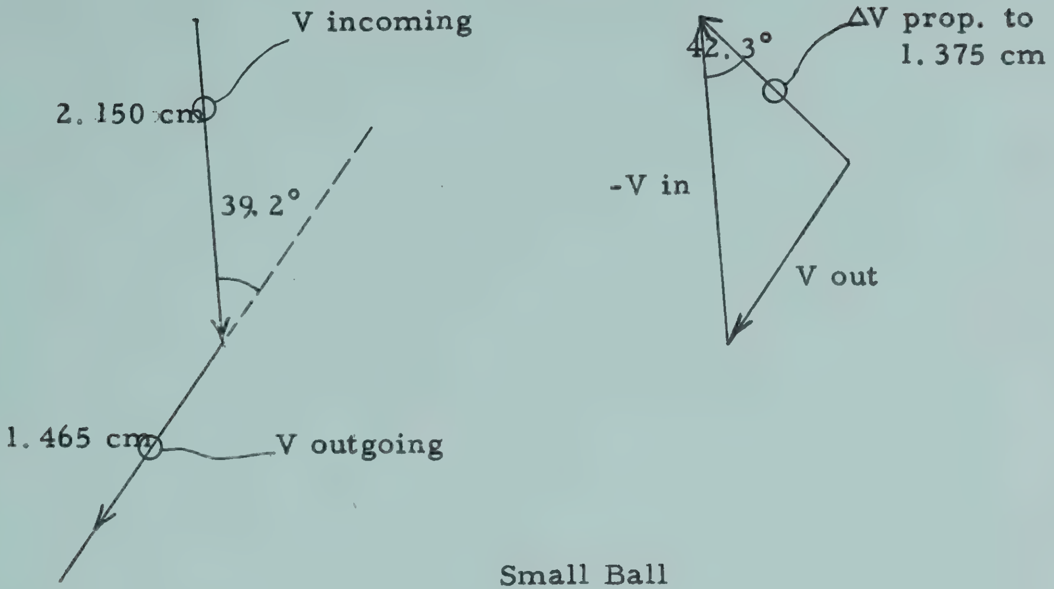
(c) The mass of the bigger ball is 201 grams. What is the mass of the smaller ball?

This problem is a graphical exercise on a collision.

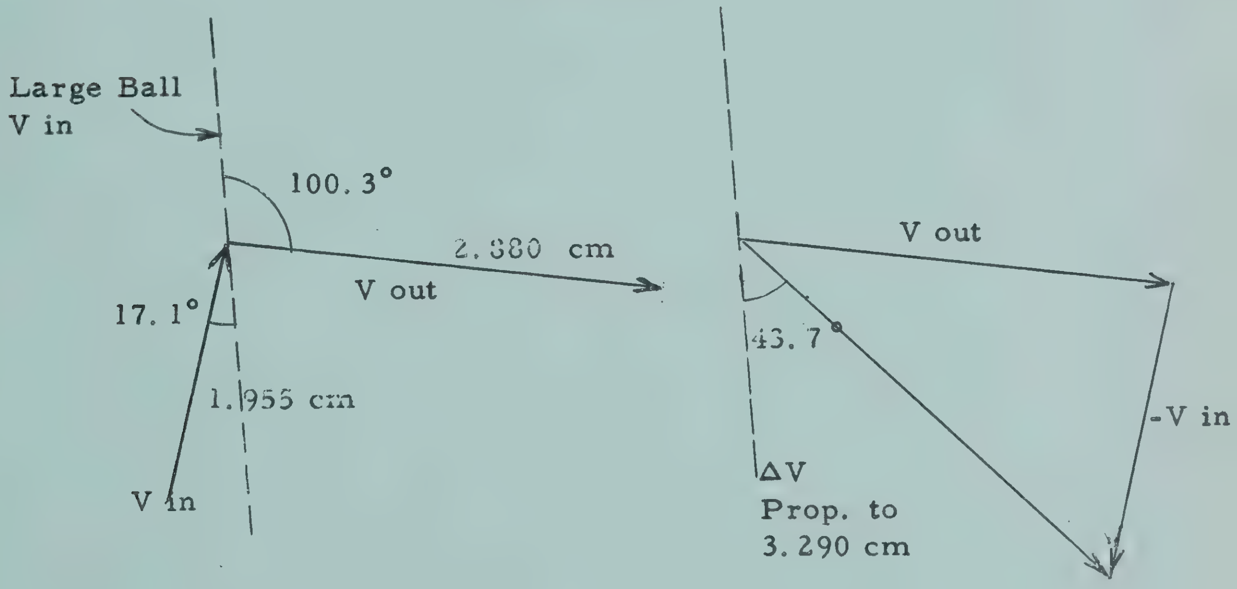
Students should be encouraged to work to 0.1 millimeter and 0.1° accuracy although the distortions in the picture are certainly greater than this. (Note the apparent change of size in the little ball as it leaves the picture). Depending on how much practice the students have had at this sort of exercise it may be well to point out ahead of time that they can plot velocity vectors proportionally even though they do not know either the real distance scale or the time between flashes. The velocity of an object may be plotted as proportional to the distance between successive images. Directions of travel can probably best be determined by drawing along one side of the line of images of the ball using tracing paper if it is not permissable to mark the text. Distances can probably best be determined by using the line of centers, measuring between the most brightly lit edges.

a) A rough drawing is shown below

Big Ball



Small Ball



The angles and lengths given in the drawing were obtained from a print of the original photograph and could be slightly different in the printed version or with a different set of measurements. The change

in velocity of the big ball ($v_{\text{outgoing}} - v_{\text{incoming}}$) is seen to lie along a line making an angle of 42.3° with the incident velocity of the big ball whereas the change in velocity of the little ball lies along a line making a 43.7° angle and in the opposite direction. The error of 1.4° is somewhat greater than the expected errors of measurement and is probably due to distortion in the photograph. Theoretically, of course, the change in velocity is proportional to the impulse and since the two impulses are equal but opposite in direction the two changes in velocity should be in exactly opposite directions.

b) The two changes in velocity are not equal. The changes in momentum are expected to be equal and opposite, i.e.

$$m_{\text{big ball}} \vec{v}_{\text{out big ball}} - m_{\text{big ball}} \vec{v}_{\text{in big ball}}$$

$$= -(m_{\text{little ball}} \vec{v}_{\text{out little ball}} - m_{\text{little ball}} \vec{v}_{\text{in little ball}})$$

$$\text{or } m_{\text{big ball}} \Delta\vec{v}_{\text{big ball}} = -m_{\text{little ball}} \Delta\vec{v}_{\text{little ball}}.$$

$$\text{Hence } \frac{\Delta v_{\text{big ball}}}{\Delta v_{\text{little ball}}} = \frac{m_{\text{little ball}}}{m_{\text{big ball}}}$$

c) The measured ratio of Δv's was

$$\frac{\Delta v_{\text{big ball}}}{\Delta v_{\text{little ball}}} = \frac{1.375}{3.290} = 0.418$$

If the big ball has a mass of 20/grams the little ball is seen to have a mass of 0.418 x 20) = 84.0 grams. Its actual mass was 85.0 grams so that the agreement is quite good.

NOTE: You may be interested in checking conservation of energy, particularly for possible future use of this problem. Using the values measured above

$$\frac{1}{2} \times 201 \times 2.150^2 + \frac{1}{2} \times 84 \times 1.955^2 = \frac{1}{2} \times 201 \times 1.465^2 + \frac{1}{2} \times 84 \times 2.880^2$$

$$465 + 160.5 = 215.5 + 348.0$$

$$625.5 = 563.5$$

Hence approximately 10% of the energy was lost to heat and rotation. Some of the rotation can be seen on the photograph, but accounts for only about 0.1% of the energy.

EXERCISE 15: A double star consists of two large masses that attract each other gravitationally. By observing the motion of both masses, we can see that they rotate around each other.

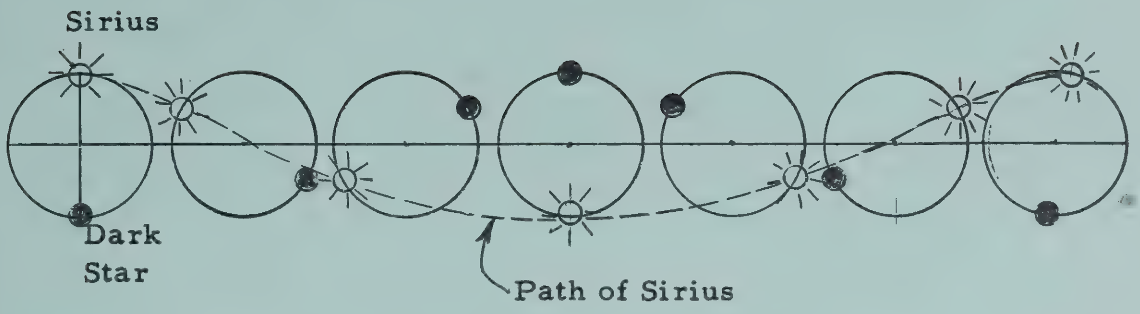
(a) What do you think happens to the momentum of each of the masses in a double star as time goes on? Explain your answer.

(b) When observed carefully, the bright star Sirius seems to wobble about, instead of having a uniform motion of the center of mass. From this and other evidence astronomers believe that Sirius has a dark companion. It is really a double star. How does this explain the peculiar observed motion?

This problem asks the student to discuss conservation of momentum in an astronomical problem.

a) The momentum of one member of a double star does not have to remain constant because there is an external force--the gravitational attraction of the other member--acting on it. (Momentum changes due to emitted light are negligible.) Therefore the momentum of each member changes, but the sum of the two momenta remains constant since we presume the double star system is far enough away from all other stars that it may be considered isolated from them. This means that when one momentum changes, the other must change by an equal amount in the opposite direction. It can be shown that the changes are periodic, i.e. repeat themselves as the two stars revolve about each other, thus each star appears to "wobble" in its path.

b) Suppose, for a moment, that Sirius and its dark companion have equal masses and are rotating in a circular orbit about each other as they together move through space. The point half-way between moves uniformly thus



EXERCISE 16: A stationary refrigerator car with mass 2×10^4 kg is rammed by a loaded gondola car with mass 3×10^4 kg. If the gondola before impact was going 1.0 m/sec, and they lock together, what is the new velocity?

Before they collide only the gondola car was moving. The total momentum of the system is the just the momentum of the gondola car or $m_g v_g = 3 \times 10^4$ kg x 1.0 m/sec = 3×10^4 kg-m/sec. After the impact we know that the total momentum of the system is the same as before. We also know that the two cars are coupled, i.e. move with the same velocity. Then

$$(m_g + m_r)\ V_{after} = \text{momentum before} = 3 \times 10^4$$

$$(3 \times 10^4 + 2 \times 10^4)\ V_{after} = 5 \times 10^4 V_{after} = 3 \times 10^4$$

$$\underline{V_{after} = 0.6 \text{ m/sec}}$$

Most students have seen the complicated banging around when two cars couple and it may be well to mention the complicated forces at work here with the very simple final result.

EXERCISE 17: When a proton (mass 1.67×10^{-27} kg) collides with a neutron (mass 1.67×10^{-27} kg) the two can stick together forming a deuteron (whose mass is 3.34×10^{-27} kg). (Actually the neutron is slightly more massive than a proton and the deuteron is slightly less massive than the sum of the two. However, these differences

are less than 1 part in 400.) With what velocity will the deuteron move if it is formed from a proton moving with a velocity of 7×10^{6} m/sec to the right and a neutron moving with a velocity of 3×10^{6} m/sec to the left?

This is a problem in conservation of momentum involving nuclear particles rather than railroad cars. It serves to illustrate the wide range of application of these principles. The teacher should read the statement at the end of this solution before assigning it.

The initial momentum is $(1.67 \times 10^{-27}) \times (7 \times 10^{6}) - (1.67 \times 10^{-27})(3 \times 10^{6})$, the minus sign coming from the fact that the particles are moving in opposite directions. The final momentum must therefore also be $(1.67 \times 10^{-27})(4 \times 10^{6})$. But it must also be $(3.34 \times 10^{-27}) \times$ (velocity of deuteron). (3.34×10^{-27}) (velocity of deuteron) $= (1.67 \times 10^{-27})(4 \times 10^{6})$ velocity of deuteron = 2×10^{6} m/sec in the direction in which the proton was travelling. Clearly m could have been used for (1.67×10^{-27}) and the students should in fact be encouraged to do so.

This problem differs in principle from the collision between two railroad cars where the "lost" energy goes into heat which is the random motion of molecules and hence carries no net momentum. Here the "lost" energy cannot go into heat (which does not exist on a microscopic scale) but actually goes into an x-ray. This x-ray carries momentum which must usually be taken into account. With the numbers given in this problem, the momentum carried off by the x-ray is about 20% of the final momentum of the deuteron.

EXERCISE 18: A fire hose with a nozzle that we can turn on and off squirts bursts of water onto a truck from behind. The truck can roll along a level surface with very little friction. The body of the truck is not leaky, and the water stays in it.

(a) Suppose the truck has a mass of 4.5 tons (4.5×10^{3} kg) and that it is at rest when the first burst of water goes into it. The water comes squirting in at 20 meters per second horizontal speed, and the first burst is half a ton. What is the momentum of the truck after receiving the first burst? With what speed is it moving?

(b) We now squirt a second burst into the truck. The water moves at the same speed, 20 meters per second, with respect to the ground. How large does the second burst have to be so that the truck gains the same amount of speed the second time?

(c) If you increase the speed with which the water is shot out so that each burst moves with the same relative velocity with respect to the truck, what would this do to your answer to part (b)?

This problem is a rather contrived one leading up to the principles of rocket propulsion.

a) This part is a straightforward momentum conservation problem. Before the water "collides" with the truck it has a momentum of

$$0.5 \times 10^3 \text{ kg} \times 20 \text{ m/sec} = 10 \times 10^3 \text{ kg-m/sec.}$$

After the collision the truck and water are moving with the same momentum i.e.

$$(4.5 + 0.5) \times 10^3 \times V = 10 \times 10^3; \; V = \underline{2 \text{ m/sec}}.$$

b) This part involves some algebra. The initial momentum is now

$$10 \times 10^3 \; \begin{pmatrix}\text{momentum of truck} \\ + \text{ first burst of water}\end{pmatrix} + M \times 20 \; \begin{pmatrix}\text{momentum of new} \\ \text{unknown mass of water}\end{pmatrix}$$

The final momentum is

$$\underbrace{(5 \times 10^3 + M)}_{\text{final mass}} \times \underbrace{4}_{\text{desired speed}}$$

These two must be equal, or

$$10 \times 10^3 + 20M = 20 \times 10^3 + 4M$$

$$16M = 10 \times 10^3$$

$$\underline{M = \frac{5}{8} \times 10^3}$$

or more than $\frac{1}{2}$ ton for the second burst.

c) If the second burst were to move with 20 meters per second with respect to the truck it would have to move with 22 m/sec with respect to the ground. Then

$$10 \times 10^3 + 22M = 20 \times 10^3 + 4M$$

$$18\ M = 10 \times 10^3 \quad \underline{M = \frac{5}{9} \times 10^3}$$

or a mass in between 1/2 ton and the answer to part (b).

It may be interesting to consider how fast the truck could ever be pushed using the method of part (a) and (b). The answer is clearly that it can never move faster than 20 m/sec. Using the method of part (c), however, it could be pushed arbitrarily fast--neglecting the limitations of relativity and, more particularly, available fire hoses!

EXERCISE 19: A rocket is out in free space shooting out a stream of exhaust gases and picking up speed in the opposite direction. What happens to the center of mass of all the matter, that which is ejected and that which is left in the rocket?

Since the rocket is in "free space" it has no outside forces acting on it and its (meaning all its material including exhaust gases) center of mass must remain fixed or move with uniform velocity.

EXERCISE 20: In an unsuccessful attempt to put a satellite in orbit, the carrier rocket moving vertically upward at 3,000 meters/sec explodes and breaks into two pieces, one of which continues upwards but at an angle of 45° with the vertical, and with a speed of 3,500 meters/sec immediately after the explosion.

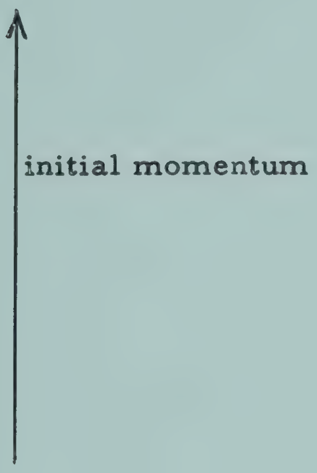
(a) What is the velocity of the second piece if its mass is 0.6 that of the first piece?

(b) What is the velocity of the center of mass of the rocket immediately after the explosion?

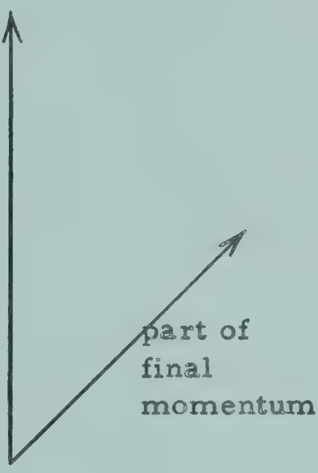
(c) What is the change in center of mass velocity in the next second?

This is an important problem on conservation of momentum and the motion of the center of mass of a system. It can be solved graphically or with components.

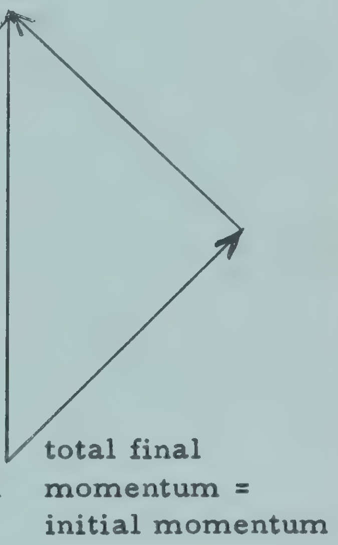
a) Let the mass of the first piece of rocket be m, the second piece 0.6m. To do a graphical solution of the problem we construct a vector proportional to the original momentum of the complete rocket i.e. proportional to 3000 m/sec x 1.6m = 4,800 m. We may make an arrow 4.8 cm vertically up as at (a). After the explosion we know that one piece had a momentum 3,500 m/sec x 1.0 m = 3,500 m 45° up and to the right as at (b). The second piece must have a momentum p such that it adds with the momentum of the first piece giving the initial momentum, as at (c). Measurement shows the momentum of the second piece to be 3,400 m at an angle of 47° to the vertical.



(a)



(b)



(c)

Then 3,400m = 0.6m (velocity) and the final velocity of the second piece is 5,660 meters/sec at an angle of 47° on the opposite side of the vertical (and going up).

Algebraically with components we find:

initial vertical component = 4,800 m kg m/sec if m is in Kgm
initial horizontal component = 0m
final vertical component of first piece = 3500 cos 45° x 1.0 m
= 2,475 m
final horizontal component of first piece = 3500 sin 45° x 1.0 m
= 2.475 m

Then $P_h + 2475\text{ m} = 0$ $P_h = -2475\text{ m}$

$P_v + 2475\text{ m} = 4800\text{ m}$ $P_v = 2325\text{ m}$

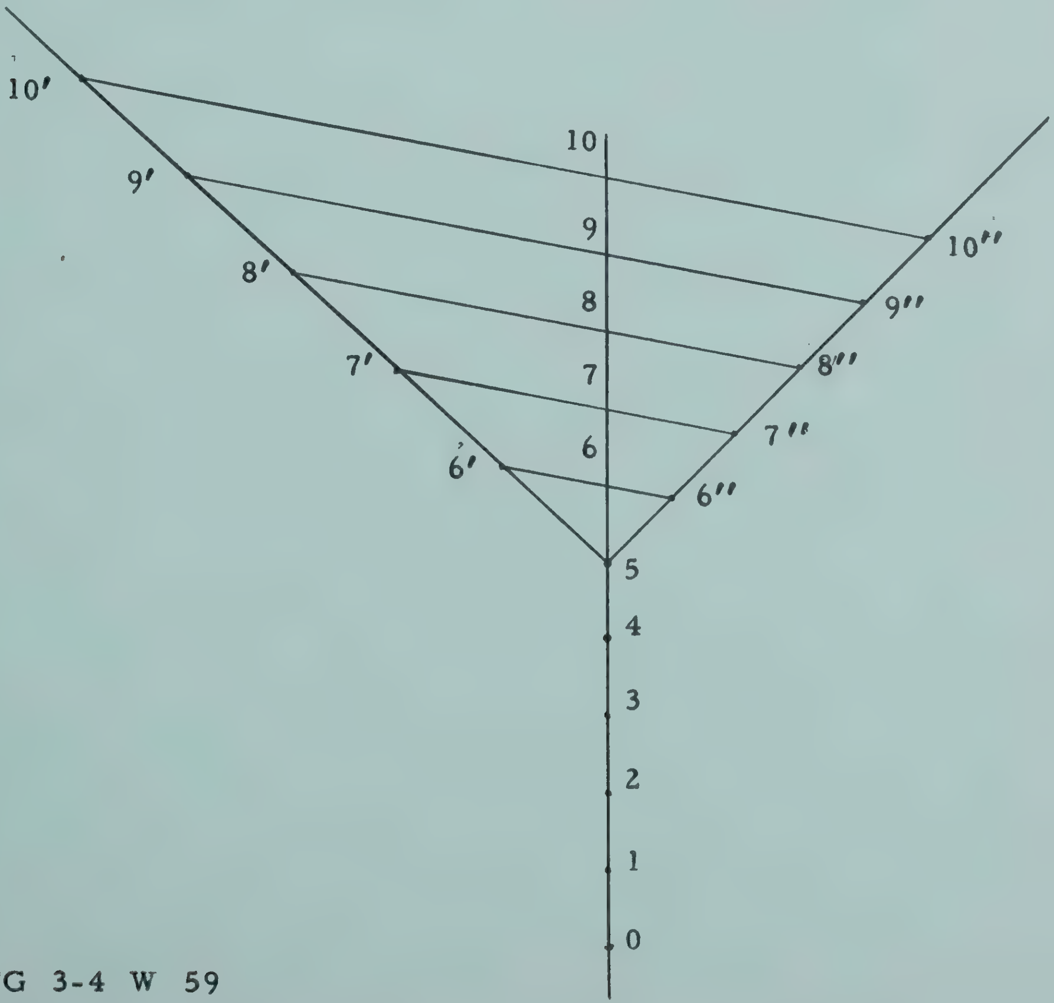
Then $P = 2475^2 + 2325^2 = 3,395\text{ m}$

and $\tan\theta = \frac{2475}{2325}$; $\theta = 46.75^\circ$

The velocity is then 5, 650 m/sec 46.75° from vertical

b) The velocity of the center of mass of the rocket was 3000 m/sec just before the explosion and since no external forces acted on it the velocity of the center of mass after the explosion is still 3000 m/sec.

It may be instructive to make a diagram like that below in which positions of the rocket and its pieces are plotted at equal time intervals. The successive positions of the rocket are shown at 0, 1, 2, 3, 4, 5. It explodes at 5 and the pieces are shown at 6′, 7′, 8′, 9′, 10′, and 6″, 7″, 8″, 9″, 10″ respectively.



The center of mass of the two pieces lies along a line joining them. Thus at time 8 the center of mass lies along $8'\ 8''$ at a point marked 8 such that m' x (dist from $8'$ to $8'$ = m'' (dist from 8 to $8''$) in this case distance 8 to $8''$ = .6 distance $8'$ to 8. The points 6, 7, 8, 9, 10 are seen to lie along the original direction and to "move" with the same velocity as before the explosion.

c) This is an important part of all such problems. Since the rocket is in the gravitational field of the earth--and presumably still close enough so that the acceleration of gravity is still 9.8 m/sec - the gravitational pull of the earth will change the center of mass velocity of the rocket by 9.8 m/sec^2. In other words, the rocket is not really an isolated system. The important point here is that the explosion takes place in a time very much less than a second and that during this time the forces are very much larger than those of gravity so that immediately after the explosion the velocities are not yet noticeably affected by gravity. If we need to take into account the gravitational effects, we can treat this problem as we have here and then ask how gravity affects each of the particles afterwards.

EXERCISE 21: A 2.0 kg brick with no horizontal motion is dropped on a 2.0 kg cart moving across a frictionless table at 0.40 meters/sec.

(a) What is the change in velocity of the cart?

(b) What is the velocity of the center of mass of the system composed of cart and brick before the brick is dropped?

This is an easy problem on momentum conservation unless some students try to include the vertical motion of brick.

The initial momentum of the cart + brick system is just the initial momentum of the cart 0.4 x 2 = 0.8 kg-m/sec. Since no horizontal external forces act during the collision the final momentum of the system is also 0.8 kg-m/sec which in turn is equal to the mass x velocity of the cart + brick

$$4 \text{ kg} \times V = 0.8; \ V = 0.2 \text{ m/sec}$$

a) The change in velocity of the cart was 0.2 m/sec.

b) The center of mass of a 2 kg brick and 2 kg cart lies along the line joining them and halfway between them. Then the center of mass point clearly moved half as fast as the cart before the "collision". It therefore moved at 0.20 meters/sec before the collision just as it did afterwards.

EXERCISE 22: A machine gun fires ten 100 gram bullets per second at a speed of 600 meters/sec.

(a) What impulse does the gun apply to the bullets in ten seconds?

(b) What impulse do the bullets apply to the gun in ten seconds?

(c) What is the average force applied to the gun?

(d) If the gun were mounted on a 2,000 kg aircraft, how much would the place slow down as a result of a 5 second burst of gunfire?

(e) Sketch rough graphs of:
    (i) Average force on gun vs. time

    (ii) Actual force on gun vs. time

(f) How are the graphs (i) and (ii) related?

This is a problem on impulse and momentum related to rocket problems.

a) Each bullet has a momentum

$$0.1 \times 600 = 60 \text{ kg-m/sec.}$$

In ten seconds 100 bullets are fired so that the gun has supplied a total impulse of 60 x 100 = 6,000 kg-m/sec or newton-sec.

b) Since momentum is conserved, the combination of the gun and the bullet can receive no net impulse. Hence the impulse the bullets apply to the gun is 6,000 newton-sec.

c) If we neglect the reaction of the gases, which we may do in a well designed gun, the total impulse applied to the gun in 10 seconds is 6,000 newton-seconds or 600 newton-seconds per second or 600 newtons average force.

d) 600 newtons for 5 seconds supplies an impulse of 3000 newton-sec which would change the momentum of the airplane by an equal amount.

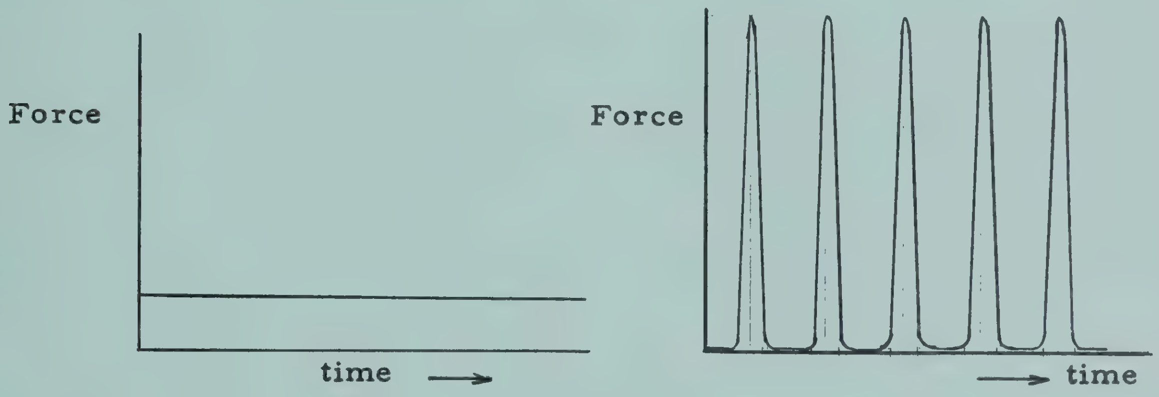
$$3000 = m\Delta v$$

$$= 2000\ \Delta v$$

$$\Delta v = \frac{2}{3}\ \text{m/sec}$$

Thus the plane would show down by 2/3 (m/sec).

e) It does not really make much sense to plot average force vs. time unless you are told over what time interval to average it. Presumably the students will plot a constant force vs. time as at (a).



(a) (b)

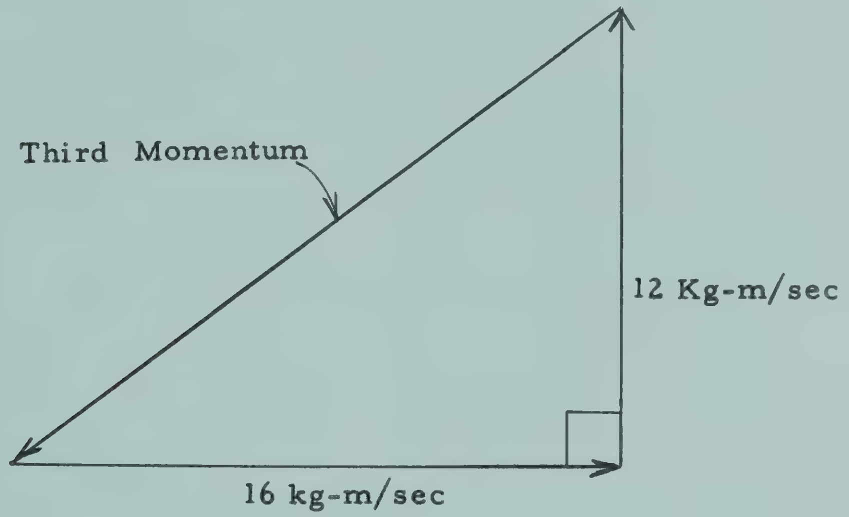
The actual force vs. time is shown at (b) in which the length of the force intervals as compared to the time between should be even shorter than shown and in which we can say very little about the actual shape of the spikes.

f) The areas under (a) and (b) are equal.

EXERCISE 23: An explosion blows a rock into three parts. Two pieces go off at right angles to each other, a 1 kg piece at 12 m/sec and 2 kg piece at 8 m/sec. The third piece flies off at 40 m/sec. Draw a diagram to show the direction in which it goes. What is its mass?

This is a standard conservation of momentum problem emphasizing its vector nature.

We must presume the rock to be at rest to begin with. It then had zero momentum before the collision and hence must have zero momentum after the collision. The momentum of the 1 kg piece is 12 kg m/sec; the momentum of the 2 kg piece is 16 kg-m/sec at right angles to the first. The third must have a momentum $\vec{P}$ given by the following vector diagram which shows that the three momenta add to zero!



The diagram shows the direction in which it flew off and from the 3-4-5 right triangle it is easily seen to have a momentum of 20 kg-m/sec. Since its velocity was 40 m/sec its mass was 1/2 kg.

EXERCISE 24: The system shown in Fig. 27 consists of a 5 kg frame (seen from above) and two masses in the middle. At the ends of the frame are stops made of putty. The frame is mounted on dry ice bearings, as are the masses, 1 kg and 4 kg respectively. The whole system has its center of mass at the center of the frame. An explosion pushes the two masses apart, the 1 kg piece traveling with a velocity of 12 m/sec. Eventually each mass is trapped by putty.

(a) What is the initial velocity of the 4 kg mass?

(b) Where is the center of mass of the whole system after 2 seconds? After 1 second?

(c) What is the velocity of the frame and the attached masses after 100 second?

(d) Describe the motion qualitatively from the time of explosion to 100 seconds later.

(e) How far does the frame move?

This is a rather complex problem on momentum conservation and center-of-mass motion which shows one rather interesting point, namely, that one part of a system can be displaced without the center of mass moving at all.

a) Momentum just be conserved in the explosion between the 1 and 4 kilogram masses. The 1 kg mass has a momentum of 12 kg-m/sec after the explosion. The 4 kg mass has an equal and opposite momentum and therefore a velocity of 3 meters/sec.

b) Since there are no external forces acting on the system the center of mass of the whole system remains fixed at all times.

c) Since there are no external forces acting and since before the explosion the momentum of the whole system was zero the momentum remains zero. When the frame and two masses are joined they all have the same velocity--namely zero.

d) At the explosion the 1 and 4 kilogram masses fly apart at 12 and 3 meters/sec respectively. The 1 kg mass hits first and sticks, moving the frame in its direction. The momentum of frame + 1 kg mass is 12 kg-m/sec, that of the mass before the collision. Next the 4 kg mass hits and transfers 12 kg-m/sec in the opposite direction, thus stopping the frame and masses, which now remain at rest. Many students will prefer your giving a definite value for the length of the frame.

e) The distance the frame moved could be calculated by doing part (d) quantitatively. However, the simplest method is to remember that the center of mass has not moved. Suppose the frame moved a distance x. Then it acts like a 5 kg mass a distance x from the old (and the new) center of mass. The 1 kg mass is (L/2 + x), where L is the length of the frame, away from the center and the 4 kg mass is (L/2 - x) away from the center. Thus

$$5x + 1\left(\frac{L}{2} + x\right) = 4\left(\frac{L}{2} - x\right)$$

$$5x + \frac{L}{2} + x = 2L - 4x$$

$$10x = \frac{3}{2}L; \; x = \frac{3}{20}L.$$

Many students who could not do this by using L will do it if you assign a particular value to L.

PSSC Teacher's Guide Volume 3

GENERAL COMMENTS ON CHAPTERS 5, 6, and 7 OF VOLUME 3

A. Importance of Energy

The final three chapters of Volume 3 deal with energy. Energy is one of the most basic concepts of physics, and the application of the law of conservation of energy is a powerful method for solving a wide range of problems. It is therefore very important for physics students to understand both energy and the conservation of energy.

Some teachers may be quite familiar with a treatment of energy that hinges, in the main, upon the conservation law. For those of you who are, it may be superfluous to read the following remarks.

B. Omission of a Definition of Energy.

No specific definition of the term, energy, is given in the text. This omission is intentional. It is believed that a better appreciation of the true significance of energy can be obtained through a more general treatment. In developing such a presentation, the text considers the following ideas:

The term energy is applied to many widely different properties of matter. It is useful to have a single term because these widely varying attributes are all connected through a single conservation law. In order to emphasize the broadness of the energy concept and to stress energy conservation, the book purposely avoids an over-simplified definition of energy such as "the capacity to do work". Instead, various forms of energy are introduced and the convertibility of energy from one form to another is discussed. "Work" assumes its rightful place as merely one mode of energy transfer. In order to get the discussion off the ground, work is defined on a quantitative basis; it then becomes possible to work out a quantitative description of several forms of energy starting from the amount of work done to produce them.

C. The Plan of Attack.

The strategy of the text is thus to introduce the three forms of energy that appear in purely mechanical situations: kinetic, potential, and thermal. With these three forms, it becomes feasible to develop a plausible discussion of the transfer of energy from one form to another showing that the total energy is always conserved.

D. Illustrative Example.

An example may serve to illustrate three difficulties that arise in defining energy as "the capacity to do work", as well as the general difficulty that such a definition does not adequately stress the conservation of energy. These difficulties are:

a. work is only one form of energy transfer;
b. there is at least one general case in which the available energy cannot all be extracted in the form of work;
c. in measuring the energy of a system by its ability to do work, you have to be careful about specifying the final state of the system after the work is done.

Consider a baseball moving horizontally with a velocity, v. With respect to the earth, it has a kinetic energy of $\frac{1}{2}mv^2$. But with respect to an observer on a moving train, it has a different kinetic energy. The amount of work that can be gotten out of it thus depends on whether it will do work on something on the ground or on the train.

If the ball were thrown horizontally off the edge of a cliff, a person at the bottom of the cliff would say that the ball had both kinetic and potential energy. The ability of the ball to do work thus depends not only on the horizontal speed, but also the height (or depth) of the system on which it is to do work. (If the man at the bottom of the cliff were to dig a hole in the ground, he could get still more work out of the ball.)

To complicate the situation even more, this person might set the ball on fire after it lands. He could then get work done by using the heat liberated by the release of chemical energy to run a heat engine. To measure the energy of the ball by its capacity to do work now requires not only that its horizontal velocity and height, but also its heat content be specified. However, unless the temperature of the exhaust gases be at absolute zero, not all of the thermal energy can be recovered as work; much of it will be lost in exhaust heat.

As a final example, the substance of the ball could be annihilated, thus liberating energy in radiant form.

E. Summary.

We have now briefly surveyed a sample of the many forms in which energy manifests itself. It is clear that no brief definition can adequately give a feeling for the true breadth of the concept. Certainly, with the exercise of sufficient care and ingenuity, you will find that you can use the "ability to do work" definition as a convenient guide. However, we feel that something is lost if the entire development is allowed to rest on this narrow definition. The important thing is to develop some feeling for the power of the conservation law in dealing with the many widely different guises of the thing we call energy. If you succeed in giving your students some appreciation for this idea, you will have given them something that is much more powerful than a neatly packaged, but incomplete, five word definition.

Chapter 5 - Work and Kinetic Energy

PURPOSE OF THE ENTIRE CHAPTER To introduce the idea of work as a measurable form of energy transfer. Using this measure, to develop kinetic energy in a quantitative way and then consider energy transfers in collisions.

CONTENT OF THE ENTIRE CHAPTER

a. Energy transfer can be measured in terms of fuel consumed or of jobs done.

b. Many kinds of jobs can be measured in terms of the product force x distance. This product is defined as work.

c. Work is done only by the component of the force along the direction of motion.

d. When the effect of a force is solely the acceleration of an object, the work done is equal to $\frac{1}{2}mv^2$. This quantity is defined to be the kinetic energy of the moving object. It refers only to the state of motion; not to the detailed mechanism by which the motion was produced.

e. The elastic collision is discussed. Its importance in the development of the concept of energy and the conservation of energy lies in the fact that the elastic collision is the only class of energy transfer in which the principle of the conservation of energy can be illustrated without knowledge of any other form of energy.

EMPHASIS OF THE ENTIRE CHAPTER If your time is short and if you wish to cover only the minimum essentials, it is possible to cover the basic ideas of this chapter rather quickly. A thorough understanding of sections 1-5 will be sufficient background upon which to base further progress through the remaining material of Vol. 3. On the other hand, if it is your intention to do a thorough job of Volume III, you will want to examine elastic collisions, simultaneous conservation of momentum and kinetic energy in elastic collisions, and elastic collisions in which the force is any unique function of the separation distance. There are many challenging and illuminating problems at the end of the chapter.

Section 1 - Work and Kinetic Energy

PURPOSE To introduce the concepts: energy, job, work, fuel, and energy transfer in a qualitative way and to begin to develop a relationship between them.

CONTENT When a job is done energy is transferred. The quantity of energy transferred can be measured either in terms of the work accomplished or of the fuel used. When proper measurements are made, and accounts kept, it is found that energy is neither created nor destroyed.

EMPHASIS Can be readily understood as part of a home reading assignment. In fact if you discuss it in class, you must be careful to avoid the trap of defining energy in an oversimplified way.

COMMENT This is not difficult material. Every student has certainly already used the word, energy, in a popular, qualitative way. This section begins to lay the groundwork for a critical, systematic examination of energy which will permit it to be developed as a useful scientific concept. In any class discussion of this material you may wish to introduce also the concept of efficiency. Two different engines may consume quite different quantities of fuel in doing the same job. What has happened to the extra fuel used by the "inefficient" engine? A great deal of energy is discharged in hot exhaust gases. The cooler the exhaust, the more efficient is the engine. Poor lubrication, extra moving parts, etc., all lead to friction forces which give rise to heat which, in this case, is an unwanted product of the consumption of fuel.

* * *

The final remark of this section (about energy conservation) will come as a real surprise to many students, and you can play this up by giving a few examples in which energy apparently disappears: the unwinding of a clock spring, the gradual shelf discharge of a battery, etc. You might point out that, in the course of the three chapters on energy, it will be established that energy is never lost. In most cases the energy which has apparently been lost is actually converted into heat (see last paragraph of section 2).

Section 2 - Work: A Measure of Energy Transfer

PURPOSE To define work so that it can be used as a meaningful, quantitative measure of the job done or the energy transferred.

CONTENT There is a close connection between fuel consumed and job done. Two jobs require double the fuel necessary for one. Fuel consumption apparently varies with magnitude of force exerted and with distance through which it acts. Define work as force x distance. 1 newton x 1 meter = 1 newton meter = 1 joule. Quantity of work done is independent of amount of time consumed in doing it. A stationary force does no work.

EMPHASIS Material of this section is very important. It is worthwhile to spend considerable classroom time reviewing the cable car problem developed in the text and introducing additional examples.

COMMENT Energy is complicated, often hard to measure quantitatively. When work is done in an energy transfer it is possible to be very specific and quantitative about the energy transferred in this particular way.

* * *

Some confusion may arise in connection with the application of a force at rest. A steel girder holds up a building. It exerts a huge force, but it moves nowhere. We need not feed it any fuel. It is not doing work. Yet, we have all, at some time or other, held a load over our head, waiting for someone to lift it from us. It is hard to hold anything up there for more than a short time. It makes us tired. We would say that we have worked. But again, the only work that we are doing is work against internal frictions (pumping blood), and generating heat by combustion. The result is only that we heat our bodies.

Again and again we find that our technical scientific vocabulary overlaps our less precise everyday language usage. Watch out for "work". We have now given it a precise definition. Don't let students confuse the word used in this sense with the one they use colloquially. Solving a problem is "hard work", but the energy consumed is very small.

Work is only one kind of energy transfer. Another is the burning of fuel which transfers chemical energy into thermal energy.

Section 3 - More About the Definition of Work

PURPOSE To point out that work is done only by that component of a force which acts along the direction of motion.

CONTENT When an object slides along a frictionless table, the table supports the object's weight by exerting an upward force m_g. The object moves horizontally. The table does no work. It consumes no fuel. This force and displacement are mutually perpendicular. Work is done only when force has a component parallel to the direction of displacement.

To move an object at constant speed around a circular path there must be a radial force acting inward. The displacement is always along the direction of the tangent to the path, thus force and displacement are perpendicular. No work is done.

EMPHASIS This is a short section, but the material is important, and difficult for many students. Plan to develop examples through classroom discussion.

COMMENT Work is done only by that component of the force which lies along the direction of the displacement. The text develops this idea by pointing out that no fuel is consumed when an object slides across a frictionless table. This is a case of a force which is perpendicular to the motion. No work is done.

Another approach to the same problem can be made from the point of view of the final state of the system. Has the state been changed by the application of a force? Consider a dry ice puck held by a string so that it moves in a circle of constant radius. Again, this is a case of a constant force acting perpendicular to the motion. Suppose a flame burns through the string after 5 revolutions. The puck flies off tangentially with the speed it had in its circular travel. Now repeat the experiment, but this time burn through the string after 10 revolutions. The force has acted much longer in this case. It has been accelerating the puck continuously. Yet the puck still flies off in exactly the same way as it did the first time. The force has accomplished nothing. It was acting perpendicularly to the direction of motion. No work was done.

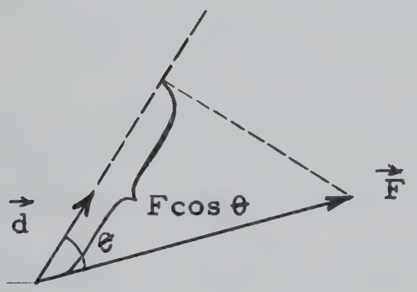
Work involves the product of a force and a displacement. These are both vector quantities. This particular kind of product is called a dot product. If $\vec{F}$ is the vector force which acts and $\vec{d}$ is the vector displacement, we denote the dot product $\vec{F}\cdot\vec{d}$. Thus we can write: Work = $\vec{F}\cdot\vec{d}$

In Figure (a), we see that in general the component of $\vec{F}$ which is along the direction of $\vec{d}$ has the magnitude $F \cos \theta$ where θ is the angle between F and d. Therefore $W = \vec{F} \cdot \vec{d} = (F \cos \theta) d$. Note also that we could group the factors differently writing: $W = F (d \cos \theta)$. We see in Fig (b) that $d \cos \theta$ is simply the component of the displacement which lies along the direction of the force. Thus we can say, equally well

Work = total displacement times component of force acting along direction of displacement

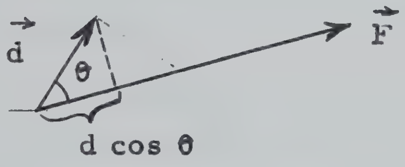
or Work = total force acting times component of displacement along direction of this force.

Fig (a)



$W = F \cos \theta \times d$

Fig (b)



$W = F \times d\cos\theta$

QUIZ PROBLEMS Suitable quiz problems can be made easily with variations on problems 4 and 12 of the Home, Desk and Lab section of the text. Another suitable problem follows. Calculate the work done by a man who pulls a sled 20 m by exerting a 20 newton force on a rope which makes a 45° angle with the horizontal.

Section 4 - Kinetic Energy

PURPOSE To introduce kinetic energy.

EMPHASIS This is very important material. Be ready to spend at least a full class period discussing kinetic energy.

CONTENT Work is a process of energy transfer. When work is done on an object in such a way that it starts the object in motion and produces no other change, then all of the energy transferred has become energy of motion. Energy of motion is called kinetic energy. When no energy is lost in the transfer process, the product $\vec{F} \cdot \vec{d}$ is uniquely related to the two parameters involved in the motion, m and v. The relationship for a constant force, accelerating an object from rest, is:

$$W = fd = mad = \frac{1}{2} mv^2 = \text{kinetic energy}$$

In lossless systems, the same work will always produce the same change in kinetic energy. It does not matter whether a small force acts through a long distance or a large force through a short distance.

An object in motion can be made to do work. In bringing an object to rest in a lossless system, the amount of work that can be extracted from the motion is equal to that which was expended in producing the motion.

COMMENT Note that this section introduces the first quantitative definition of any form of energy. To give students a good feeling for kinetic energy and its relationship to work, it will be worth while to follow through a number of examples in class. The text suggests a pattern. First, to convince the doubters, you might work through in detail the examples of a 1 kilogram mass accelerated by a 5 newton force acting through 10 meters, and by a 10 newton force acting through 5 meters. Show that the same work is done and the same kinetic energy is developed in the two cases.

* * *

Three specific examples, illustrative of some of the principles presented in this section, are given here. The first develops kinetic energy as a property of the motion of an object; a property which is quite distinct from momentum. It shows how the same work performed in two different

cases produces the same change in kinetic energy, but quite different changes in momentum. The second example emphasizes the different properties of two objects moving with the same momenta but widely different energies. The third example shows that the kinetic energy of an object and the work done on an object both depend upon the velocity of the person who is making the observations.

Example 1) Let a force of 1500 newtons act through a distance of 200 meters upon a 1500 kg. car which starts from rest.

$$W_1 = F_1 X_1 = 1500 \times 200 = 3 \times 10^5 \text{ joules}$$

To find the final speed of the car after it has traveled these 200 meters, we note that the acceleration was:

$$a_1 = \frac{F_1}{m} = \frac{1500}{1500} = 1 \text{ m/sec}^2 \text{ so that}$$

$$v_1^2 = 2\, a_1 x_1 = 400 \text{ m}^2/\text{sec}^2$$

$$v_1 = 20 \text{ meters/sec.}$$

The momentum of the car will be:

$$p_1 = mv_1 = 1500 \times 20 = 3 \times 10^4 \text{ Kg} \times \text{m/sec.}$$

the kinetic energy will be:

$$(\text{K. E.})_1 = \frac{1}{2} mv_1^2 = \frac{1}{2}\ 1500 \times 400 = 3 \times 10^5 \text{ joules,}$$

which is equal to w_1, the work which was done (see above).

Now let a 1500 newton force act again, in the same direction, through 200 more meters.

$$W_2 = F_2 x_2 = 1500 \times 200 = 3 \times 10^5 \text{ joules}$$

To find the final speed we note that the acceleration is again:

$$a_2 = \frac{F_2}{m} = \frac{1500}{1500} = 1 \text{ meter/sec}^2$$

The final speed, v_2, is given by

$$v_2^2 = v_1^2 + 2a_2 x_2 = 400 + 2 \times 1 \times 200 = 800 \text{ m}^2/\text{sec}^2$$

$$\therefore v_2 = 28.2 \text{ m/sec.}$$

The new momentum of the car will be:

$$p_2 = mv_2 = 1500 \times 28.2 = 4.23 \times 10^4 \text{ Kg. m./sec}$$

Thus the change in momentum during the second acceleration interval was:

$$(\Delta p)_2 = p_2 - p_1 = 1.23 \times 10^4 \text{ Kg. m/sec}$$

Whereas as a result of the initial acceleration:

$$(\Delta p)_1 = p_1 - p_0 = 3 \times 10^4 \text{ Kg. m./sec.}$$

The final kinetic energy will be:

$$(\text{K.E.})_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\, 1500 \times 800 = 6 \times 10^5 \text{ joules}$$

Thus the change in K.E. was the same during both acceleration intervals while the change in momentum was quite different. In each case,

$$\Delta(\text{K.E.}) = W = 3 \times 10^5 \text{ joules}$$

Now suppose that the brakes are applied in such a way that they exert a retarding force of 3000 newtons. How far will the car go before it stops? Its deceleration will be

$$a = \frac{F}{m} = \frac{-3000}{1500} = -2 \text{ m/sec}^2$$

Its initial speed was 28.2 m/sec

$$\therefore v^2 = v_2^2 + 2\,ax$$

The final speed, v_2, is to be zero therefore;

$$x = 200 \text{ meters.}$$

Then the work done on the brake drums can be shown to be

$$W = Fx = 3000 \times 200 = 6 \times 10^5 \text{ joules}$$

This is just what was put into the car to get it going. Of course, this final work is dissipated in the form of heat in the brake drums. As a matter of interest, if the brake drums contain 20 lbs. of iron with a specific heat of 0.113, $3 \cdot 10^5$ joules of energy will raise their temperature by 100° C., to a point well above the boiling point of water.

Note that in the two acceleration intervals, the increments of energy were the same, corresponding to equal work done. The increments of momentum were different. Why should this be? In the first period, the car accelerated from zero speed to 20 m/sec with constant acceleration. Therefore the elapsed time must have been:

$$v_1 = 20 = v_0 + a_1 t_1 = 0 + 1 \times T_1$$

$$\therefore T_1 = 20 \text{ seconds}$$

The elapsed time during the second acceleration must have been:

$$v_2 = 28.2 = v_1 + a_2 t_2 = 20 + 1 \times t_2$$

$$t_2 = 8.2 \text{ seconds}$$

Thus although identical forces acted through identical distances, the times of application were very different. This is because for the second acceleration there was a large initial velocity and the distance was therefore covered in much shorter time.

As was shown in chapter 4, change of momentum stems from an impulse. In the two intervals in question, the work done was the same, but the impulse, force x time, was quite different.

Example 2) In order to drive home the difference between the two measures of motion, momentum and energy, it is often helpful to cite the example of a 100 gram bullet and a 1500 kilogram car, each striking a block of wood. The bullet might have an initial speed of 300 meters per second; the car, .02 meters (2 centimeters) per second. Thus both have an initial momentum of 30 Kg meters/sec, but the bullet's kinetic energy would be $(K.E.)_b = 4.5 \times 10^3$ joules whereas for the car, $(K.E.)_c = 0.3$ joules. This difference in kinetic energies results in the car giving the block of wood a slight nudge while the bullet, with the same momentum but much more energy, tears a hole into the wood.

Example 3) Note that the kinetic energy of an object is a relative thing. With respect to a viewer in a moving train, a baseball has a very different velocity and therefore kinetic energy than it has with respect to a ground observer. It follows that changes in kinetic energy are also different when viewed from different frames of reference.

Picture a man riding on a flatcar of a freight train. Suppose it is moving at 10 meters per second. Now he throws a baseball in the direction in which the train is moving. He throws it so that it attains a speed of 10 meters/second with respect to the flatcar. If the mass of the ball is 1 kilogram, its kinetic energy with respect to the flatcar is $\frac{1}{2} 1 \cdot 100 = 50$ joules. The man could have thrown it by exerting a force of 50 newtons through a one meter distance. But now, what appears to have happened to an observer on the ground? The ball was initially moving with a speed of 10 meters/second as a result of the motion of the flatcar. It's original kinetic energy was therefore 50 joules. After the man threw it, it had a speed of 20 meters per second. Its new kinetic energy was therefore 200 joules, and its change of kinetic energy, in the throwing process, was 200 - 50 = 150 joules. Where did the work come from that produced this energy change? The force which was acting on the ball was still 50 newtons. However, the distance through which it acted is quite different as seen from the ground. It can be calculated as follows:

On the train the man's arm moved through 1 meter. If his force was constant, the acceleration was constant. His arm's final speed was 10 m/sec. Thus the average speed was 5 meters/sec and the time of the acceleration was $t = \frac{s}{v} = \frac{1}{5}$ seconds. Now in 1/5 seconds the train itself moved 1/5 x 10 = 2 meters with respect to a ground observer. Therefore in pushing the ball, the man's arm moved 1 meter as a result of his own

motion, 2 meters as a result of the train's motion, a total of 3 meters with respect to the ground observer. Thus the work done from his point of view was 50 newtons x 3 meters = 150 joules, just equal to his observation of the change in kinetic energy. This is not a fictitious amount of work. The man did only 50 joules in pushing with 50 newtons force through 1 meter. But the man's feet pushed back on the train, the train forward on his feet, with this same 50 newton force. While exerting this force on the man's feet the train traveled two meters. Thus the train itself supplied 100 joules of the work done on the ball. This work came from the energy liberated from the burning of its fuel.

Section 5 - The Transfer of Kinetic Energy from One Mass to Another

PURPOSE To describe a mechanism by which kinetic energy may be transferred from one object directly to another.

CONTENT In a two-body collision, kinetic energy is transferred from one object to another. Since the force of interaction in any real example is extremely complicated, the problem is most easily understood by considering an idealized case. Two objects are proposed, m_1 and m_2, which exert no force on each other as long as their separation is greater than some distance, d. When their separation is anything less than d, they repel each other with a constant force F. This problem is worked out in detail in the text.

EMPHASIS This is important and difficult material. It should be covered first as part of a home reading assignment. Then the example illustrated in figures 8 and 9 should be reviewed in the classroom.

APPROPRIATE LABORATORY Experiment 550 (See TG section on lab experiments).

COMMENT It may seem to some students that this example is really not such a special case. A simple illustration of its lack of generality is that of a collision between two lead balls (or balls of plasticene). In such a collision the objects are deformed and their distance of separation at the end of the interaction is different from what it was at the beginning. Thus the two bodies will not have moved the same distance with the equal and opposite forces acting upon them.

* * *

Figure 6 was made using balls as pendulum bobs with long suspensions, not rolling balls.

QUIZ PROBLEM A ball of 200 grams mass, moving at 6 m/sec, approaches a second ball head-on. The second ball has a mass of 100 grams. While the balls are within 5 cm of each other, they repel each other with a force of 40 newtons. The second ball leaves the collision with a speed of 8 m/sec.

a) How much kinetic energy does the second ball gain in the collision? (3.2 kg-m^2/sec^2 or 3.2 joules)

b) What is the kinetic energy of the first ball after the collision? (0.4 kg-m^2/sec^2 or 0.4 joules)

c) How far did each ball move during the collision, i.e. while they remained within 5 cm of-each other? (8cm)

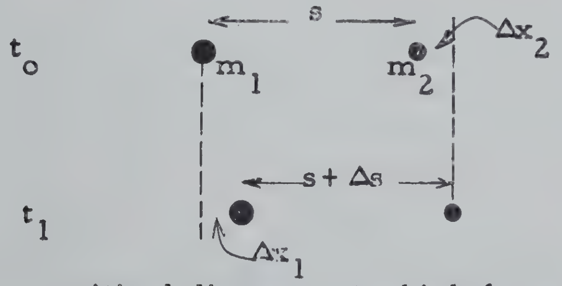
Section 6 - Another Look at the Simple Collision

PURPOSE To introduce the idea that kinetic energy is not constant at every instant, even in the ideal interaction discussed in Section 5.

CONTENT The ideal interaction of ~~the~~ Section 5 is examined in detail, and it is found that during the interval when m_1 and m_2 are interacting, the total kinetic energy of the two is less than the kinetic energy before (and after) the interaction. Quantitatively, the kinetic energy which disappears from the system during any interval of the interaction is given by $F\Delta S$ where F is the constant force of interaction and ΔS is the change in the separation distance between m_1 and m_2 during the interval in question. For a given force of repulsion, when the separation distance decreases the total kinetic energy decreases. As the separation distance increases, the total kinetic energy increases.

EMPHASIS This also is important and difficult material; certainly requiring classroom discussion if it is to be developed at all.

COMMENT This rather complicated argument is developed very carefully in the text. In order to facilitate understanding the geometry of the problem, the following discussion is presented. The figure is extracted from Figure 8 of the text.



positions of m_1 and m_2 at one instant during collision ($s < d$)

Same thing, an instant later

d = critical distance at which force of repulsion begins to act.
Δx_1 = distance moved by m_1 during interval between t_o and t_1.
Δx_2 = distance moved by m_2 during interval between t_o and t_1.
s = distance of separation of m_1 and m_2 at time, t_o.
Δs = the change in s during the time interval between t_o and t_1.

From the figure, it can be seen that

$$s + \Delta x_2 = \Delta x_1 + s + \Delta s$$

$$\text{or, } \Delta s = \Delta x_2 - \Delta x_1$$

Now, as developed in the text, when m_1 moves through Δx, it does so against the retarding force F. Therefore it loses kinetic energy. The change is equal to the work done:

$$(\Delta E_k)_1 = - F\Delta x_1$$

For m_2, the force, F, is an accelerating force, and the change in its kinetic energy is given by:

$$(\Delta E_k)_2 = + F\Delta x_2$$

The total change in kinetic energy of the two masses is simply the sum of the individual changes:

$$(\Delta E_k)_{total} = F\Delta x_2 - F\Delta x_1 = F(\Delta x_2 - \Delta x_1) = F\Delta s$$

Thus the kinetic energy does change during the collision. The change is equal, quantitatively, to the negative of the work done on the masses. The distance of separation, s, becomes smaller while the repulsive force tends to make it become larger. Then the work done on the masses is $-F\Delta s$.

QUIZ PROBLEM A block having a mass of 5 kg slides on a frictionless surface with a speed of 1.0 m/sec toward a second block which is at rest. The second block has a mass of 1.0 kg. When the two blocks are within 10 cm of each other, they repel each other with a force of 50 newtons.

a) What is the magnitude and direction of the acceleration of each block while they are colliding, i.e., closer together than 10 cm?

$a_1 = 10 \text{ m/sec}^2$, backward

$a_2 = 50 \text{ m/sec}^2$, forward

b) The beginning of the collision is at the instant when the first block first gets within 10 cm of the second block. From their positions at that instant, how far will each block move in 1/60 sec? In 1/30 sec?

$$\begin{array}{lll} & \underline{1/60 \text{ sec}} & \underline{1/30 \text{ sec}} \\ \Delta x_1 = & \frac{11}{12} \cdot \frac{1}{60} \text{ m, and} & \frac{5}{6} \cdot \frac{1}{30} \text{ m} \\ \Delta x_2 = & \frac{5}{12} \cdot \frac{1}{60} \text{ m, and} & \frac{5}{6} \cdot \frac{1}{30} \text{ m} \end{array}$$

c) Calculate the change in the total kinetic energy 1/60 sec after the beginning of the collision, and 1/30 sec after the beginning of the collision.

1/60 sec: $(\Delta E_k)_{total} = 50(\frac{5}{12} \cdot \frac{1}{60} - \frac{11}{12} \cdot \frac{1}{60})$

$$= -50 \cdot \frac{1}{2} \cdot \frac{1}{60} = -\frac{25}{60} = -\frac{5}{12} \text{ newton-meters}$$

1/30 sec: $(\Delta E_k)_{total} = 0$

Section 7 - Conservation of Kinetic Energy in Elastic Collisions

PURPOSE To broaden the special example of a kinetic-energy-conserving collision to include all elastic collisions.

CONTENT If the force between two objects is only a function of their distance of separation, the total kinetic energy after an interaction is complete will be the same as it was before the interaction started.

EMPHASIS The elastic collision is an extremely important and useful phenomenon, but it is not easy to understand. It will be difficult to get this idea across to many of your students.

COMMENT We have established that the change in kinetic energy during any short interval of the interaction of two objects is given by $F(t)\,\Delta s$ where $F(t)$ is the force which is acting and Δs is the change in the separation distance. Note the F is written here as $F(t)$ showing that in general it will be different from time to time during the interaction. When F and Δs are in the same direction the total kinetic energy is increasing, when they are oppositely directed it is decreasing. If the interaction force is determined only by the separation of the two objects, then we can write $F(t)$ as $F(s)$. Then the change in the total kinetic energy, as two mutually repelling objects part from a separation distance, s, to $s + \Delta s$, will be exactly equal and opposite to the change in their total kinetic energy which occurred as they approached each other from a separation $s + \Delta s$ to a separation s. This is true because the forces were identical at these two instants, because F was a unique function of S. Whenever this is true, kinetic energy will be conserved over the full course of a collision (not at every instant during the collision). Collisions between objects whose interaction forces are of this type are called elastic collisions.

* * *

You may be asked to explain why most collisions are inelastic. The example of the lead balls that we gave earlier is a simple one. There, the two balls of radius, r, begin to interact when the separation of their centers is 2r. But, as they part, the interaction force goes to zero before this separation is 2r. This is the result of the deformation of the balls. F is not a unique function of s, and as much work is not done on the balls by the interaction force as they part as was done by the balls in coming together. Thus

kinetic energy is lost in such a collision. It has been dissipated as heat and has been used to rearrange the internal structure of the balls.

A more subtle example is that of a rubber ball. We know that a dropped rubber ball does not rise to its initial height. The collision is not perfectly elastic. But the ball does return perfectly to its original shape. This is another case where we must watch our vocabulary. Colloquially, the word elastic is used to refer to the property of a rubber ball that returns it to its original shape. This, as we have seen in the case of the lead balls, is a necessary condition for our newly defined elastic collision. But it is not a sufficient one. Actually rubber is a material in which there are appreciable internal friction forces associated with any deformation. Thus as the ball deforms when it hits the ground the force that develops is the sum of the upward force corresponding to the static deformation, plus the upward force necessary to overcome the internal frictions as the bottom edge of the ball is pushed upward toward the center. As the ball leaves the ground, the force that acts on it is the upward force corresponding to the static deformation plus the downward force necessary to overcome the internal frictions associated with the downward directed motion of the bottom edge of the ball away from the ball's center. Thus the net force acting on the ball at any separation distance, s, as it approaches the floor is greater than that which acts on it at the same s as it leaves the floor. Kinetic energy is lost. It is dissipated into internal heat in the ball. You might ask your students if any of them have ever felt a squash ball or a handball after it has been batted around for a while. The temperature rise is striking.

Section 8 - Kinetic Energy and Momentum

PURPOSE To demonstrate the use of the laws of conservation of energy and momentum in the solution of problems involving elastic collisions.

CONTENT The problem of the head-on elastic collision is solved (in the appendix to Chapter 5) using the conservation laws. The application of this method by Chadwick in his discovery of the neutron is presented.

EMPHASIS This section does not contribute any important new ideas to the development of the concept of energy. From this point of view, it can be treated very briefly or even omitted entirely. However it does present one of the very powerful applications of the energy conservation principle. If you have the time you can profitably spend several days discussing the method and doing problems.

COMMENT A wide variety of problems in modern physics basically concern what is called the two-body problem. Problems dealing with the orbit of a planet around the sun, the dynamics of a rocket, and the collision between two protons all are examples of the two-body problem. Fortunately this relatively simple physical situation, the interaction of two objects, can be applied rather broadly. The general problem which involves the interaction of more than two bodies can still not be solved exactly, even in this day of fast electronic computers.

Two body problems dealing with the motions of the planets can be solved by applying Newton's Law and thus obtaining solutions for the motion along successive very small intervals of an orbit. This kind of solution is possible because the interaction force is well defined at all points of space by the Law of Universal Gravitation. Even so, the procedure is a tedious one at best. In the case of the propulsion of a rocket, or of the collision of two protons, a solution by application of $F=ma$ is impossible because the interaction force is not known. We can still make considerable progress with these problems by applying the conservation laws. When the collision is elastic, we can use the conservation of momentum and of kinetic energy. When energy is dissipated, stored, or released during the course of the interaction, this can be taken into account in working out the solution.

* * *

It would probably be helpful to your students if you built up the application of the conservation laws somewhat in the following way. Start with a head-on collision of m_1 and m_2, m_2 being initially at rest and m_1 approaching with velocity, v_1. The total momentum before the collision must then be m_1v_1.

After the collision, m_1 and m_2 will have velocities v_1' and v_2', respectively (both in the same direction as v_1). The principle of momentum conservation tells us that (A) $m_1v_1 = m_1v_1' + m_2v_2'$. However this does not help us very much. That is, if we know values for m_1, m_2, and v_1 we can still not predict the final values of v_1' and v_2'. There is an infinite number of combinations which will satisfy the above equation. For any arbitrary v_1' we can find a corresponding value of v_2'. To see this clearly, consider the case where $m_1 = m_2 = m$. Then our equation becomes:

(B) $mv_1 = m(v_1' + v_2')$

or (C) $v_2' = v_1' - v_1'$

A few sample solutions would be:

$v_1' = 0 \qquad v_2' = v_1$

$v_1' = v_1 \qquad v_2' = 0$

$v_1' = \frac{v_1}{2} \qquad v_2' = \frac{v_2}{2}$

and so forth. Thus we have no unique solution to the problem. In mathematical language, the difficulty is simply that we have two unknowns, v_1' and v_2', but only one relationship between them, that dictated by momentum conservation. If we add a second equation, we can remove the indefiniteness of the solution. The equation that we have available for an elastic collision is the one which expresses the conservation of kinetic energy.

(D) $\frac{1}{2} mv_1^2 = \frac{1}{2} mv_1'^2 + \frac{1}{2} mv_2'^2$

(E) or $v_1^2 = v_1'^2 + v_2'^2$

but we had, from momentum conservation, the relation

(C) $v_2' = v_1 - v_1'$

and substituting in (E) we get.

$$v_1^2 = v_1'^2 + (v_1 - v_1')^2 = v_1'^2 + v_1^2 - 2v_1 v_1' + v_1'^2$$

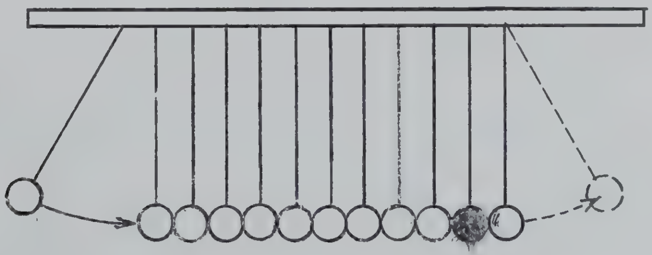
or $v_1'^2 - v_1 v_1' = 0$

therefore $v_1' = v_1$

and $v_2' = 0$

This is now the only possible solution. Notice that it has been obtained without any detailed knowledge of the force law between m_1 and m_2. We have only assumed the interaction to be elastic.

There is a nice standard demonstration that can be done to illustrate the uniqueness of the solution to this type of problem. Get a set of pool balls or golf balls and suspend each from a string tied to a bar so that they hang in contact with each other as in figure 1.



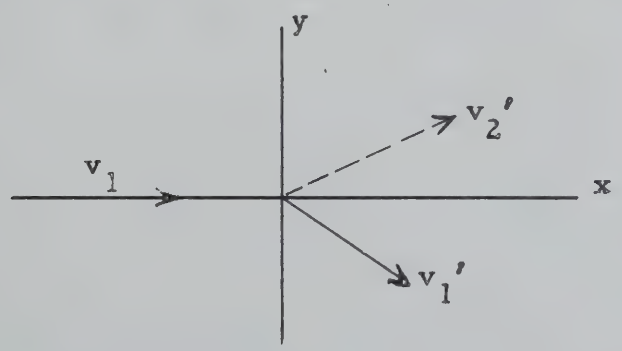
Now draw back one ball and let the class guess what will be the solution to the problem. If the one ball has velocity, v, when it hits what will happen at the other end? With momentum conservation only, we could have one ball bounce off with velocity, v; two balls with $\frac{v}{2}$, three with $\frac{v}{3}$, etc., etc. Applying conservation of kinetic energy we find that only the one ball solution works. If the balls are fairly elastic, this will turn out to be the case experimentally. Likewise, if two balls are drawn back initially, the only possible solution is for two to bounce off, etc.

* * *

If your students are doubtful about the usefulness of this technique for the more general case of a two-body collision which is not directly head-on, you can indicate the method of attack in the following way:

Let the direction of motion of the bombarding object, m_1 be along the x-axis of a coordinate system. Let the x, y plane of your coordinate axes include the trajectory of one of the particles after the collision. You can then be sure that the trajectory of the second particle also lies in the plane of your coordinate axis. This follows from momentum conservation.

Before the collision the total momentum lay along the x-axis. Therefore it must lie there after the collision, also. This can be true only if the second particle, also, has its trajectory in the x, y plane after the collision, (figure 2).



Furthermore, if we consider the x and y components of v_1' and v_2' we can write, to conserve momentum;

$$m_1 v_1 = m_1 v'_{1x} + m_2 v'_{2x}$$

$$0 = m_1 v'_{1y} + m_2 v'_{2y}$$

The equation for energy conservation is the same as it was before

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

For an experiment of the type that Chadwick performed, m_1, v_1, v'_{1x} and v'_{1y} were all unknowns. Thus one set of observations in which the angle and energy of a recoil proton were recorded would yield a system of three equations involving four unknowns. If now a second measurement is obtained in which the proton recoil is measured at some different angle, we shall have the same m_1 and v_1 as before, v'_1 and v'_2 corresponding to the second measurement will be two new unknowns, making a total of six. But we have three more equations describing the second event, thus six equations for six unknowns. It is then possible to calculate any of them, in particular the mass, m_1, the neutron mass.

Section 9 - Work and Kinetic Energy When More than One Force Acts

PURPOSE To indicate a method for calculating the amount of work done when many objects are interacting.

EMPHASIS This is not very important material. It is a detailed trimming which can be omitted completely without any serious loss.

CONTENT When an object, under the influence of many forces F_a, F_b, . . ., moves in the x direction the work done is given by the sum:

$$W_{total} = W_a + W_b + \ldots\ldots = (F_{ax} + F_{bx} + \ldots\ldots)\, x$$

where W_a = work done by a, etc.; F_{ax} = the x component of F_a, and x is the displacement in the x direction.

Note that F and x are vectors, W a scalar.

Section 10 - Loss of Kinetic Energy in a Frictional Interaction

PURPOSE To call attention to the effect of the action of friction in most real (as opposed to ideal) collisions.

EMPHASIS Heat will be treated in more detail in Chapter 7. For the time being it is introduced only to help give a more complete and realistic view of the interaction of two objects. Treat briefly.

CONTENT When macroscopic bodies interact, not only is there an effect upon motion of each body as a unit, but also there is a tendency for them to be distorted. A bouncing ball is one example, a sliding block is another. The sliding block is a less obvious case, but it can be understood by observing a hairbrush which is pushed, bristles down, across a rug. Bristles temporarily couple with tufts of knap, are distorted and then released to snap back into place. This is what is happening to clusters of molecules on the bottom of a block which is sliding along a rough surface. These distortions, when released, give rise to local oscillations of the molecules. On this microscopic scale, the energy is kinetic energy, but not in the usual sense of bulk motion of the block. The energy is found in the increased chaotic motions of molecules. In fact it is sensed by us only as an increase in temperature of the regions in question. This particular guise of energy is called heat. It is considered to be a form of energy because of the consistent way it fits the conservation principle pattern.

Section 11 - Conclusion

PURPOSE To summarize the material presented in this chapter.

EMPHASIS It is certainly important to summarize, in some way, all of the ideas that have been separately developed and then pulled together within the energy concept.

The time such a summary will take will depend on your previous presentation of the subject matter.

CONTENT When a force is applied to a moving body, the transfer of energy to the body is given by the work, the distance the object moves times the component of force acting in the direction of the motion. When the force which acts between two bodies depends only on their distance of separation, kinetic energy will be conserved in any collision, and we call such a collision an elastic collision. When the interaction force is not a unique function of the separation, the collision is called inelastic and kinetic energy is not conserved. If you try to bounce a ball of putty, kinetic energy will be lost. The missing energy is dissipated in the form of heat. An instantaneous inspection of an elastic collision during the course of the interaction reveals the kinetic energy is not conserved. The missing energy in that case is stored in the force field and is then called potential energy. As the interacting objects fly apart it is reconverted into kinetic energy. Chapter 6 deals in some detail with potential energy.

For Home, Desk and Lab

This is a fairly extensive set of problems, but not too many considering the number of ideas that are introduced in the chapter.

Section	Easy	Medium	Hard	Good for class Discussion	Lab Experiments
Introd.		1		1	
2	5, 28	2, 29	3	2, 5, 28	
3		4, 12	13	4	
4	7, 8, 9	6, 10, 14, 11		6, 10	
5		15		15	Exp 550 (To be during the section
6			18	18	
7		16, 17	32	16	
8	19, 20 21, 26	24	18, 22	20, 22, 24	
9	23	31	33	23	
10	28				
11		25, 27, 30		27, 30	

Problem 29 is very complicated unless treated superficially, you may not wish to assign it.

Short answers, For Home Desk and Lab, Chap 5 Vol 3

Note: Some teachers have asked for a condensed list of answers for preparing lists of answers to selected problems to hand out to students. The following is such a list.

1. Word answers
2. (a) 6×10^6 joules, (b) 6×10^6 joules
3. (a) 6 gals. (b) more, but not necessarily twice as much.
4. (a) none, (b) none, (c), 120 joules, (d) - 1500 joules.
5. (a) 0.2 joules, (b) 0.2 joules
6. (a) 30.0 joules (b) 30.0 joules, (c) 5.48 m/sec
7. Both 1200 joules
8. (a) 120 joules, (b) 11 m/sec
9. (a) 1.4 joules, (b) 1.2 m/sec
10. (a) -32 joules, (b) 8 meters, (c) 32 joules
11. (a) 3.86×10^5 joules, (b) at least as much, (c) no way of knowing
12. (a) 39.4 joules, (b) 158 newtons, (c) none
13. 40 newtons perpendicular to the force
14. (a) 90.0 joules, (b) 160 joules, (c) 70 joules into heat
15. (a) $\Delta x_1 = .16t - t^2$, $\Delta x_2 = 3t^2$, (b) 0.04 secs., [illegible]
    (c) 0.08 m/sec, 0.24 m/sec.
16. 2.67 joules
17. (a) 4 joules, (b) 14.5 joules, (c) 38 joules, (d) 40 joules

18. 0.02 joules/second

19. 0.1 m/sec forward, 0.1 m/sec backward

20. 0.22 °/o

21. (a) $4 \frac{m_2}{m_1} E_k$, (b) E_k, (c) $4 \frac{m_1}{m_2} E_k$

22. (a) 0.20 joules, 0 (b) 0.178 joules, 0.022 joules, (c) 0.0890 joules, 0.0445 joules, (d) 1.66 cm

23. $\vec{p}' = \vec{p}$, $\sum E'_k = E_k + W_{\text{explosion}}$

24. (a) 0.21, (b) 0.68, (c) 1.12

25. Yes

26. (a) 3.3×10^7 m/sec, 0, (b) 4.4×10^6 m/sec, (c) 0. -2.86×10^7 m/sec

27. (a) 5×10^4 kg m/sec, (b) $\frac{2}{3}$ m/sec, (c) 5×10^4 joules, 1.667×10^4 joules, No.

28. Yes, it heats your hands.

29. Don't assign

30. 78 joules

31. (a) 5 newtons, (b) 12.5 joules, (c) 4.5 joules, 8 joules, equal

32. v, 0

33. (a) 0, (b) $4.5\ mv^2$, (c) $\frac{1}{2} mv^2$, (d) $4mv^2$, (e) $3\ mv^2$

EXERCISE 1: Coal is burned in a city electric power plant to make steam to run a steam turbine that drives an electric generator. The city water department uses this electric supply to run an electric motor to pump water from a well into a standpipe on a hilltop. List, in order, all the energy changes that take place.

a. The chemical energy of the coal is changed to the thermal energy of the flame.

b. The thermal energy of the flame gives thermal energy to the water which results in production of steam. The thermal energy of the steam takes the form of kinetic energy of the steam molecules which in turn impart kinetic energy to the turbine by colliding with the vanes of the turbine. The turbine shaft is connected to the generator shaft and turns it, thus generating electrical energy.

c. The electrical energy is converted to kinetic energy by the electric motor which rotates the pump and gives kinetic energy to the water causing it to flow through the pipes up into the tower.

d. As the water rises in the pipe into the tower it is moving upward against the force of gravity and acquires potential energy. The potential energy of the water in the standpipe can be converted into kinetic energy as it flows out of the standpipe through pipes and out of a faucet.

EXERCISE 2: A cable pulls a car up a mountain with a force of 4×10^3 newtons at a velocity of 5 m/sec. It takes the car 5 minutes to reach the top.

(a) How much work is done in getting the car to the top of the mountain?

(b) How much work would be done to get the car up the mountain if it travelled 2.5 m/sec.?

(a) Since the car takes 5 minutes, or 300 secs to get to the top at a velocity of 5 m/sec, it travels 1500 m. A force of 4×10^3 newtons exerted on the car for a distance of 1500 m does 4×10^3 nt x 1500 m = 6×10^6 newton-meters or $\underline{6 \times 10^6 \text{ joules}}$ of work.

(b) If it requires the same force to pull the car at the slower speed of 2.5 m/sec up the mountain, then the work done in getting the car up the mountain at the slower speed would be the same as before: 6×10^6 joules, since the force and distance are both the same. Sources of friction like bearings, pulleys, etc., usually exert less drag at slow speeds than at high speeds, so it would probably take less force, and therefore less work, to make the same trip at the slower speed. (This problem relates directly to Sec. 2 of the text, the second part being an illustration of the ideas in the 5th paragraph).

EXERCISE 3: A rotary snow plow uses four gallons of gasoline to plow 10 miles of road in 20 minutes.

a. About how many gallons would be used in plowing 15 miles at the same speed?

b. Do you think it would take twice as much fuel to plow 15 miles in half the time? Why?

(a) Plowing fifteen miles requires the engine to exert the driving force for 1 1/2 times as far and therefore the engine does 1 1/2 times as much work as it does in plowing 10 miles. Since the engine does 1 1/2 times as much work, it must use 1 1/2 times as much fuel, or 6 gallons.

(b) To plow 15 miles in half the time, requires the plow to go at 60 miles per hour (some plow!). While the distance is the same, 15 miles, the force required to drive the plow through the snow at 60 mph must certainly be greater than the force required to drive the plow through the snow at 30 mph. (For example the snow would be thrown farther by the higher speed of plowing). The work done would be greater but probably not twice as much, since it is the product of the force times the distance and the force must certainly be greater than at the slower speed. (This problem relates to Section 2, and particularly to the ideas in the fifth paragraph of Section 2.)

EXERCISE 4: How much energy is transferred to a 15 kg. suitcase while

a. you hold it for 5 minutes while waiting for a bus?

b. you run with it a horizontal distance of 10 meters in two seconds at constant velocity in catching the bus?

c. you lift it 0.8 meters getting into the bus?

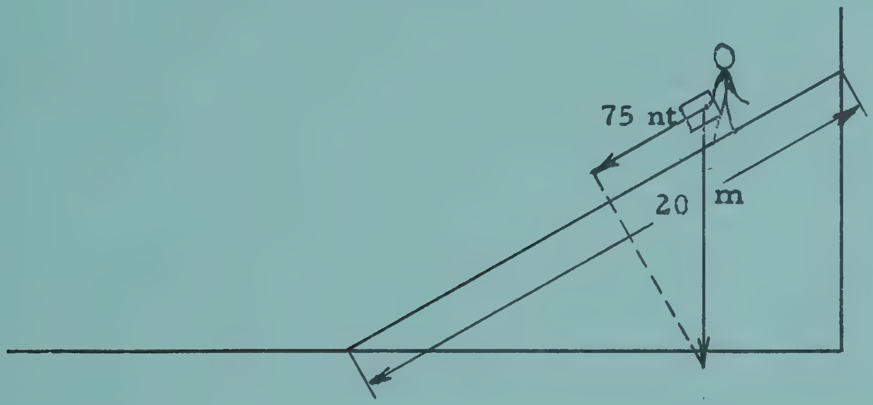
d. you carry it 20 meters down a ramp that makes an angle of 30 degrees with the horizontal?

(a) Despite its tiring effect on you, you do no work while holding a suitcase since work is defined as force x distance moved. If you had placed the suitcase on a stand while you waited for the bus, you would not think of the stand as having done any work.

(b) Similarly, although not so obviously, you do no work on the suitcase while running with it along a horizontal path. (Neglecting air friction. A negligible amount of work will be required to "push" the suitcase through the air). Work is only done when the force acts in the direction of the motion. As before, you can reason that if you had the suitcase on frictionless wheels, you would not think that the sidewalk was doing any work to hold up the suitcase while it rolled along on its wheels.

(c) When you lift the suitcase 0.8 meters, you are exerting a force in the direction of motion; the force is 15 kg x 9.8 nt/kg = 150 nt. Since the force is in the direction of motion, you do 150 nt x 0.8 m or 120 joules of work on the suitcase.

(d) When you carry the suitcase 20 meters down a ramp, its weight has a component of force along the direction of your motion down the ramp; it therefore does work on you. The component of the weight parallel to the ramp is $\frac{1}{2}$ x 150 newtons; the distance down the ramp is 20 meters; the work done by the suitcase on you is 75 newtons x 20m = 1500 joules. A literal answer to part (d) would say that the energy transferred to the 15 kg. suitcase is -1500 joules. (This energy is transferred to you and to the ramp in the form of heat.) (This problem relates to Section 3.)



EXERCISE 5: A meter stick, mass 0.2 kg, is lying on a table near two blocks 10 cm. high (Fig. 16).

(a) If you lift the stick, holding it horizontal, and put it on the blocks, how much work have you done?

(b) If you lift one end and set it on one block, and then lift the other end, setting it on the other block, how much work have you done in moving the stick?

(a) If you lift the stick, holding it horizontal, you are lifting the whole weight of the stick and must be exerting a force of 0.2 kg x 9.8 nt/kg or 2.0 nt. This force, moving the stick 10 cm in the direction of the force, does 2.0 nt. x 0.1 m = 0.2 joules of work.

(b) If you lift the stick one end at a time and put it up on the blocks, you have done the same amount of work, but in two parts. In the

first step you lift half of the weight of the stick, the table supporting the other half of the weight; you thus do $\frac{1}{2}$ x 2.0 nt x 0.1 m = 0.1 joules of work. Then when you lift the other end you again are lifting half of the weight, and the other half is being supported, or held up, by the block on which it is resting. In this step you also do 0.1 joules of work and the total work is 0.2 joules as before. You can make the point that here is an example where a transfer of energy that takes place in two steps is the same as if it were done all in one step. (This problem relates to Section 2).

EXERCISE 6: A force of 10.0 newtons acts on a 2.00 kg roller skate initially at rest on a frictionless table. The skate travels 3 meters while the force acts.

(a) How much work is done?

(b) How much energy is transferred to the skate?

(c) What is the final speed of the skate?

(a) The work done on the skate is 10.0 nts x 3 m = 30.0 joules.

(b) The energy that has been transferred to the skate equals the work done on it, 30.0 joules.

(c) The final speed of the skate is gotten by knowing that the kinetic energy of the skate, $\frac{1}{2}mv^2$, equals the energy that has been transferred to the skate.

$$\frac{1}{2}mv^2 = 30.0 \text{ joules, or } v = \sqrt{\frac{2 \times 30.0}{2.00}} = \underline{5.48 \text{ m/sec.}}$$

(This problem relates to the first part of Section 3).

EXERCISE 7: Compare the work done to lift a 6 kg mass 20 meters and the kinetic energy the same mass has when it has fallen a distance of 20 meters from rest.

The work done in lifting a 6 kg mass 20 meters is:

$$6 \text{ kg} \times 9.8 \text{ nt/kg} \times 20 \text{ m} = \underline{1200 \text{ joules}}.$$

When a 6 kg mass falls 20 meters under the force of its weight, its velocity is given by

$$\begin{aligned} v^2 &= 2as \\ &= 2gh \\ &= 2 \times 9.8 \text{ nt/kg} \times 20 \text{ m} \\ &= 400 \text{ (m/sec)}^2 \end{aligned}$$

$$\begin{aligned} \text{Its kinetic energy} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 6 \text{ kg} \times 400 \text{ (m/sec)}^2 \\ &= \underline{1200 \text{ joules}} \end{aligned}$$

The kinetic energy gained by a body in falling from a certain height is thus shown to be equal to the work done in raising it that height. (This problem relates to the first part of Section 3).

EXERCISE 8: A force of 30 newtons accelerates a 2 kg object from rest for a distance of 3 meters along a level, frictionless surface; the force then changes to 15 newtons and acts for an additional 2 meters.

a. What is the final kinetic energy of the object?

b. How fast is it moving?

(a) The kinetic energy of the object equals the energy transferred to it by the net force that acts on it. The energy transferred to the object equals the work done on it by the force, 30 nt x 3 m + 15 nt x 2 m = 120 joules.

(b) Its speed can be determined by using the fact that its kinetic energy is $\frac{1}{2}mv^2$;

$$\begin{aligned} v &= \sqrt{\frac{2 \times E_k}{m}} \\ &= \sqrt{\frac{2 \times 120}{2}} \\ &= \underline{11 \text{ m/sec}} \end{aligned}$$

(This problem relates to Section 3, particularly paragraphs 5 and 6)

EXERCISE 9: A force of 4 newtons applied at an angle of 45 degrees with the horizontal accelerates a 2 kg dry-ice disc, initially at rest, for a distance of 0.5 meters on a horizontal table.

(a) Find the work done on the disc.

(b) What is the velocity of the disc?

(a) The horizontal component of the 4 newton force has a magnitude of 4 x 0.7 = 2.8 newtons. Since the disc can only move in a horizontal plane, it is only this component of the 4 newton force that does work on the disc. The work done on the disc is 2.8 nt. x 0.5 m = 1.4 joules.

(b) The velocity of the disc is $v = \sqrt{\frac{2 \times E_k}{m}}$

$$= \sqrt{\frac{2 \times 1.4}{2}}$$

$$= \underline{1.2 \text{ m/sec}}.$$

(This problem relates to Section 4).

Note that the vertical component of the 4 newton force does no work on the disc; it merely lifts the disc, or presses it down on the table, depending on whether the force is directed upward or downward at 45°. In either case, the vertical component of the applied force, 2.8 newtons, is small compared to the weight of a 2 kg object, 19.6 nts.

EXERCISE 10: A 3 kg mass moving 5 m/sec is acted upon by a force of 4 newtons opposite to the direction of motion. If the object is slowed to a speed of 2 m/sec, what is

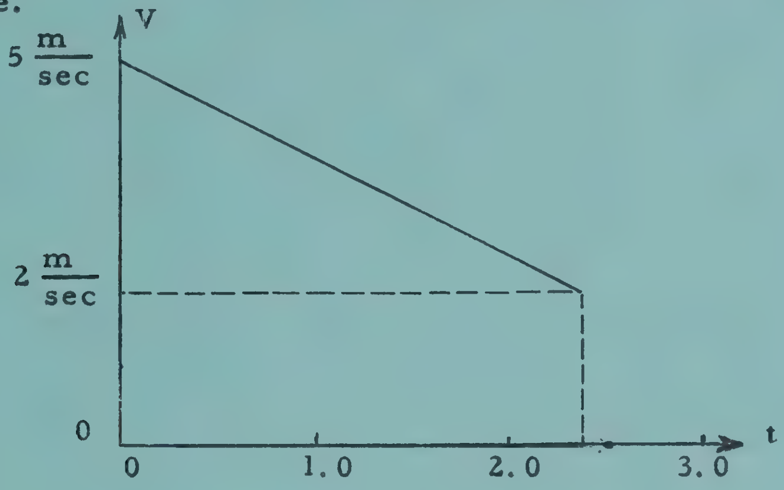
(a) The change in kinetic energy?

(b) The distance moved while the force acts?

(c) The energy gained by the system supplying the force?

(a) The kinetic energy of the mass before the force acts upon it is $\frac{1}{2} \times 3 \times 25 = 38$ joules. The kinetic energy of the mass after the force acts upon it is $\frac{1}{2} \times 3 \times 4 = 6.0$ joules. The change in kinetic energy is therefore -32 joules.

(b) The distance that the mass moves while the force acts can be found by plotting a graph of the velocity versus time and finding the area under the graph. At the beginning, the velocity is 5 m/sec. A force of 4 newtons acting on a mass of 3 kg in a direction opposite to its direction of motion produces a constant deceleration of $a = \frac{F}{m} = \frac{4}{3} = 1.3 \text{ m/sec}^2$. The time required for the velocity to decrease from 5 m/sec to 2 m/sec at a rate of 1.3 m/sec every second is $\Delta t = \frac{\Delta v}{a} = \frac{3}{1.3} = 2.3$ sec. The graph of velocity vs time is shown in the figure.



The area under the graph is $2 \times 2.3 + \frac{1}{2} \times 3 \times 2.3 = \underline{8 \text{ meters}}$.

(c) The energy gained by the system supplying the force is the product of the force times the distance moved = 4 nt x 8 m = 32 joules. Note that the energy gained by the system supplying the force is the same as the decrease in the kinetic energy of the mass. (This problem relates to Section 4, particularly to the last two paragraphs).

EXERCISE 11: An automobile of mass 1000 kg is moving at 100 km/hour. We read this speed on the speedometer.

(a) What is its kinetic energy?

(b) How much work was done to provide this kinetic energy?

(c) What force acted on the car to provide this kinetic energy? Through what distance did it move?

(a) The kinetic energy of a mass of 1000 kg moving at 100 km/hr, or 10^5 m/3600 sec = 27.8 m/sec, is $\frac{1}{2}$ x 1000 x 773 = $\underline{3.86 \times 10^5 \text{ joules}}$

(b) At least this much work must have been done to provide this kinetic energy; probably much more work was done since automobile engines lose a lot of energy in the form of heat in the exhaust gases, heat in the bearings, etc.

(c) There is no way of knowing what force acted on the car, or through what distance it moved; we only know that the product of the force and the distance is 3.86 x 10^5 joules. Any combination of force and distance whose product is 3.86 x 10^5 joules would produce that speed. (This problem relates to Section 4.)

EXERCISE 12: A 2 kg stone whirls around on the end of an 0.5 m string with a velocity of 2 revolutions per second.

(a) What is its kinetic energy?

(b) What is the centripetal force on it?

(c) How much work is done by the centripetal force in one revolution?

(a) To find the kinetic energy of the stone we have to know its speed as well as its mass. The circumference of the circle around which it is whirling is 2π x 0.5m = 3.14 m: It traverses the circumference in $\frac{1}{2}$ sec, so the speed is 3.14 m/$\frac{1}{2}$ sec = 6.28 m/sec. It's kinetic energy is then $\frac{1}{2}$ x 2 x 39.4 = $\underline{39.4 \text{ joules}}$.

(b) The centripetal force is $F_c = \frac{mv^2}{r} = \frac{2 \times 39.4}{0.5}$ = 158 newtons.

(c) No work is done by the centripetal force since it always acts at right angles to the direction of motion and thus has no component in the direction of motion.
(This problem relates to Section 4).

EXERCISE 13: A body moves along a curve with a constant kinetic energy of 10 joules. Part of the curve is the arc of a circle of radius 0.5 meters. What is the net force acting on the body while in this part of the curve? What is the direction of the force?

Since the centripetal force is $F_c = \frac{mv^2}{r}$ and the kinetic energy, E_k, is $\frac{1}{2}mv^2$, then $F_c = \frac{2E_k}{r} = \frac{2 \times 10}{0.5}$ = 40 newtons. Since this part of the curve is circular, the force must be perpendicular to the motion.
(This problem relates to Section 4).

EXERCISE 14: A man pulls with a string on a 20.0 kg mass initially at rest on the floor. He exerts a force of 20.0 newtons horizontally and the mass moves through 8.0 m. The mass then has a velocity of 3.00 m/sec.

(a) What is its final kinetic energy?

(b) How much energy has been transferred from the man?

(c) How do you explain the difference in your answers to (a) and (b)?

(a) The final kinetic energy of the mass is $\frac{1}{2} \times 20.0 \times 9.00$ = 90.0 joules.

(b) On the other hand, the man has done 20.0 x 8.0 = 160 joules of work.

(c) The difference between these two figures must be due to 70 joules of energy transformed into heat through friction.
(This problem relates to Section 10).

EXERCISE 15: Fig. 8 illustrates the interaction of two bodies m_1 and m_2 where $m_1 = 3m_2$. The repulsive force F is constant as long as the separation is less than the distance d, and the force is zero when the separation is greater than d. Suppose F = 6 newtons, d = 20 cm, $m_1 = 3$ kg, $m_2 = 1$ kg, and the initial speed of m_1 is $v_1 = 16$ cm/sec.

(a) Find expressions for Δx_1 and Δx_2 (the distances moved by m_1 and m_2 during the interaction) as a function of time, t, by using Newton's Law and $\Delta x = v_i t + \frac{1}{2}at^2$ where v_i is the initial speed (the speed v_1 for m_1 and zero for m_2).

(b) At the end of the interaction the masses are again d apart and $\Delta x_1 = \Delta x_2$. How long a time does the interaction last?

(c) Find expressions for v_1' and v_2', the speeds at the end of the interaction, in terms of v_1. Remember that for constant acceleration $v = v_i + at$.

Curiosity may impel someone to ask whether block 1 hits block 2. If block 2 were fixed so that it could not move, block 1 would have to approach block 2 against a repulsive force of 6 newtons for a distance of 0.2 m, thus losing 1.2 joules of energy. It possesses a kinetic energy of only $\frac{1}{2} \times 3 \times (0.16)^2 = .04$ joules, so it would stop before hitting block 2, even if block 2 did not move.

(a) The acceleration of block 1 is $\frac{F}{m_1} = -2\ \text{m/sec}^2$, the minus sign expressing the fact that block 1 is decelerating; Δx_1 is then

$$\underline{\Delta x_1} = 0.16t - \frac{1}{2} \times 2 \times t^2 = \underline{0.16t - t^2}$$

The acceleration of block 2 is $\frac{F}{m_2} = 6\ \text{m/sec}^2$; Δx_2 is then

$$\underline{\Delta x_2} = 0 + \frac{1}{2} \times 6 \times t^2 = \underline{3t^2}.$$

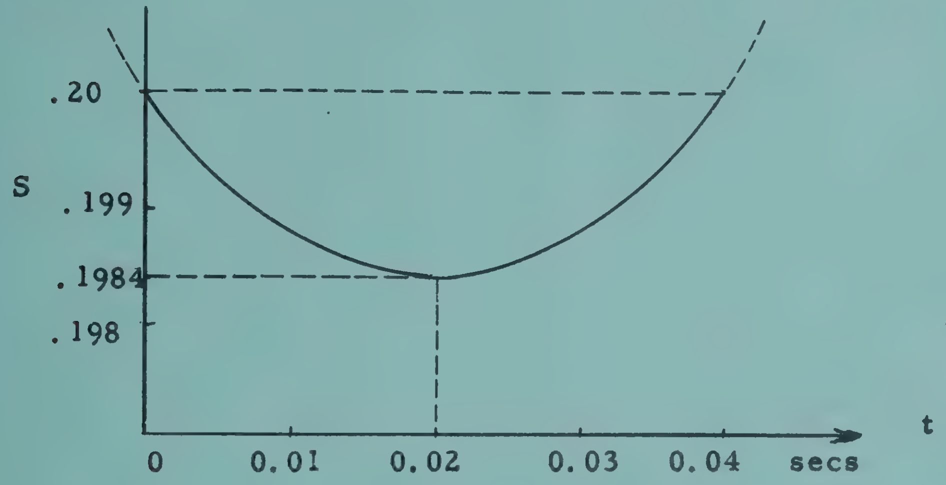
The separation distance is $s = d - \Delta s$

$$= .20 - (\Delta x_1 - \Delta x_2)$$

$$= .20 - (0.16t - 4t^2)$$

$$= 4t^2 - 0.16t + .20$$

(b) The graph of the separation during the time the two masses are less than 0.20 m apart looks as follows:



From this graph it can be seen that the closest the two masses get together is 0.1984 m. After that they start to move apart again because block 2, being lighter, accelerates more rapidly than block 1 under the force of repulsion. At 0.04 seconds, when they are again 0.2 m apart, the repulsive force no longer acts. At that instant the slope of the curve of s vs. t is 0.16 m/sec. This then is the rate at which they continue to separate after the "collision".

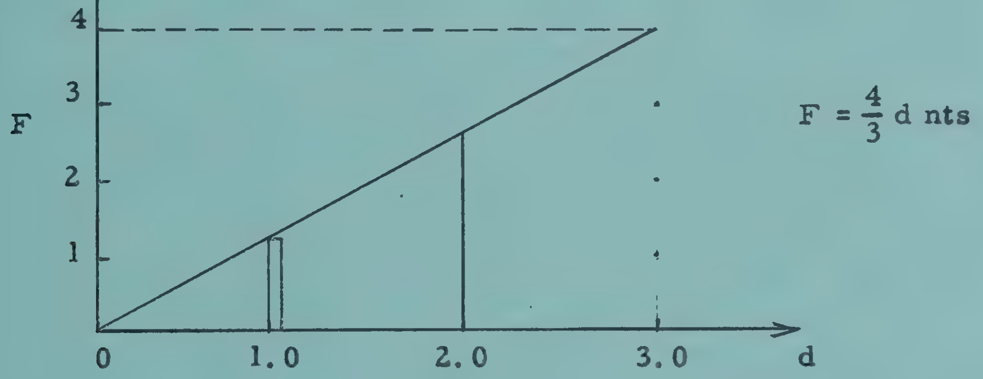
From this curve it is seen that the interaction lasts for 0.04 seconds. Without working out this curve the duration of the interaction can be calculated by setting $\Delta x_1 = \Delta x_2$, i.e., $0.16t - t^2 = 3t^2$. This equation is satisfied, i.e., $\Delta x_1 = \Delta x_2$, when $t = 0$ and when $t = 0.04$ secs.

(c) Since $v = v_i + at$, at $t = 0.04$ sec, $v_1' = 0.16 - 2 \times 0.04 = \underline{0.08 \text{ m/sec}}$, and $v_2' = 6 \times 0.04 = +\underline{0.24 \text{ m/sec}}$. The difference between these two velocities is the rate at which the two masses are separating, 0.16 m/sec. This figure agrees with the speed of separation deduced from the curve of s vs. t.

This problem deals quite directly and thoroughly with the material in Section 6. It could be extended for class discussion to examine the loss in kinetic energy of block 1 and the gain of energy of block 2 and show that the one equals the other.

EXERCISE 16: How much kinetic energy is lost by a 10.0 kg mass as it moves 2 meters against a retarding force that increases linearly by 4.00 newtons for every 3.00 meters the mass moves. The force is zero at the beginning.

To find the work done by the mass in pushing against a retarding force that changes with distance, we have to break the distance up into small intervals and calculate the work done by the force in each of these intervals.



For example, suppose we were to calculate the work done by the mass in moving from 1.0 to 1.1 meters against the retarding force. At 1.0 m the force is 1.33 newtons; at 1.1 m the force is 1.46 nt. The average force in this interval is 1.4 nts. The work done in going from 1.0 to 1.1 meters is then approximately equal to 1.4 nt. x 0.1 m = 0.14 joules. But this is just the area under the curve from 1.0 to 1.1 m. The total work done is just the sum of the work done in each one-tenth of a meter and is equal to the area under the graph of force vs. distance from 0.0 m to 2.0 meters or $W = \frac{1}{2} \times 2.67 \times 2.0 = \underline{2.67 \text{ nt.-meters}}$

$= \underline{2.67 \text{ joules}}$

(This problem relates to Section 7).

EXERCISE 17: A 3-kg mass accelerates from rest according to the force function shown in Fig. 17. What is the kinetic energy of the object at s = 2 m? At s = 5 m? At s = 13 m? At s = 15 m?

By the argument given in the answer to problem 16, the work done in each distance interval equals the area under the force vs. distance curve for that interval. So the answers are:

(a) $W = 4$ joules $= E_k$

(b) $W = 14.5$ joules $= E_k$

(c) $W = 38$ joules $= E_k$

(d) $W = 40$ joules $= E_k$

(This problem relates to Section 4).

EXERCISE 18: Two 3 kg bodies interact. At a given moment the first body is moving to the right at 0.50 m/sec and the second body is moving to the right at 0.30 m/sec. Over a short time interval the speed of approach of one body as measured from the second is 0.20 m/sec. If the force of interaction is repulsive and equal to 0.10 newtons, what is the rate at which the total kinetic energy decreases?

The question should specify that the desired rate at which the total kinetic energy decreases is the rate during the interval in which the speed of approach of one body as measured from the second is 0.20 m/sec. In a short time interval, say 1/100 second, the first body moves 0.005 m against a force of 0.1 newtons, thus losing 0.0005 joules of kinetic energy in 0.01 sec, a rate of energy loss of 0.05 joules per second. The second body, presumably being overtaken by the first, is being accelerated by the repulsive force, gaining kinetic energy. In 1/100 sec it goes 0.003 m and is acted on by a repulsive force of 0.1 nts, thus gaining 0.0003 joules of kinetic energy in 1/100 sec, a rate of energy gain of 0.03 joules per second. The total kinetic energy of the pair must be decreasing at a rate of 0.05 - 0.03 joules = 0.02 joules per second.

(This problem relates to Section 7).

EXERCISE 19: A 1.5 kg body is at rest. It is "hit" head-on by a body of mass 0.5 kg moving with a speed of 0.2 m/sec. The interaction force depends only on the separation of the two bodies. What is the final velocity of each body? In what direction does each move after the interaction?

This problem can be worked out from scratch using the approach described in the appendix to the chapter. For students who do not go through that development, this problem serves as an example of the use of the equations that are derived in that appendix. They are presented in the second paragraph of section 8. For this problem, $m_2 = 1.5$ kg, $m_1 = 0.5$ kg, $v_1 = 0.2$ m/sec.

$$\underline{v_2'} = \frac{2m_1}{m_1 + m_2} V_1 = 2 \times \frac{0.5}{2.0} \times 0.2 = \underline{0.1\ \text{m/sec}}.$$

Similarly,

$$\underline{v_1'} = \frac{m_1 - m_2}{m_1 + m_2} v_1 = \frac{0.5 - 1.5}{2.0} \times 0.2 = -\ \underline{0.1\ \text{m/sec}}.$$

The significance of the positive sign in the answer for v_2' is that the second ball, the struck one, goes off in the same direction the incident ball was travelling before the collision. The significance of the negative sign in the answer for v_1' is that the incident ball, which is lighter, bounces back in the collision so that its velocity is oppositely directed from what it was before the collision.
(This problem relates to Section 8).

EXERCISE 20: A moving electron "hits" a stationary proton head-on. The forces between the two depend on the separation only. (The mass of a proton is 1840 times the mass of an electron.) What fraction of the electron's kinetic energy is transferred to the proton?

This problem, as Problem 19, gives an example of a collision where a light body strikes a heavy one, only this time, the light body is much lighter than the heavy one. All we need to know about the masses is their

ratio, since we can write

$$v_2' = \frac{2m_1}{m_1 + m_2} V_1 = \frac{2\,\frac{m_1}{m_1}}{\frac{m_1}{m_1} + \frac{m_2}{m_1}} V_1 = \frac{2}{1 + \frac{m_2}{m_1}} V_1$$

$$\text{and } v_1' = \frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}}$$

In this problem, body 1 is the electron, so $\frac{m_2}{m_1} = 1840$, and

$$v_2' = \frac{2}{1841} v_1 \qquad\qquad v_1' = \frac{1-1840\ v_1}{1 + 1840} = -\frac{1839}{1841} v_1$$

$$= 1.086 \cdot 10^{-3}\ v_1 \qquad\qquad v_1' = -.999\ v_1$$

so the electron recoils with .999 of its incident speed and the proton acquires a forward speed of about 1/1000 that of the incident electron.

The ratio of the kinetic energy of the proton after the collision to that after the collision of the electron before the collision is

$$\frac{\frac{1}{2} m_2 v_2'^2}{\frac{1}{2} m_1 v_1^2} = \frac{m_2}{m_1}\left(\frac{v_2'}{v_1}\right)^2 = 1840\ (1.086 \times 10^{-3})^2 = 1840 \times 1.179 \times 10^{-6}$$

$$= 2.169 \times 10^{-3}$$

(This problem relates to Section 8).

EXERCISE 21: An object of mass m_1 with a kinetic energy E_k collides head-on with a stationary object of mass m_2. How much kinetic energy is transferred if the force of interaction depends only on the separation and

(a) m_1 is much greater than m_2?

(b) $m_1 = m_2$?

(c) m_1 is much less than m_2?

To calculate the energy transferred from body 1 to body 2 we need to know $v_2' = \frac{2m_1}{m_1 + m_2} v_1$.

(a) If $m_1 \quad m_2$, $v_2' \approx 2v_1$, so

$$\frac{1}{2}m_2{v_2'}^2 = \frac{1}{2}m_2 \times 4v_1^2 = 4 \times \frac{m_2}{m_1} \times \frac{1}{2}m_1v_1^2 = \underline{4\frac{m_2}{m_1}E_k}$$

(b) If $m_1 = m_2$, $v_2' = v_1$, $\frac{1}{2}m_2v_1^2 = \frac{1}{2}m_1v_1^2 = \underline{E_k}$

(c) If $m_1 \ll m_2$, $v_2' = 2\frac{m_1}{m_2}v_1$; $\frac{1}{2}m_2{v_2'}^2 = \frac{1}{2}m_2 \cdot 4\frac{m_1^2}{m_2^2}v_1^2$

$$= 4 \times \frac{m_1}{m_2} (\frac{1}{2}m_1v_1^2,$$

$$= \underline{4\frac{m_1}{m_2}E_k}$$

The purpose of this problem is to show that for equal masses, all of the kinetic energy is transferred, i.e., the incident mass comes to rest, and that for large ratios of masses, the fraction of the energy transferred is 4 x the ratio of the masses.
(This problem relates to Section 8).

EXERCISE 22: A 5.0 kg body is at rest. A 10 kg body approaches it with a velocity of 0.20 m/sec. The interaction force is zero when the separation is greater than 0.10 meters and is 4.0 newtons when the separation is less than this distance.
(Note that some of the answers in this problem are easier to get if you use the conservation of momentum).

(a) What is the kinetic energy of the masses before the interaction?
(b) What will be the kinetic energy of each mass after the interaction is complete?
(c) What will be the kinetic energy of each mass when the separation is at a minimum? Recall that the speeds at minimum separation are equal.
(d) What is the minimum separation?

(a, b) Before the interaction only the 10 kg body is moving and it has a kinetic energy of 0.20 joules. The velocity of the 5.0 kg mass after the interaction is $\frac{2 \times 10.0}{15.0} \times 0.20$ m/sec = 0.267 m/sec. Its kinetic energy thus becomes $\frac{1}{2} \times 5.0 \times (0.267)^2$ = 0.178 joules. Since this kinetic energy must have come from the first mass, the kinetic energy left with the first mass must be only 0.200 - 0.178 = 0.022 joules.

(c) (Comment on Comment in statement of problem, part (c)): If the speeds are not equal, the masses must be approaching or leaving each other, i.e. the spacing cannot be a minimum. When the spacing is a minimum, the speed of the "struck" mass must have increased to the point where the incident mass is no longer catching up to it; therefore at that instant the speeds must be equal).

When the separation of the masses is at a minimum, their velocities are equal at some value v''. The momentum of the two masses together

is $(m_1 + m_2)v''$ and is equal to $m_1 v_1$ by conservation of momentum.

So $v'' = \frac{m_1}{m_1+m_2} v_1$. The kinetic energy of the 10 kg mass at this time is then $\frac{1}{2} m_1 v''^2 = \frac{1}{2} m_1 (\frac{m_1}{m_1+m_2})^2 v_1^2$

$$= \frac{1}{2} m_1 v_1^2 \left(\frac{m_1}{m_1+m_2}\right)^2 = .20 \times \frac{100}{225} = .0890 \text{ \underline{joules}}$$

The kinetic energy of the 5 kg mass is similarly

$$E_{k_2} = \frac{1}{2} m_2 v''^2 = \frac{1}{2} m_1 v_1^2 \quad \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.20 \times \frac{50}{225} = \underline{0.0445} \text{ joules}$$

The total kinetic energy is .0890 + .0445 = .1335 joules. (Note that we can not use conservation of kinetic energy here to solve for v'' since kinetic energy is not conserved during the interaction.

(d) Now that we know the kinetic energy gained by the 5 kg mass and the kinetic energy lost by the 10 kg mass, we know the net kinetic energy lost by the pair and therefore the work done against the interaction force.

$$-(\Delta E_{k_1} + \Delta E_{k_2}) = F\Delta s$$

$$\text{So, } \Delta s = -\frac{(\Delta E_{k_1} + \Delta E_{k_2})}{F} = -\frac{(-.111 + .0445)}{4.0} = \frac{.0665}{4.0}$$

at the minimum, $\Delta s = 0.0166$ m = 1.66 cm.

To check this we should see that the net gain in kinetic energy during the second half of the collision equals the work done by the force of interaction in separating the masses from 20-1.66 = 18.34 cm back to 20 cm.

$$F \cdot \Delta s = 4 \times .0166 = .0664 \text{ joules}$$

$$\Delta E_{k_1} = .022 - .0890 = -.067 \text{ joules}$$

$$\Delta E_{k_2} = .178 - .045 = .133 \text{ joules}$$

$$F \cdot \Delta s = \Delta E_{k_2} + \Delta E_{k_1} = .133 - .067 = .066 \text{ joules}$$

* * *

Parts (c) and (d) can be solved without using the conservation of momentum as follows: at the minimum separation, the velocities are equal at some value, v''.

$$v'' = 0.2 \text{ m/sec} + a_1 t = 0.2 - \frac{4}{10} t = 0.2 - .4t$$

$$\text{and } v'' = 0 + a_2 t = \frac{4}{5} t = 0.8t$$

So $0.2 - .4t = 0.8t$, and $t = \frac{0.2}{1.2} = 0.167$

and $v'' = 0.8 \times 0.167 = 0.133$

$$E_{k_1} = \frac{1}{2} \times 10 \times (.133)^2 = \underline{.089} \text{ joules}$$

$$E_{k_2} = \frac{1}{2} \times 5 \times (.133)^2 = \underline{.044} \text{ joules}$$

$$\text{Then } \Delta s = \frac{\Delta E_k}{F} = \frac{0.2 - .133}{4} = .0166 \text{ m} = \underline{1.66 \text{ cm}}$$

(This problem relates to Section 8).

EXERCISE 23: An antiaircraft shell has a kinetic energy E_k and a momentum $\vec{p}$. Just then it explodes. What can you say about

(a) The momentum of the pieces?
(b) The kinetic energy of the pieces?

Since no external force acts on the shell, the total momentum of the shell must remain unchanged by the explosion. The vector sum of the momenta of all of the pieces is therefore equal to the momentum (vector) of the shell just before it exploded. (See question at the end of Experiment 520). The total kinetic energy of the pieces must equal the kinetic energy of the shell plus the work done by the explosion on the pieces.

EXERCISE 24: (a) In Fig. 6 make the necessary measurements and calculate the ratio of the kinetic energy before and after impact of the billiard ball marked with the dot.

(b) What fraction of the initial energy of the dotted ball is gained by the undotted ball?

(c) What is the ratio of the total kinetic energy before the collision to the total kinetic energy after the collision?

(a) By measuring distances on the print, we found that the dotted ball moved 1.14 cm/flash. After the collision the dotted ball moves 0.52 cm/flash. Since the kinetic energy is proportional to v^2, the ratio of the kinetic energy of the dotted ball after the impact to that before the impact is

$$\frac{(0.52)^2}{(1.14)^2} = 0.21$$

(b) Again using the fact that kinetic energy is proportional to v^2 and also the fact that the balls have equal masses, we find that the fraction of the initial energy of the dotted ball gained by the undotted ball is

$$\frac{(0.94)^2}{(1.14)^2} = .68$$

(c) After the collision the sum of the kinetic energies of the two balls is 0.89 of the initial kinetic energy. 11°/o of the energy must have gone into heat. Thus, the kinetic energy before the collision is 1.12 E_k after the collision.

EXERCISE 25: A 3-kg ball with a speed of 6 m/sec strikes another 3-kg ball moving 6 m/sec in the opposite direction. After collision each ball has a velocity equal in magnitude but opposite in direction to its initial velocity. Is this an elastic collision?

This must have been an elastic collision since the kinetic energy of each ball is the same after the collision as before. If any kinetic energy had been lost, the collision would have been inelastic. Ask the students why the kinetic energy remains the same for each ball even if the velocity has changed direction. (Answer: The kinetic energy is proportional to the square of the speed and is therefore the same for positive or negative speeds.)

EXERCISE 26: In one of the experiments that led to the determination of the mass of the neutron, Chadwick measured the velocity of protons that had been hit head-on by neutrons. The speed of the protons was 3.3×10^7 m/sec.

(a) What was the speed of the neutrons before and after collision with the protons?

(b) Chadwick also measured the speed of nitrogen atoms hit head-on by the neutrons. What was it?

(c) What was the velocity of the neutrons after each kind of collision?

$v_2' = 3.3 \times 10^7$ m/sec.

(a) $v_1 = \frac{m_1 + m_2}{2\,m_1}\, v_2' = \underline{3.3 \times 10^7 \text{ m/sec}}, \quad v_1' = \frac{m_1 - m_2}{m_1 + m_2} = \underline{\underline{0}}.$

(b) $v_2' = \frac{2m_1}{m_1 + m_2} v_1 = \frac{2m_1}{m_1 + 14m_1} \times 3.3 \times 10^7 \text{ m/sec} = \underline{4.4 \times 10^6 \text{ m/sec.}}$

(c) v_1' for proton target $= \frac{m_1 - m_2}{m_1 + m_2} v_1 = \underline{0}$ since $m_1 = m_2$

v_1' for nitrogen target $= \frac{m_1 - 14m_1}{m_1 + 14m_1} \times 3.3 \times 10^7 = -\,\underline{2.86 \times 10^7 \text{ m/sec.}}$

(In the latter case the neutrons bounce back.)

EXERCISE 27: An empty freight car A with a mass of 2.5×10^4 kg (25 tons) coasts along a horizontal track at 2 m/sec until it couples to a stationary car B whose mass is 5×10^4 kg. There is little friction, and the brakes are off.

(a) What is the initial momentum of car A?

(b) What is the speed of the two cars as they move along the track after the interaction?

(c) What is the total kinetic energy before and after the impact? Is this an example of an elastic collision?

$v_1 = 2$ m/sec, $m_1 = 2.5 \times 10^4$ kg.

$v_2 = 0$, $m_2 = 5 \times 10^4$ kg.

(a) $p_1 = m_1 v_1 = \underline{5 \times 10^4 \text{ kg m/sec}}$

(b) $p' = p_1, \therefore (m_1 + m_2) v' = m_1 v_1$

$$v' = \frac{m_1}{m_1 + m_2} v_1 = \frac{2.5 \times 10^4}{7.5 \times 10^4} \times 2 = \underline{\frac{2}{3} \text{ m/sec.}}$$

(c) K.E. (before) $= \frac{1}{2} m_1 v_1^2 = \underline{5 \times 10^4 \text{ joules}}$

K.E. (after) $= \frac{1}{2} (m_1 + m_2) v'^2 = \underline{1.667 \times 10^4 \text{ joules}}$

3.33×10^4 joules were lost in the collision and therefore it must have been an inelastic collision.

EXERCISE 28: Rub the palms of your hands together briskly. Do you do any work? What happens to the energy?

When you rub your hands together you do work since each arm is exerting a force on its hand in the direction of motion. The energy expended by your muscles goes into heating your hands.

EXERCISE 29: This problem is terribly difficult and should not be assigned. You may want some of your better students to think about it.

EXERCISE 30: A 100-gm light bulb dropped from a high tower reaches a velocity of 20 m/sec after falling 100 m. About how much energy has been transferred to the air?

The weight of the bulb, the downward gravitational force, does an amount of work equal to the force times the distance the bulb drops, 0.1 kg x 9.8 nt/kg x 100 m. equals 98 joules. After the bulb has fallen 100 m, its kinetic energy is only 20 joules. The difference between the work done on the bulb and its kinetic energy represents work done by the bulb against the force of air friction. It amounts to 78 joules and goes into warming the air and the bulb.

EXERCISE 31: A force of 3 newtons and a force of 4 newtons are applied simultaneously to a 4-kg mass initially at rest. The two forces act at an angle of 90 degrees with each other for 2 sec.

(a) What is the resultant force?
(b) Use the resultant force to calculate the total work done.
(c) What is the work done by each force separately? How does the sum of these works compare with the answer to (b)?

(a) The resultant force is 5 newtons at an angle of 37° measured from the direction of the 4 newton force toward the 3 newton force.
(b) The resultant acceleration of the 4-kg mass is 1.25 m/sec^2. During the two seconds that the forces act, the mass goes a distance of 2.5 m. The work done by the resultant force is 5 x 2.5 = 12.5 joules.
(c) In the direction of the 4 newton force, the 4 newton force causes an acceleration of 1 m/sec^2. With this acceleration, in 2 secs the mass moves 2 meters. Thus, the work done by the 4 newton force is 8 joules. In the direction of the 3 newton force, the 3 newton force causes an acceleration of 0.75 m/sec^2. In 2 seconds the mass moves 1.5 meters. The work done by the 3 newton force is thus 4.5 joules. The sum of the work done by the two forces regarded separately is 12.5 joules, the same as the work done by the resultant. Note particularly that while the resultant of the two forces is found by vector addition, the resultant of the individual contributions to the work done on the mass is found by scalar, or arithmetic, addition.

EXERCISE 32: A horseshoe magnet of mass m stands on end on a frictionless table, and a steel ball bearing of mass m is rolled toward it from far away with velocity v and goes through it and on far beyond. Assume the force of attraction F changes with distance and is the same in front of the magnet and behind. (See Fig. 18.) What is the final velocity of the ball? What is the final velocity of the magnet?

This problem involves an elastic collision, although the students may not see it as a collision. Furthermore, this collision is novel in that the forces are attractive. It is only because the ball can scoot between the legs of the magnet that the ball can go away from the collision instead of sticking to the magnet. Now, why is it elastic? Because the force between the two objects depends only on the separation distance.

Since it is elastic, the formulas for v_1' and v_2' derived in the appendix to Chapter 5 apply. Note, however, that this problem represents an example of the special case mentioned in the last paragraph of the appendix; in addition to the formulas for v_1' and v_2' given in the text, there is another valid solution, namely, that $v_1' = v_1$, and $v_2' = 0$, i.e., the momentum is unchanged and no energy transfers from one body to the other over the complete interaction. The unlikely situation that is mentioned in the appendix, namely that m_1 must pass right through m_2 in this case, is just what happens in this problem.

So, the final velocity of the ball is the same as its initial velocity. The final velocity of the magnet is zero.

EXERCISE 33: A two-stage rocket is moving with velocity v in a region where gravity is negligible. It consists of a final stage, or head, of mass m, and a tail of mass 2m which includes an explosive charge to separate the head. The charge is carefully designed so that after separation the tail has the least possible energy. (Assume that the gases from the explosion stay with the tail.)

(a) What is the least possible energy the tail can have?

(b) In this case, how much energy has the head got after the explosion? Give the answer in terms of m and v.

(c) How much kinetic energy did the head have before the explosion?

(d) How much energy has the head gained?

(e) How much energy is supplied by the fuel in the explosion?

(f) Why is the answer to (e) different from the answer to (d)?

(a) Since kinetic energy is always a positive quantity, its least possible value is zero.

(b) By conservation of momentum, we see that the initial momentum of the whole rocket, $p = 3mv$, is equal to the final momentum of the head, $p' = mv'$. Solving this equality for v', we find,

$$p = p'$$

$$3m\,v = mv'$$

$$v' = 3v.$$

We then find the kinetic energy of the head after the explosion to be

$$\frac{1}{2} \times m \times (3v)^2 = \underline{4.5\ mv^2}$$

(c) Before the explosion, the head had a kinetic energy of $\underline{\frac{1}{2}mv^2}$.

(d) The head has gained $4mv^2$ of kinetic energy.

(e) In the explosion process, the tail lost mv^2 of kinetic energy, while the head gained $4mv^2$. The difference of $3mv^2$ had to be supplied by the explosive charge.

(f) The difference between the answer to (e) and the answer to (d) lies in the kinetic energy given up by the tail of the rocket during the explosion.

Supplement to Problem Solutions:
For Home, Desk and Lab, Problem 29.

EXERCISE 29: I push a finger steadily across a glass table top so that it moves at a constant speed of 3 cm/sec under a 1-newton horizontal force.

(a) What work is done in 1 sec by the force my arm exerts on the finger? What energy does it transfer into the finger each second?

(b) How large and in what direction is the force of friction on my finger? What work is done by the force of friction on my finger each second? In what direction does it transfer energy--in or out of my finger?

(c) What is the net force on the finger? What work does the net force do per second on my moving finger? What happens to the kinetic energy of the finger?

(d) What is the force exerted horizontally on the table by my finger? How far does the table move? What work on the table does this force do each second? In which direction does it transfer energy--in or out of the table?

(e) What happens in the region where finger and table are in contact?

(a) I push my finger across the table top 3 cms in 1 second, exerting a 1 newton horizontal force, so the energy transferred is 1 x 0.03 = 0.03 joules.

(b) The frictional force on my finger, as-a-whole, is in the opposite direction. It therefore transfers energy out, of the center of mass, instead of in, at exactly the same rate, in other words at 0.03 joules per second.

(c) The net force on the finger is zero, as we know, because it goes at constant velocity. It does no work, and does not increase the center of mass kinetic energy.

(d) The force exerted by my finger on the table is the same as by me on the finger, but the table, as-a-whole, does not move any distance. Consequently, work is not done by this force in the usual meaning, and the kinetic energy of the table as a whole does not increase. The energy transferred out of my finger, as answered in section (b), therefore must go into something else. It goes into the region where my finger and the table are in contact and

(e) we find heat there.

The point to be emphasized in this problem is that work is done by me when I push my finger along the table. No change in kinetic energy occurs. What has happened to the energy that has been transferred

from me by the mechanical work that I have done? The only other change which has occurred in the objects with which the problem is concerned is the heating of finger and table. The student is then led to suspect that this is another form of energy which must eventually be fitted into the overall picture. This is analogous to the implicit suggestion of the idea of potential energy in the problems of elastic interactions within which kinetic energy is found not to be conserved.

There are some difficult points in the complete understanding of this problem. The above discussion should suffice, but students may be puzzled by the intricacies of friction forces. Here, then is a discussion of some of these details. It is hoped that you can avoid them entirely.

The finger is getting hot. Yet in the answers to a) and b) it is stated that energy is being transferred out of the finger as fast as it is being transferred in. This seems to be inconsistent with the principle of energy conservation. The explanation lies in the fact that frictional forces are really very complicated when viewed in detail. The interacting spots of the surfaces are not rigidly attached to the bodies that are sliding over one another. Thus although it is true, as stated in section 9 of the text, that the work done at each interacting spot on the center of mass will be

$$w_a = F_{ax}x, \ w_b = F_{bx}x, \text{ etc.}$$

This is not necessarily the complete story. The trouble is that the force components, F_{ax}, F_{bx}, etc. do not usually act on the same point of the body and relative motions of the points of application are possible. Thus while the body as a whole may move a distance, x the points of application of the many small contributing forces which total up to the "friction force" may move through quite different distances.

Consider a "simple" macroscopic example. A violinist draws his bow across a violin string. In doing this he moves it through a distance L, applying a force F_v. He has then done an amount of work, $W_v = F_vL$. The motion is opposed by the frictional force associated with the bow sliding across the string. This force must be equal and opposite to F_v since the bow does not accelerate. However this friction force is really the time-average of a varying force between bow and string. In a simple model, the bow can be thought of as pushing the string forward until the elastic restoring force of the string exceeds the force of static friction of string on bow. The string then starts to slide backwards while the bow continues to move forwards. The relationship between force x distance and force x time becomes very complicated. It is no longer obvious that the sum of the small contributing local (F_iX_i) will be the same as F_vx.

In the case of the finger on the table, we know that this identity must break down locally, in the region of the contact. We know it, because energy, in the form of heat, accumulates in the finger. Then, since mechanical work is the only important mode of energy transfer in this problem, the negative work done by the friction force on the finger in the region of the interaction must be less than the positive work done by me in pushing my finger along.

Actually, since work is a scalar and force a vector, it is best to think of the total work done by a number of forces as the sum of the individual works, not as the vector sum of the forces times the displacement distance. The latter is satisfactory when you are interested only in center of mass motion and kinetic energy, but consider the example of a coil spring held between the jaws of a pair of pliers. Squeeze the pliers. A force +F acts at one end of the spring through a distance +x. A force, -F, acts on the other end of the spring pushing it through a displacement -x. The net force acting on the spring is zero. Therefore the acceleration is zero and the change in bulk kinetic energy is zero. Yet the work done on the spring is

$$(+F)\quad(+x)\quad+\quad(-F)\quad(-x)\quad=2Fx$$

Note that it is not desirable to go into details of the microscopic picture at this stage of the development. Heat will be studied microscopically in chapter 7. At that time, this problem might be discussed again.

Chapter 6 - Potential Energy

PURPOSE OF CHAPTER: The concept of energy is broadened to include "stored" or potential energy. We then study the exchange of energy between the two forms, kinetic and potential, the conservation of mechanical energy for "conservative" forces, and various specific forms of potential energy.

CONTENT OF CHAPTER: Most of the mathematical relations and equations necessary for Chapter 6 have been developed in Chapter 5. In Chapter 6 we re-examine the elastic collision and see that when kinetic energy disappears it may be thought of as stored. The case of a mass striking a spring, compressing it, and then being thrown back by the spring is treated in Section 1. An expression for the potential energy of a stretched or compressed spring is developed and used to state the law of conservation of mechanical energy. We start with this example for several reasons: (1) Only one mass is involved (2) It is easy to "locate" the stored energy (it is "in" the spring) in contrast to the puzzling problem of energy stored in a field of force. (3) It is easy to follow the energy transfers in detail. (4) Students can easily imagine such a collision in contrast with the unfamiliar collision of two particles acting at a distance through "force fields." (5) The utility of the energy method is apparent, there being no temptation to find the velocity by starting from the acceleration as is the case with a constant force.

Section 2 takes up the more complicated case of two bodies which interact. The physical origin of the force is not specified, although the gravitational force would serve as an example. In fact, Section 2 principally sets the stage for the discussion of gravitational potential energy in Sections 3 and 4, and of electrical potential energy in Vol IV.

In Sections 3 and 4, the gravitational potential energy of a pair of objects is considered. Since calculation of the gravitational potential energy of an object near the surface of the earth is simplest (it involves the work done by a constant force) it is treated first (Section 3). The more complicated case involving distances so great that the gravitational force varies from place to place is important when considering planetary motion or satellites, and is treated in Section 4. Section 5 is a prelude to enlarging the conservation of energy to include heat.

COMMENT: The major objective of Chapter 6 is to give the students a good feel for potential energy as stored energy; and a thorough understanding of the "bookkeeping" of energy conservation. They will benefit from considerable drill in which they keep track of the energy of a system, when it is in the form of kinetic energy, when it is in the form of potential energy, which body does work on which when energy is transformed. The analogy to keeping financial records is a good one, and the sign of the energy transfer (is the work of A on B positive or negative?) is just as crucial as in banking where a sign error changes a deposit into a withdrawal! The major difficulty students have arises from confusion between two forms of energy bookkeeping: in the first, the total energy is equated to the sum of the kinetic and potential; in the second, the gain (or loss) in kinetic energy is equated to the loss (or gain) in potential energy. The class needs lots of practice in both viewpoints. Many examples with simple situations and numbers are needed, and will contribute to developing a genuine physical feeling for the conservation of energy. Teach the general principle by repeated specific examples.

PSSC Teacher's Guide Chap 6 Vol 3

Section 1 - The Spring Bumper

PURPOSE: To introduce the concept of potential energy and how to use it; to set the stage for more general forces in terms of a familiar example, the spring.

CONTENT: (1) Mechanical energy is conserved whenever the force between two bodies depends only on their separation. (2) If the force to compress a spring a given amount is "history independent", so are the work to compress the spring and the energy which can be realized by releasing the spring. (3) The potential energy of a spring, U is given by the area under the curve of force versus compression. $U = \frac{1}{2}kx^2$ (4) $\frac{1}{2}mv^2 + U =$ constant (for conservative forces.)

EMPHASIS: A very important section which must be thoroughly grasped before later sections can be properly appreciated.

COMMENT: Do not stress the notion of the non-elastic spring, but be sure the class understands what property we want: for the spring to be elastic it must be able to do as much work on us when we release it as we needed to do on it to compress it. Students should know that they can compute the potential energy of a spring from the area under the force-compression curve. They will memorize $U = \frac{1}{2}kx^2$ automatically, but the important thing for the students to know is where it came from. Discuss lots of examples and follow the energy transfers.

* * *

The key objective to be attained is a familiarity with energy conservation and transfer gained through discussion of numerous examples in which the form of the energy (kinetic or potential) is followed, and the signs of the transfer recognized. In this section we try to avoid the abstraction of gravitational potential energy or that of any other "action at a distance" force field.

* * *

You can discuss many variations of the mass and spring. For example, let the mass strike the spring, but at the moment of maximum compression catch the spring with your hand. Then the mass stays at rest. If you slowly release the spring, the potential energy of the spring does work against your hand. How does the work your hand does in releasing the spring compare with the original kinetic energy of the mass? (It is exactly the negative since you "absorb" the potential energy of the spring; it is minus, because in releasing the spring, your hand moves in the opposite direction from that of the force it exerts on the spring).

You can let one mass strike and compress the spring, and then after you have caught it, as before, substitute a different mass. Follow the acceleration of this second mass in detail and show that the final kinetic energy is still the same.

Another variation: suppose the mass hits the spring, rebounds, but leaves the spring vibrating. How does the kinetic energy of the mass compare with its value before the collision? It is less by just the energy left in the spring. Ask your class whether the energy left in the spring is potential or kinetic. Of course, it goes back and forth between the two forms, just as the energy of a mass oscillating on a massless spring goes back and forth from kinetic energy (when the spring is unstretched and the mass going its fastest) to potential energy (when the mass is momentarily at rest and the spring compressed or extended the maximum amount).

* * *

Incidentally, when an object of mass M collides with a spring of mass m and bounces back, the spring will, in general, be left vibrating. For M to leave with all its incident kinetic energy, it must be much more massive than the spring. We can see this result by noting that one end of the spring does not move, but the other moves at the same velocity, v, as M. Therefore, when the spring and mass break contact, M has kinetic energy $\frac{1}{2} mv^2$, but the spring has kinetic energy somewhere between $\frac{1}{2} mv^2$ (the value it would have if all parts of the spring were in motion at velocity v) and 0 (the value if no part of the spring could move).

* * *

The tuning fork is another case of a system which may possess both potential and kinetic energy (in this case due to the vibrational motion), as does the Slinky used in wave demonstrations. Note that when a wave travels down the Slinky, the kinetic and potential energy (of the deformed spring) move along, and the wave may be viewed as a flow of energy.

* * *

The use of a spring necessitates a slight generalization of the concept of a conservative force developed in Ch. 5, namely, a force between two bodies which depends only on their separation. What we wish of the spring is that on release we get back the work done in compression. We may express the requirements by a physical statement such as "there must be no friction". Or we may make a somewhat more abstract statement "the compression of the spring must depend solely on the force, but not on other factors, for example, the history of previous compression." Although the second statement may seem remote from requirements on "friction" it is not. If the compression depends only on the force, the work we do in compressing the spring is exactly the negative of the work we do when we release the compression. And, we get back all the energy we put into compression - there is no energy lost in frictional effects. (All of this means "elastic.")

It is instructive to consider a specific example which you can easily perform and which further illustrates the relation between the abstract requirement and its physical consequences. If we fasten one end of a four foot piece of No. 22 copper wire in a vise and grasp the other end with a pair of pliers, we can easily stretch the wire. In a typical case, the wire will stretch for about 8 inches before breaking. If stretched only 4 inches and then released the wire does not return to its original length. (Like pulling taffy, it stays pulled). We call this plastic deformation. Had the stretch been only $\frac{1}{32}''$, the wire would have returned to its original length on release, a case of elastic deformation. From an atomic viewpoint, an elastic deformation simply increases the interatomic spacing by the same percentage as the gross length changes. On the other hand, with plastic deformation rows of atoms slide over one another (a phenomenon appropriately called slip). During slip, we expect, and find, "frictional" energy loss entirely analogous to rubbing together two pieces of metal.

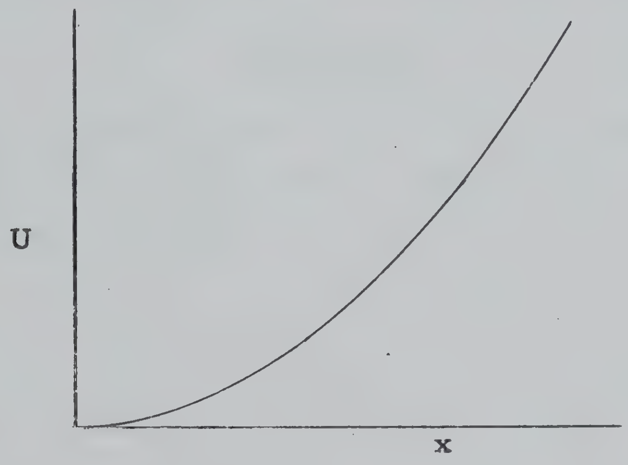
* * *

We must be careful to realize that the requirement for a spring to be perfectly elastic has nothing to do with whether or not the force-compression curve is a straight line. The only requirement is that the force be a single-valued function of the compression. Thus, the spring might get progressively "stiffer", the more it is compressed, without there being any "frictional" effects. (See Problem 4 and Figure 18.)

In practice we usually find that a deviation from linearity occurs when plastic flow sets in.

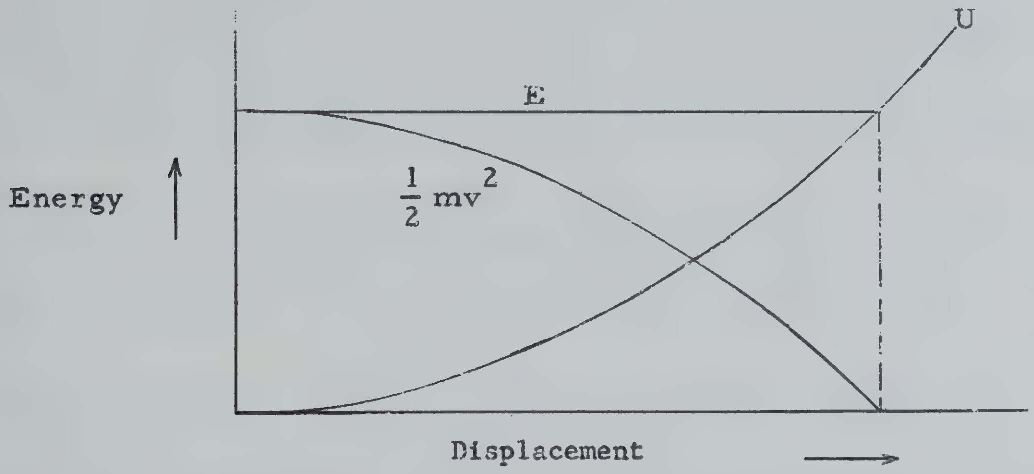
* * *

Figure 6 may warrant a little discussion. The graph is a plot of the potential energy of a peculiar spring as a function of compression. Let us consider here the same type of plot for the ideal spring whose F vs x graph is shown in Figure 5. For this spring, we have $F = kx$ and $U = \frac{1}{2}kx^2$ as shown. Thus the potential energy graph would resemble the following:



Now, if we wish to represent all the energies involved in the motion of a mass, m, as it compresses this spring, we can make another graph on which energy is on the ordinate axis and displacement is represented along the abscissa. Then we can plot the three curves together on this graph. The potential energy curve will be the same as that plotted above. The total energy, $E = \frac{1}{2}mv^2 + U$, must be a constant if the spring is

elastic. When U = 0 (at x = 0), $E = \frac{1}{2}mv^2$. Thus the horizontal straight line which represents E is drawn at this level, the value of the kinetic energy when the compression is zero. The kinetic energy is then given at each point by the difference between the horizontal line depicting E and the curve depicting U; $\frac{1}{2}mv^2 = E - U$.



Note that since a negative value of $\frac{1}{2}mv^2$ has no meaning, we draw the line representing E only out to the point at which it intersects U. At that point, $\frac{1}{2}mv^2 = 0$, and therefore v = 0.

Section 2 - Potential Energy of Two Interacting Bodies

PURPOSE: To generalize the previous section; to deal with more abstract forces in preparation for gravitational potential energy.

CONTENT: (1) With forces which depend only on separation, the kinetic energy which disappears during a collision is stored as potential energy and can be recovered. (2) The potential energy depends only on the separation and not on the details of the collision. (3) Potential energy is stored "in the force field"--the imaginary spring. (4) The money analogy

EMPHASIS: Very important. Treat thoroughly with strong reference back to Section 1. It may be useful to make some reference ahead to Section 3 and 4, since gravitational forces are the only specific examples of an "action at a distance" force that have been considered in detail.

COMMENT: In section 2 we consider potential energy of a pair of bodies acting not by direct contact but rather at a distance, as with a gravitational interaction. The mathematical aspects have been worked out in the previous chapter where the kinetic energy during a collision was followed in detail. In apparent contrast with the spring, there is no visible place where the potential energy is located, and students may be troubled. The fact that the kinetic energy which is missing at one part of the collision is later returned is all that we require for introducing the concept of potential energy. Whether you wish to say "the kinetic energy is destroyed, but later on an equal amount is created" or whether you prefer "the kinetic energy is converted to potential energy, and then reconverted to kinetic energy" is immaterial as far as the physical consequences are concerned. The advantage of the second viewpoint is that it emphasizes that the total work available at the middle of the collision is the same whether we let it appear as kinetic energy, or whether we reach in, grab the particles, and make them separate slowly (in which case the energy appears as work against us). The example of the cage is to emphasize the concept of storage of energy for later use. "Storage" is the key significance of the word "potential" in the term potential energy.

Where is the potential energy located? For the collision of Section 1 we say "in the spring". We could consider the force between the bodies A and B as being an "imaginary spring" as suggested in the text. If one objected that here the spring is only imaginary, we must reply that the imaginary spring is just as real as the forces between A and B, forces which exist with no visible agent of transmittal.

Of course, we do not need to be able to "locate" the potential energy, no matter how natural that desire may be. All that is necessary for the concept of potential energy to be useful is that we be able to use it to predict the work we can realize (or must supply) when the bodies go from one separation to another. In fact, the separation of the bodies gives us as clear an indication of the potential energy of the system A-B as does the extension of the spring in section V. As explained in the text, even the real spring must, on the atomic level, be understood in terms of force acting at a distance--the "imaginary spring."

One may be led into difficulties if one tries too hard to locate the potential energy, perhaps associating it with one or the other of the two bodies. It should be considered a property of the two bodies when together. In fact, the work needed to bring up B while holding A fixed is identical to that of bringing up A while holding B fixed, providing only that we bring them to the same final separation in the two cases. As a preview of Section 3, when we say a body has gravitational potential energy mgh, we really mean the body and the earth have a mutual potential energy mgh, relative to the value at h = 0.

* * *

The several paragraphs (Col 1, pg 6-5) starting with "consider what happens when two large masses hit with a bang" give a preview of the next chapter on heat and kinetic theory and begin to give an atomic picture of of loss of mechanical energy. The statements are quite brief indicating that this is not the main treatment. Such a treatment will come in Ch 7, Section 3. The statements in this chapter are to be thought of as keeping us aware that sometimes mechanical energy is not conserved.

* * *

In connection with application of potential energy to collisions which are not all on a line, emphasize that a force does work only when it has a component in the direction of its displacement. Thus, the work by the force of attraction on one body comes from the component of its displacement in the direction of the force acting on it.

* * *

The money analogy should prove helpful to students. Perhaps the lesson of the analogy for a teacher of physics is that handling energy transfers is as easy or as hard for students as accounting--the principles are simple and seemingly should not cause trouble--all that is needed is a clear head and keeping one's wits. But how many of us cringe at working out our own bank accounts!

Section 3 - Gravitational Potential Energy Near the Surface of the Earth

PURPOSE: To apply the energy concepts to the most familiar force of man's earthly experience, the force of gravity.

CONTENT: (1) When a mass m moves in the gravitational field of the earth, we may ignore the motion of the earth provided the mass is small with respect to that of the earth. (2) The potential energy of position is mgd, where d is the distance of the mass m above some arbitrary level of zero potential energy. (3) The total energy is conserved.

EMPHASIS: This section is very important and should be stressed by means of many examples.

COMMENT: The students should be made aware that the control of their potential energy of position has been one of the first learned experiences--to keep it as potential, and not let it convert to kinetic except on demand, is one of the first great achievements of infancy. Later on, in childhood coasting and sledding down hills, the potential energy is put to good use. In so many ways man is dependent on the conversion of gravitational potential energy to kinetic--the run-off of rain water, the felling of trees, the community water supply and so on. There are many of these every day examples so common-place they are taken for granted and never mentioned-because useful work is obtained by the conversion. Fortunately, the processes go by themselves and do not require the expenditure of work.

* * *

In this section one is apt to lose sight of the fact that the potential energy has meaning because there are two interacting objects, the earth and another mass. At the outset is is proved that, in most of the situations involving the motion of a mass in the field of the earth, the kinetic energy of the earth can be neglected. For this reason one is apt to think of the mass as having the potential energy. Actually it is a mutual affair; the mass has potential energy by virtue of its position with respect to the earth. The potential energy belongs to the system rather than separately to either of the partners of the system.

* * *

The text develops quantitatively the idea that the decrease in potential energy is equal to the work done by the pull of gravity and this in turn is equal to the increase in kinetic energy. Also the negative of this idea: the increase in P.E. is work done against the pull of gravity and this is a decrease in kinetic energy.

The starting point for the development is the kinematic relationship, $v^2 = 2gd$, which has been derived Vol 1, Ch 5. This equation leads to the expression for the potential energy. Incidentally, the formula at the top of the first column on page 6-8 might better be written; $U_h = U_o + mgh$, the first subscript h having been omitted in the text. One can regard the arbitrary constant, U_o, in the general formula as a means for simplifying the potential energy term in any particular problem. One usually chooses $U_o = 0$ to make the potential energy be zero at the origin of the distance scale.

The development of the remainder of the section is a recasting of that of Ch 5, Sec 4 (Kinetic Energy) in terms of the potential energy. It was shown there quite generally that the work done by a force acting on a mass measures the transfer of energy from the outside and equals the increase in kinetic energy of the mass. Here the force is mg and the work done is (mg) (d). But mgd is the change in potential energy when the mass changes its height by an amount d. Therefore, the gain in kinetic energy as a body falls is equal to the decrease in its potential energy. Just as in Ch 4, Sec 4 (The Law of Conservation of Momentum) the equality of the loss of momentum of one body to the gain of momentum for the other body could be restated as the law of conservation of momentum for the system, so here, in precisely the same way, this main result can be rephrased to state that the total energy of the body is conserved--the law of conservation of mechanical energy. The law actually applies to the earth plus mass system. But, as we have seen that we may ignore the kinetic energy of the earth, the total energy is just the sum of the kinetic energy of the body and the potential energy. The mass of the earth enters only implicitly in the potential energy through the quantity "g".

* * *

In using the energy concept we lose information about the details of the motion such as arrival times and directions. But in losing these details we gain in simplification and in our understanding of the fundamental aspects of the motion.

* * *

When the horizontal component of velocity is constant, as in projectile motion, it is a great temptation to think of the energy associated with one component as distinct from another. For example, the total energy is $E = E_k + U_h = \frac{1}{2} mv^2 + mgy = \frac{1}{2} m(v_x^2 + v_y^2) + mgy$. In projectile motion v_x is constant and so we may write $\frac{1}{2} mv_y^2 + mgy = E - \frac{1}{2}mv_x^2 = E^1$ where E^1 is a new constant. The maximum height to which the projectile rises is found by noting that at the top of the flight $v_y = 0$ so that $y_{max} = E^1/mg = (E - \frac{1}{2}mv_x^2) /mg$.

It is correct to speak of the contribution of the horizontal component of the velocity to the kinetic energy but it is incorrect to think of this part of the energy as the "horizontal component of energy". Energy is a scalar and has no associated direction. It is conserved in toto but the parts associated with horizontal or vertical components of the motion are not separately conserved. To see this, consider the example of a particle rolling from rest down an inclined plane. $\frac{1}{2} mv_x^2$ is not a constant.

Section 4 - Gravitational Potential Energy in General

PURPOSE: To acquaint the student with the potential and kinetic energy considerations involved in the motions of planets, satellites and, more generally, for problems in atomic physics in Vol 4.

CONTENT: (1) Gravitational Potential Energy is $\frac{GMm}{r}$ (2) A way of testing the correctness of any expression for the potential energy is given, (it is not pointed out in the text that it is a general method). (3) The escape velocity of a satellite is computed. (4) The concept of binding energy is defined.

EMPHASIS: The general considerations and not specific results should be stressed.

COMMENT: While the specific results are of great interest in such specialized fields as practical rocketry and nuclear reactions (we will review this material in Vol 4), generality of the ideas permits one to regard many distinct types of motion from a common and unified point of view.

The discussion of the previous section is generalized to include distances which are not necessarily small compared to the earth's radius. In passing, we note again that the potential energy is a mutual quantity depending on the positions of the two masses relative to one another. If they fall toward each other from rest the ratio of the kinetic energies is the inverse ratio of the masses.

In the first paragraph the word "field" appears again (See definition Ch 2, Sec 1) and should not be stressed. If, however, it is questioned just say at this time it is a word used to describe the property of a region of space where a mass will experience a force. In this case the gravitational field of force is that region surrounding one of the masses. When another mass is placed there, it will experience a gravitational force. In general, whenever an object experiences a force when it is not in contact with the agent causing the force we say that it is in the field of force of that agent (electric charge, electric current, gravitational mass . . .)

The question asked in the first paragraph (What is the gravitational potential energy of a satellite?) must be answered by anyone who needs to know part of the fuel requirement necessary to put a satellite into an orbit. Another part of the fuel requirement is, of course, to supply the kinetic energy of the satellite in the orbit. Still another large part of the required fuel is needed to accelerate and lift the remaining fuel. The total energy requirement will be considered at the end of this section.

The second and third paragraphs serve to collect the earlier results, one from Chapter 2, that the force between two masses M and m is $\frac{GMm}{r^2}$, and from Chapter 6 that the work done by an interaction force as the distance between the masses changes from r to r' is the area under the force-separation curve from r to r'. An expression for this area is stated without proof in the fourth paragraph. The result is that the gain in kinetic energy as the two masses move closer together, changing the separation distance from r to r', is the same as the decrease in gravitational potential energy and this is $GMm\left(\frac{1}{r'} - \frac{1}{r}\right)$. For those of you who remember the integral calculus the result obtained is as follows. The force is the gravitational force of attraction and since it acts in the direction of the center whereas dr is directed outward, $dw = F \cdot dr = -Fdr$. Therefore

$$W = -\int_r^{r'} \frac{GMm}{r^2}\,dr = GMm\left[\frac{1}{r}\right]_r^{r'} = GMm\left(\frac{1}{r'} - \frac{1}{r}\right)$$

(Note that dr is the symbol for an increment in r. Whether r is actually increasing or decreasing is specified by the relative magnitudes of r and r'.) This result is checked in the fifth paragraph by straightforward algebra. That this method is generally useful is pointed out in Appendix II of this section. One substitutes $\Delta r = r' - r$, so that, $r - r' = -\Delta r$, and, $r' = r + \Delta r$. The last step has not been explicitly spelled out; it is to note that if Δr is small with respect to r then the *sum* $r + \Delta r$, is for practical purposes just r, so that $r(r + \Delta r) = r^2$. We do not, of course, neglect Δr when it occurs as a *factor*, because it is not zero even though it is small compared with r.

The text next takes up the example of the escape velocity of a satellite. There may be some confusion because Figure 15 shows the point P' (that is, the *final* position of the mass m) to the *left* of the point P (the *initial* position

of the mass m) which means that the mass has come closer to the earth. In the example of the escaping satellite we want the final position to be far away not close to the earth. It may disturb the class to have the same formula be used when P' is to the right of P.

If this or a similar question is raised about the troublesome point of the sign convention, you might simply remind the class that the equation for the work done has algebraic significance. It is not simply a relationship between absolute values. It will give a positive value for work done by the gravitational field (accompanied by an increase in kinetic energy). Note that in this section the text refers to work done by the gravitational force. This is not to be confused with the work done by a person who lifts a mass, m through a height, h. He does work, +mgh, which increases the potential energy of the system, earth and mass. Now, when he releases the mass, the gravitational force does work, +mgh, thereby converting the potential energy into the kinetic energy with which the mass strikes the ground.

Remembering that the work that is expressed by the formula

$$W = GMm\left(\frac{1}{r'} - \frac{1}{r}\right)$$

is work done by the gravitational field, you can demonstrate its algebraic significance by substituting values of $r' > r$ and $r' < r$ and comparing the results with your knowledge of the physics of the problem.

If your background in mathematics is such that you may be concerned about the sign of the area under the force vs. distance curve, it should be noted that as Fig. 15 is drawn, a negative area under the curve is associated with a positive quantity of work done by the gravitational field.

The origin of the potential energy scale has implicitly been put at infinity so that U_r approaches zero as r approaches infinity. Now the only way that a quantity can increase numerically toward an ultimate numerical value of zero is to have that quantity be less than zero, that is, be negative. And note that, sure enough, $U_r = -\frac{GMm}{r}$, although again this equation is only implied in the text.

While we are discussing signs, positive and negative, two further comments about the notation may help. One is that r is always positive; it is the separation distance between two bodies; it is never negative. A quantity x such as the displacement of a spring from its position of equilibrium can be both positive and negative. These, of course, are conventious but they are adhered to so strictly that they are taken for granted by physicists and are frequently overlooked. The other comment is to explain the statement that $\frac{1}{2}mv'^2$ must be greater than zero. Frequently, when you are busy trying to understand the physics, a statement like this one can put you at a loss.

The reason is that v'^2 is always positive v' is itself positive or negative. The fact that $\frac{1}{2} Mv'^2$ is never negative is the basis for the development in the text. It is implicit whenever one uses the energy to determine the maximum height to which a projectile will rise or in general to determine the bounds of a motion.

This portion of the text concludes with the numerical value of the escape velocity. Note that at the top of page 6-12 there is a typographical error; the unit is Km/sec, rather than RM/sec. This numerical value will be calculated in Appendix I of this section of the guide.

The concluding paragraph of this section introduces a new concept, the binding energy.

The completely logical derivation is as follows. We return to the original relationship (the equation just under Fig 15)

$$\frac{1}{2} mv'^2 - \frac{1}{2} mv^2 = GMm \left(\frac{1}{r'} - \frac{1}{r}\right)$$

and rearrange it as follows,

$$\frac{1}{2} mv'^2 - GMm \frac{1}{r'} = \frac{1}{2} mv^2 - GMm \frac{1}{r}$$

This equation expresses the conservation of mechanical energy. On each side we have the sum of the kinetic and potential energies at one particular position. If this sum turns out to be a negative quantity the system is said to be "bound". The significance of this situation is that there is not enough energy in the system to carry the interacting members outside their range of interaction. That is to say, the system cannot, without having energy added, be brought to the state of zero potential energy. (If K.E. + P.E. = negative, then for P.E. = 0, K.E. would be negative. This is not a useful solution to the problem.) The energy which must be added to the system in order to bring it to the P.E. = 0 state is called the binding energy. Its magnitude can be determined by solving the equation

$$\epsilon + \text{K.E.} + \text{P.E.} = 0$$

$$\text{Thus } \epsilon = -\text{P.E.} - \text{K.E.} = GMm\frac{1}{r} - \frac{1}{2}mv^2$$

It is this amount of energy that is called the binding energy, and which must be supplied to bound systems (those that can't achieve infinite separation) in order to "blow them apart." Clearly, a system can be bound only if the interaction force between the members is attractive.

Appendix I - Section 4

In this appendix the numbers given in the text are computed and the question of the first paragraph is answered.

1. The escape velocity is derived in the text to be: $\sqrt{\;=\Delta 2gr}$. (In appearance, this is identical to the velocity that would be achieved by an object moving a distance r from rest under a constant acceleration g. This identity is a meaningless coincidence.) We use:

$g = 9.8$ m/sec^2 (acceleration due to gravity)

$r = 6.4 \times 10^6$ m (radius of earth)

we compute

$v = \sqrt{2 \times 9.8} \times 6.4 \times 10^3$ m/sec

or, $v = 11.2$ Km/sec, (escape velocity)

but 1 Km = 0.62 miles,

$v = 11.2 \times 0.62$ miles/sec =

or $v = 6.9$ miles/sec (escape velocity)

To compute the binding energy of the earth to the sum we collect here some of the pertinent constants of our solar system. (See page 3-9, Vol 3)

$G = 6.670 \times 10^{-11}$ nt m^2/kg^2 (Gravitational Constant)

$M = 2.0 \times 10^{30}$ kg (Mass of sun)

$m = 6.0 \times 10^{24}$ kg (Mass of earth)

$R = 1.5 \times 10^{11}$ m

1 yr = 3.2×10^7 sec (Number of seconds in a year)

$V = 2\pi R/1$ year $= 3.0 \times 10^4$ m/sec (Speed of earth in orbit)

Then,

$\frac{mv^2}{2} = 2.7 \times 10^{33}$ joules (kinetic energy of earth relative to the sun)

$-\frac{GMm}{R} = -5.3 \times 10^{33}$ joules (potential energy of earth relative to the sun)

$\epsilon = 2.6 \times 10^{33}$ joules (binding energy of earth to the sun "a little bit more than 2×10^{33} joules")

the figure computed in the text has come from more accurate values for the constants of the solar system.

In the next example, we compute your binding energy to the earth, we shall neglect your motion due to the rotation of the earth. Then

$v = 0$

$r = 6.4 \times 10^6$ m (radius of earth)

m = YOUR mass, say, 50 to 70 kg

$g = 9.8$ m/sec^2 (acceleration due to gravity)

Now since

$$\frac{GMm}{r^2} = mg = \text{YOUR weight}$$

then

$\epsilon = \frac{GMm}{r} = mgr = 3.1$ to 4.4×10^9 joules which is approximately the same as the number in the text for the binding energy of YOU to the earth--the disagreement being the uncertainty in m, which depends markedly on whether you are a mere slip of a girl or a great hulk of a man.

Actually this is not the true value of your binding energy because we have neglected the kinetic energy associated with your motion with the earth's surface. This kinetic energy can be evaluated as follows:

$$E_k = \frac{1}{2} mv^2 = \frac{1}{2} 60 \left(\frac{2\pi r}{T}\right)^2 = \frac{1}{2} 60 \left(\frac{2\pi \times 6.4 \times 10^6}{24 \times 60 \times 60}\right)$$

$$= 5.5 \times 10^6 \text{ joules}$$

This is only a 1/10 percent correction to the binding energy calculated above.

This is a good spot to take up the questions of the satellite raised in the first paragraph of Section 4. Let us consider a 1 ton satellite that has a 2 hour period of revolution around the earth and assume a circular orbit (as has been done in the earth-sun example above). Eliminating v between

$$\frac{mv^2}{r} = \frac{GMm}{r^2}, \qquad \text{(Newton's Second Law)}$$

and

$$T = \frac{2\pi r}{v}, \qquad \text{(Period of Rotation)}$$

(since the earth is now the center of motion, we let M stand for its mass, and m for the mass of the satellite) we have

$$r^3 = \frac{GMT^2}{4\pi^2} \qquad \text{(Kepler's Third Law) (of Ch 3)}$$

For

$T = 2$ hours (Period of Satellite)

$M = 6.0 \times 10^{24}$ kg (Mass of Earth)

$G = 6.670 \times 10^{-11}$ nt m^2/kg^2

then $r = 8.1 \times 10^6$ m (Radius of Orbit of Satellite about Earth)

and putting this value of r into the equation for T we have, solving for v,

$$v = 7.1 \times 10^3 \text{ m/sec} \qquad \text{(Speed of Satellite in its Orbit)}$$

Since the radius of the earth is $6.4\ 10^6$ m this orbit is 1700 km or about 1000 miles above the surface of the earth. Thus we find (taking $m = 10^3$ kg for the mass of the satellite)

$$\frac{mv^2}{2} = 2.5 \times 10^{10} \text{ joules,} \qquad \text{(kinetic energy of satellite in orbit relative to earth)}$$

$$\frac{GMm}{r} = 5.0 \times 10^{10} \qquad \text{(potential energy of satellite in orbit relative to earth)}$$

and

$$\epsilon = 2.5 \times 10^{10} \text{ joules orbit} \qquad \text{(binding energy of satellite in orbit)}$$

It would take this much energy to remove the satellite from the influence of the earth.

Now let us find the energy required to put the satellite into this orbit in two ways, first in an indirect way and then in a more direct way.

We note that the binding energy of the satellite to the earth, when it is at rest before launching, is found in the same way as the binding energy of YOU to the earth. It is (again neglecting the very small initial kinetic energy)

$$\epsilon \text{ ground} = \frac{GMm}{r_{earth}} = 6.3 \times 10^{10} \text{ joules}$$

The difference between this binding energy and the binding energy in orbit must be the total energy required to put the satellite into the orbit from the ground.

$$\text{Energy required} = \epsilon_{ground} - \epsilon_{orbit} = 3.8 \times 10^{10} \text{ joules.}$$

It is $GMm\left(\frac{1}{r_{\text{earth}}} - \frac{1}{r_{\text{orbit}}}\right) = 1.3 \times 10^{10}$ joules. The kinetic energy energy that it must have in this orbit to keep from falling to the earth we have seen before is 2.5×10^{10} joules and therefore the total energy required is $(1.3 + 2.5) \times 10^{10}$ joules, as above.

A rough approximation of the amount of fuel required can be gotten by assuming that all of the energy released when kerosene (the usual fuel) is burned is converted to kinetic energy of motion of the satellite. One pound of kerosene releases 2.1×10^7 joules when it is burned so that to obtain 3.8×10^{10} joules we would need 1.8×10^3 lbs or about 1 ton of fuel. This number, of course, is just the minimum amount of fuel because we have neglected many things, such as the mass of the launching engines which are necessary to harness the energy, the conversion efficiency of the engines and so forth.

The principal quantity that we have neglected in the preceding fuel estimate is the momentum. If we refer to the graph on pages 4-13 and assume as on page 4-14 $v_E \approx 2 \times 10^3$ m/sec for the exhaust gases then V/VE = 7· 1/2 = 3 for this satellite. This ratio infers that $\frac{M_a}{M_o} \approx 0.03$.

Since M_a = 1 ton then $M_o \approx 33$ tons. So one sees that the momentum consideration greatly increases the amount of fuel required compared to the estimate based on energy only.

Appendix II - Section 4

When a mass moves under the action of a linear restoring force $F = - kx$, we know that its potential energy is $U = \frac{1}{2}kx^2$, and since the change of kinetic energy is minus the change of potential energy and is measured by the work done transferring energy from one form to the other, we obtain

$$W = \Delta E_K = \frac{1}{2}k\ (x^2 - x'^2)$$

Now let us suppose that somebody gave us this expression and we wanted to know if it was correct, just as somebody gave us an expression for the change of potential energy or kinetic energy in a gravitational interaction. The procedure we use to test whether our expression for the change of potential energy is correct is to cast it into a form where it is a product of some expression times the small change in distance from x' to x. Since we are dealing with the work, this expression must be the force times the small change in distance, and we can check whether or not the force is what it should be. For the example of the potential energy for a linear restoring force we therefore proceed as follows.

$$W = \frac{1}{2}k\ (x - x')\ (x + x')$$

In this expression we recognize that the change in distance is $\Delta x = x'$ and therefore

$$W = - k\ (\frac{2x + \Delta x)}{2}\ \Delta x$$

For a small change in distance Δx, the Δx inside the parentheses may be neglected compared to $2x$, the distance, and the result is

$$W = - kx\ \Delta x$$

This result agrees with our expectation, for it says that the force must be $F = - kx$.

Suppose, on the other hand, we were given a wrong expression for the potential energy, one which really belongs to a different force. By going through the same procedure, we would then find the force to which it belongs. For example, suppose that somebody asserts that the potential energy associated with a linear restoring force is really $U = kx$. From this we would conclude that the work W changing the kinetic energy by a small amount when the mass goes from x to x′ is

$$W = \Delta E = k\,(x - x')$$

$$= -\,k\Delta x$$

and we see immediately that the force corresponding to this potential energy is just a constant $F = -\,k$.

We should have known that we would get the constant force because our expression $U = kx$ is of the same form as the potential energy in a constant gravitational field.

In other words, any expression for a potential energy or for the work in moving from one place to another implies a definite pattern of force, and the potential energy belongs to that pattern of force and to none other. A possible method of finding the potential energy associated with any force pattern is to investigate all possible expressions that might be chosen to represent potential energy and pick out the one which reproduces the actual force.

Section 5 - Conservation of Mechanical Energy

PURPOSE: To point out briefly that the concepts of potential and kinetic energy can be extended to more complex mechanical systems, including those in which we do not have detailed knowledge of the nature of the interaction forces. Furthermore, we believe that for systems in which other forms of energy play no part the mechanical energy is always conserved.

CONTENT: (1) A simple toy that nevertheless is a more complex system than any previously considered is discussed in terms of the energy. (2) The conservation of mechanical energy is made plausible.

EMPHASIS: The conservation of mechanical energy as a general principle should be stressed. Do try to discuss many examples of systems.

COMMENT: The first paragraph defines Newtonian force as the interaction force between two masses provided: 1. that the masses can be regarded as points, 2. that the force on one mass is equal and opposite to the force on the other, and 3. that the direction of the force is along the line joining the two mass points.

The question is then asked: can one use the notions of kinetic and potential energies in systems in which the mass cannot be regarded as being concentrated at a point and/or the forces of interaction are not Newtonian?

An example of this kind of system (the toy merry-go-round) is then presented in the second paragraph. A plausibility argument is given which shows that the conservation of mechanical energy (except for frictional effects) still seems to be a valid concept.

You can point out to your students that the descending mass of the toy exerts a Newtonian force on the string and that this force can be considered to be transmitted undiminished around the pulley to the surface of the shaft. But the mass of the merry-go-round does not appear to be concentrated at the axis of rotation because the inertial properties of a rotating mass depend markedly on where the mass is located with respect to the axis. The students have probably had some experiences with this effect (whirling ice skater speeding up as he brings his arms in close to the axis of rotation, the heavy wheel in gyroscope and momentum toys) but they have probably not had an opportunity to compare the rotational inertia effects of two wheels of the same mass but with different mass distributions.

In order to compare two wheels that have the same mass you can make the following simple calculation. Consider a bicycle wheel which is idealized as having all its mass at the rim. Replace the merry-go-round with this wheel, and now suppose at a certain instant it is turning N times a second. The mass M of the wheel is distributed uniformly around the rim at a distance R (cm) from the axis of rotation. The speed in the path of a point on the rim is then $2\pi R \times N$ cm/sec because in one second a point will have gone N times around the circumference, that is, $2\pi R$ cm in one turn or $2\pi RN$ cm each second. Now consider the mass of the rim to be made up of 1001 pieces (or any number you want). Then the mass of each piece is M/1001, and each piece is so small that it is practically a point mass so that its kinetic energy is

$$\frac{1}{2}\,(\text{mass})\,(\text{speed})^2 = \frac{1}{2}\,\frac{M}{1001}\,(2\pi RN)^2.$$

Since kinetic energy is a scalar quantity, the total kinetic energy of the wheel is the sum of the energy of each piece. These energies are all equal and there are 1001 pieces so that the total kinetic energy of the wheel is

$$\frac{1}{2}M\,(2\pi RN)^2 \text{ or } \frac{1}{2}MR^2\,(2\pi N)^2.$$

This expression shows that two wheels each of the same mass M and each turning at N revolutions per second can have markedly different kinetic energies according to whether or not the mass is close to or far from the axis of rotation. It should be clear from this example what is meant in the text by "viewing the merry-go-round as a collection of small masses and finding their speeds, we can calculate the kinetic energy of each piece."

One can also see from this example what will be the inertial effect of different mass distributions.

When the mass has descended a given distance, the interaction force (force of gravity) has been allowed to do work on the system [the potential energy of the system has therefore decreased] and the kinetic energy has increased to a given value. This value is the same whenever the decending mass has fallen through the same vertical distance from rest. A given kinetic energy of rotation of the wheel can be realized in two contrasting ways. If the wheel has R large, then N must be small; and if R is small, then N must be large. But N is the rate that the wheel is turning, and so it is a direct measure (via the shaft radius r) of the speed of the falling mass, the speed is $2\pi rN$ cm/sec. This

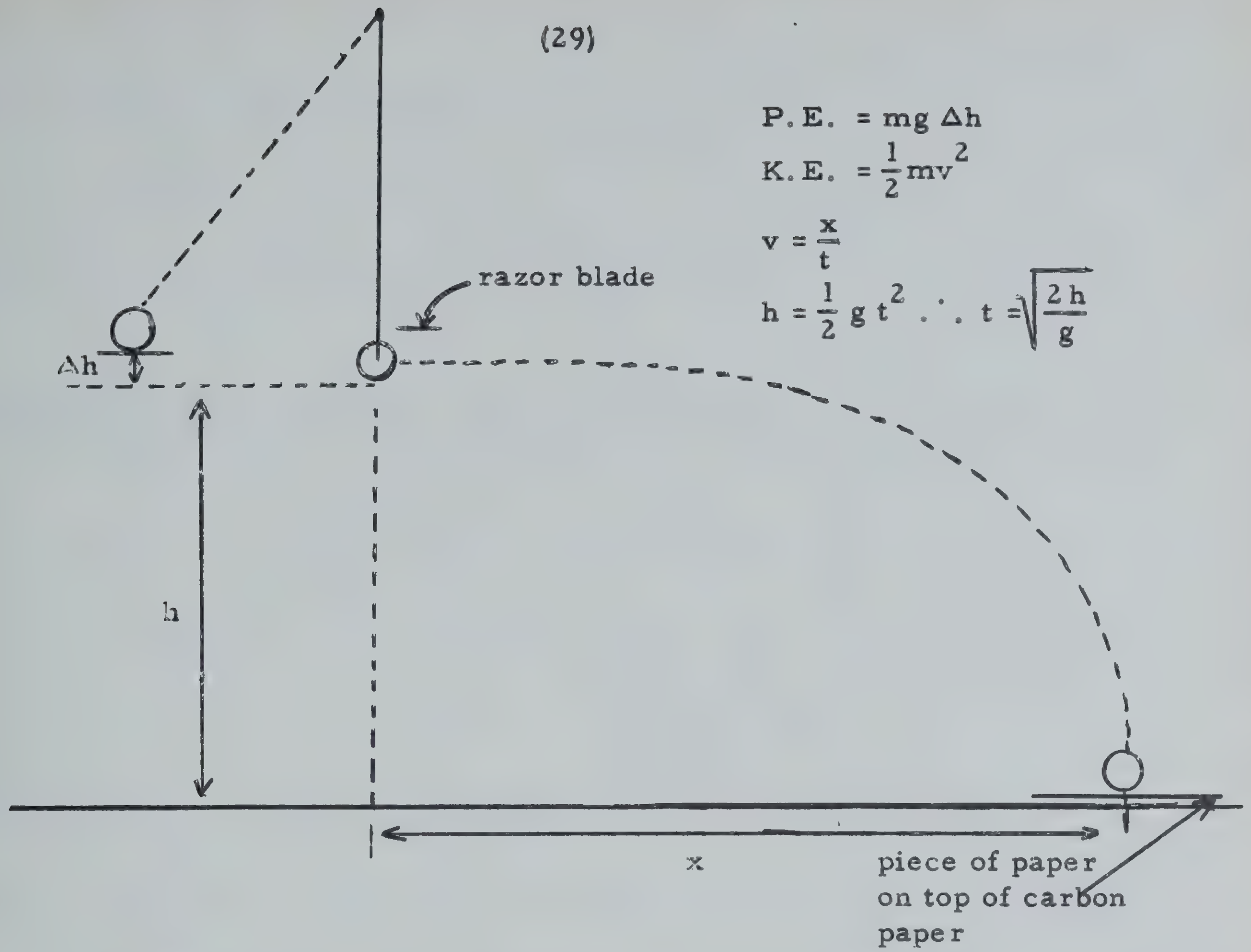
means that after the mass has fallen a given distance. it will be moving faster or slower according to whether R is small or large. Thus a wheel of the same mass if R is large will accelerate slower than if R is small. All of this is in direct accord with the various experiences mentioned above. A wheel with large R is harder to turn than one of the same mass with small R.

Returning now to the text the remaining paragraphs simply state that we believe that mechanical energy is conserved, that is, the sum of kinetic and potential energies for an isolated system will remain constant when frictional effects are negligibly small. All the evidence we have verifies this.

Classroom Demonstration Experiment:

An interesting experiment that you can do as a classroom demonstration shows nicely the conversion of potential energy into kinetic energy and involves some rather nice, simple techniques.

A measurable amount of potential energy is stored in a pendulum bob by swinging it a known distance up from its equilibrium position. The bob is released. At the bottom of its swing the original increment of potential energy will have been completely converted into kinetic energy, $\frac{1}{2}mv^2$. The motion is entirely horizontal. If we could measure this horizontal speed, we could calculate the kinetic energy and check the energy balance. In order to measure this speed, the bob can be hung from a thin thread, and a razor blade can be carefully and rigidly mounted so as to cut the thread just when it reaches the vertical. Then the speed can be calculated by measuring the horizontal distance traveled by the bob before hitting the floor. This distance must be divided by the time of flight which is determined, by calculation, from the known height of the bob above the floor when in the equilibrium position.



$$P.E. = mg\,\Delta h$$

$$K.E. = \frac{1}{2}mv^2$$

$$v = \frac{x}{t}$$

$$h = \frac{1}{2}g\,t^2 \quad \therefore \quad t = \sqrt{\frac{2h}{g}}$$

It is advisable to try this out before doing it in class. The main source of error lies in the energy that is dissipated in the cutting operation. Be sure that your razor blade is rigidly mounted and use as heavy a bob as possible. In class you might make several measurements using different values of Δh.

PSSC Teacher's Guide Chap 6 Vol 3

For Home, Desk and Lab

Note: In all of the following problems we assume that the springs are massless and elastic, unless otherwise noted.

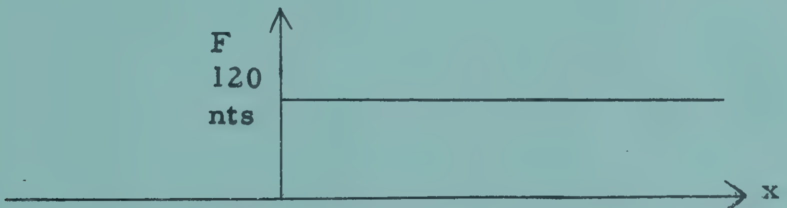
Section	Easy	Medium	Hard	Experiments	Class Discussion
1	2, 6, 12	1, 3*, 5 7, 8*, 10	4, 11		4, 10
2		13*, 18	9, 14		9, 13
3	15, 19, 20	17, 18	16*	560, 570 (To be done after Sec 3 is completed)	15, 16, 19, 20
4	22	23, 24, 26, 28	21, 25		24
5	27				27

Note: Problem 25 is easier if 24 is assigned first. Problem 26 is easier if 23 is assigned first. *Indicates problems particularly recommended.

EXERCISE 1: A mass of 4 kg sliding with a speed of 3 m/sec on a frictionless horizontal table collides with a queer kind of elastic spring bumper. The bumper exerts a constant force of 120 newtons on the mass as it moves in (compressing the spring) and out again until the spring is back where it was.

(a) Is this an elastic collision? How do you know?
(b) What is the kinetic energy at the beginning of the interaction?
(c) How much is the spring compressed?
(d) What is the ratio of kinetic energy to potential energy when the spring has been compressed 10 cm?

This is a moderately easy problem that requires a little thought and an understanding of the energy principles. Plot of force exerted by bumper versus distance of compression.



a. The Collision is elastic, because the force depends only on the distance. This step function is the simplest form of relation between quantities but a form not usually encountered. This case is analagous to a gravitational force where x might be the distance above the floor and F the gravitational force acting on a mass. Usually, when one thinks of one thing depending on another thing, one thinks of it changing as the other changes, but the simplest relation is when one changes and the other does not. One would be apt to say "well the force doesn't depend on the distance because it doesn't change with the distance." But then one would be falling into a semantic trap, a trap of words. This kind of force is seen in gravitational attraction, it is constant with distance (for ordinary distances.)

b. The kinetic energy at the beginning of the interaction is $\frac{1}{2}mv^2$ initial $= \frac{1}{2} \times 4 \times 9$ kgm^2/sec^2 = 18 joules.

c. The "spring" will continue to be compressed until there is no more energy of motion-that is, no more kinetic energy. The kinetic energy is reduced to zero when the speed is zero. This energy has been transferred to the spring as potential energy. The work done in compressing this spring is the constant force, F, multiplied by the distance X. This work is the potential energy, so at the instant that the mass stops moving the initial kinetic energy in b) is equal to the final potential energy (120 nts) (X meters)

$$120 \text{ x nt m} = 18 \text{ joules}$$

$$X = \frac{18}{120}\text{ m} = \frac{3}{20}\text{ m} = \underline{0.15\text{m or } 15 \text{ cm}}.$$

d. In (c) we used conservation of energy in evaluating the complete change from one form to another. In this question we may use conservation of energy to solve a problem which can also be done using kinematics. We say that when the spring has been compressed 10 cm, the potential energy has increased from zero to 120 x 0.10 joules = (12 joules). This increase has been at the expense of kinetic energy of motion (the mass has slowed down) and the mass has lost 12 joules of kinetic energy. It still has 18 - 12 or 6 joules of kinetic energy. The ratio, therefore, [getting back to the question] of the kinetic to the potential energy is 6 to 12 - that is, $\frac{6}{12}$ or there is half as much kinetic energy as potential energy at this instant in the interaction. Let us compare (and you might do this for your students) the above calculation with a more elementary one using kinematics only and pretending that we do not know about conservation of energy.

1. As before, the work done in compressing spring 10 cm is

$$\begin{aligned} FX &= 120 \cdot 0.1 \text{ nt m} \\ &= 12 \text{ joules} \\ &= \text{potential energy at the instant} \end{aligned}$$

2. The force F acts on the 4 kg mass for a distance of 0.1 m. The speed was initially 3 m/sec. What is the speed at the instant when the spring is compressed 0.1 m? Since we know the force, we can compute the acceleration (really the deceleration) from Newton's Second Law, it

is a = Force/Mass = $\frac{120}{4}$ = 30m/sec^2. The connection we seek relates speeds with acceleration and distance; it is

$$v^2 = v \text{ initial}^2 + 2as$$

$$v^2 = 9 - 2.30 \cdot 0.1 = (9-6)\ m^2/sec^2$$

$$\text{or} \quad v^2 = 3\ m^2/sec^2$$

Thus the kinetic energy at the instant when the speed has the value corresponding to this value for v^2 is $\frac{1}{2}mv^2$ or $\frac{1}{2} \cdot 4 \cdot 3$ kg m/sec^2 = 6 joules.

3. The ratio of kinetic energy to potential energy is, as before, 6/12 or $\frac{1}{2}$. You can emphasize that the energy concept at this stage in the development seems little more than a mathematical manipulation i.e. we have really used $v^2 = v^2$ initial + 2as but in a form multiplied by mass/2. $\frac{1}{2}mv^2 = \frac{1}{2}mv^2$initial - mas = $\frac{1}{2}$mv initial^2 - FX. The advantage is that the quantities are all of the same type in the energy; we do not need to bother about the details of the process, i.e. calculate acceleration, etc.

EXERCISE 2: A 3-kg mass moving with a speed of 2 m/sec collides with a spring bumper which exerts a force F = 100x, where F is the force in newtons and x is the compression in meters.

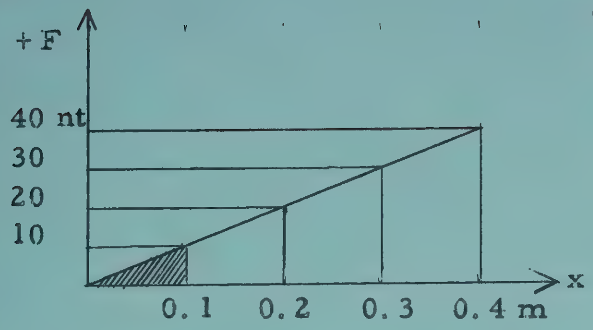
(a) Draw a graph of F against x from x = 0 to x = 0.4 m.

(b) What is the potential energy stored in the spring when x = 0.1 m? What is the kinetic energy of the mass at this point?

(c) What will happen if the spring is compressed 0.1 m by hand and then the 3-kg mass is placed in contact with the spring and the hand is removed?

This problem is moderately easy and is similar to problem 1. The principle of conservation of energy is used to obtain the kinetic and potential energies at an intermediate stage of the interaction.

a.



(+F = + force exerted by bumper
= - force exerted by external agent)

b. The work done by an external agent in compressing the spring a distance of 0.1 m is the shaded area under the force-compression curve shown in part a). If we choose the zero of potential energy to be the uncompressed spring, then the potential energy of the compressed spring is the same shaded area shown above. The initial kinetic energy is $\frac{1}{2}(3)(4)$ or 6 joules. The increase in potential energy of the spring occurs at the expense of the kinetic energy of the mass. The kinetic energy must therefore decrease by the same amount that the potential energy has increased at the point x = 0.1 m. The shaded area is $\frac{1}{2}(0.1)(10)$ or 0.5 joules. The potential energy of the spring has increased from zero to 0.5 joules. Therefore the kinetic energy has decreased from 6 to 5.5 joules, because energy is conserved.

c. In compressing the spring the hand does work on the spring $= \frac{1}{2}(0.1)(10) = 0.5$ joules and the potential energy of the spring increases by this amount, that is, from zero to 0.5 joules. This energy will become available to accelerate the mass when the hand is removed but we need not be alarmed at trying to solve the motion problem with this non-constant acceleration of the spring. We can say very simply, that when the spring has returned to its normal position, the interaction is finished, and the transfer of potential energy to kinetic energy is complete. Therefore, the kinetic energy $= \frac{1}{2}mv^2 = 0.5$ joules. Then,

$$v^2 = \frac{1}{3}\ m^2/sec^2$$

and $$v = \frac{\sqrt{3}}{3}\ m/sec$$

or $$v = 0.577\ m/sec$$

EXERCISE 3: The force-compression curve of a spring is shown in Fig. 17.

(a) How much work is done in compressing the spring 0.3 m?

(b) What is the potential energy of the spring when compressed this amount?

(c) Place a 2-kg mass at rest against the spring when it is compressed 0.3 m. Let go. What is the kinetic energy of the mass as it passes the point where the spring is compressed 0.2 m?

This problem is conceptually the same as problem 2. It is less easy than problem 2 because there is a more difficult calculation to perform.

(a) The work to compress the spring is the area under the force-separation curve. In this case it is the sum of the area of the triangle and the area of the trapezoid, these figures being the two parts of the area if it is cut at 0.2 m.

$$\text{Area of triangle} = \frac{1}{2}(0.2)(2) = 0.2 \text{ nt-m}$$

$$\text{Area of trapezoid} = \frac{1}{2}(.1)(2+4) = 0.3 \text{ nt-m}$$

$$\text{Sum} = 0.5 \text{ nt-m} = \text{work of compression}$$

(or in joules = 0.5 joules).

(b) Potential Energy of the spring = work done by the applied force = 0.5 joules (from above).

(c) The gain in kinetic energy is equal to the final minus the initial kinetic energy or $(1/2)mv^2 - 0$. The gain in kinetic energy is the loss in potential energy and this is the area under the force separation curve from $x = 0.3$ to $x = 0.2$ m. This area is just the area of the trapezoid, and this is $\frac{1}{2}(2 + 4)(0.1) = 0.3$ joules.

Therefore

$$\frac{1}{2}mv^2 = 0.3 \text{ joules} = \text{kinetic energy}$$

EXERCISE 4: Pretend you are given two springs of different sizes. How would you set them up so that the force-compression curve of the system looks like Fig. 17?

This problem is a challenge to the student with a flair for inventing mechanical devices. It is possible that it might even encourage an attempt to figure out how two coupled springs behave.

We show two such devices that would give the desired behavior of Figure 17. The first of these uses springs in parallel; the second uses springs in series.

One might in an off-hand way believe that two springs, of force constants k_1 and k_2, when connected in series ( ) would give the desired behavior. This is not the case as the following analysis will show.

Let us solve the problem of determining the behavior of a complex (series) spring whose two parts have force constants, k_1 and k_2.

We write

$F_1 = k_1x_1$ for the first spring

$F_2 = k_2x_2$ for the second spring

where x_1 is amount that first spring has been compressed by the force F_1, similarly for x_2 and F_2. When the springs are considered in combination then

$x = x_1 + x_2$ (the overall compression)

So far we have only invoked the elastic property of each spring separately.

Now we must invoke Newton's third law (Section 8, Chapter 4, Vol 3) by noting that if we push on one end of a spring with a force, F, then the spring pushes back on us with a force, -F, and the other end of the spring exerts the force, F, on the wall or whatever object it is interacting with. This means that in this example $F_1 = F_2 = F$.

We substitute the first two equations into the equation for x

$$x = \frac{F_1}{k_1} + \frac{F_2}{k_2} = \frac{F}{k_1} + \frac{F}{k_2} = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

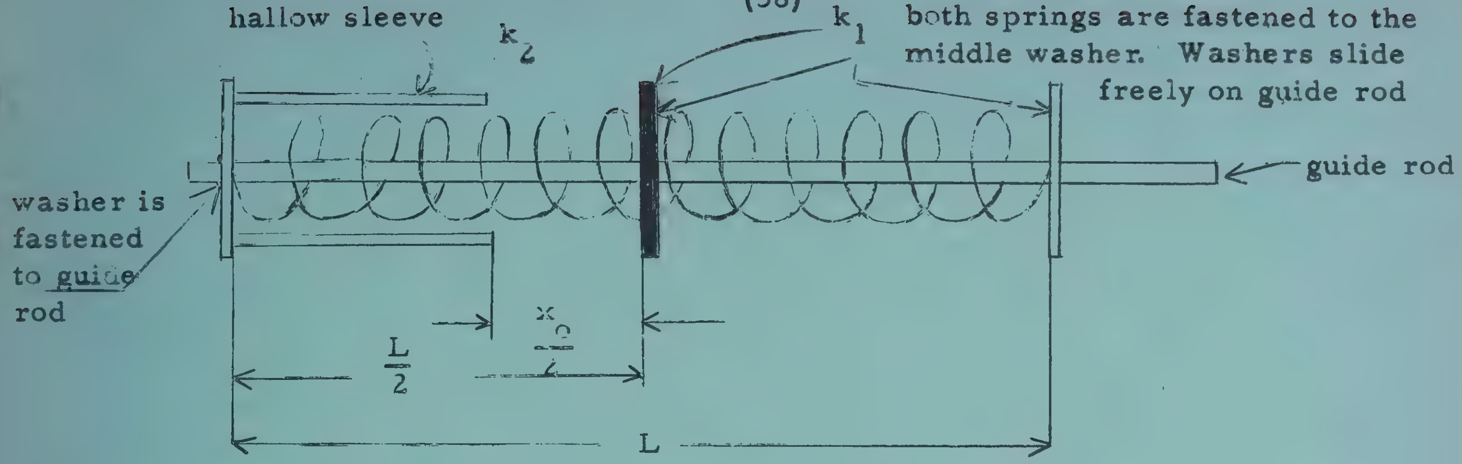
If we equate x to the last expression and solve for F, we have

$$F = \left(\frac{k_1 k_2}{k_1 + k_2}\right) x.$$

This shows that the series combination behaves as though it were a single simple spring with a force constant

$$k = \frac{k_1 k_2}{k_1 + k_2} \; .$$

This analysis shows that one cannot achieve the desired behavior only by hooking springs together. In addition there must be some sort of mechanical device that "turns on (or off)" at the desired compression, x_o, some extra spring of a different spring constant. The first figure shows one of many possible arrangements that would give the desired behavior. It is designed for compression only and has a simple way of "shorting out" or removing one of the component springs.


hallow sleeve
k_2
k_1
both springs are fastened to the middle washer. Washers slide freely on guide rod
guide rod
washer is fastened to guide rod
x_o
$\frac{L}{2}$
$\frac{x_o}{2}$
L

In this example the stiff spring constant, k_2, is 20 nts/m from the graph of the steep portion of Fig 17. We put the effective spring constant, k, equal to 10 nts/m from the graph of the first slope. We solve for k_1 $\left(\frac{1}{k_1} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}\right)$, and find k_1 is 20 nts/m. These values are, of course, for the first mentioned design in which the springs act together until the desired compression (x_o = 0.2 m) is reached. After that one spring is rendered inoperative. You notice that the two sections of the spring have identical spring constants. Thus one could achieve the desired behavior with a single spring and simply arrange to block out one half the spring at the desired compression.

When the springs are connected in parallel as shown in the next figure, the force initially arises from the compression of the longer spring. When the second spring begins to act then the force is the sum of the forces exerted by each spring. The first continues to exert a force k_1x; the second exerts a force $k_2(x-x_o)$. The total force can be expressed as follows,

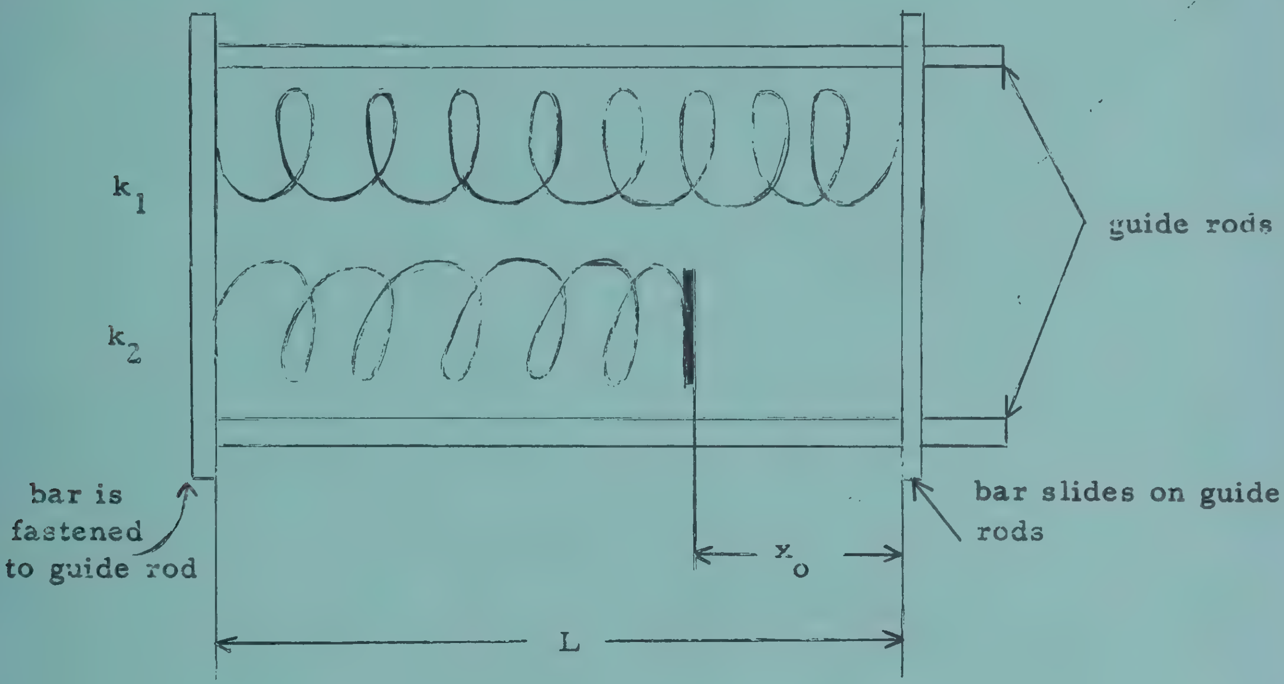
$$F = k_1x + k_2(x-x_o) = (k_1 + k_2)(x - x_o) + k_1x_o$$

and this shows that the force constant k (slope of the line F versus x) for

springs acting together in parallel is

$$k = k_1 + k_2$$

In this exercise then, k_1 = 10 nts/m as only one spring acts for the first part of the travel. For the steep portion when both springs act, k = 20 nts/m, which shows that k_2 = 10 nts/m.



In case some student gets intrigued with springs as a result of this problem, the general considerations for a single spring are as follows: The spring constant, k_ℓ, is inversely proportional to the length, ℓ, of the spring. A short spring is stiffer than a long spring of the same material and the same diameter and turns/unit length. This assertion can be seen easily as follows. Consider a spring of length L stretched by an amount x so that its total length is now L + x. The stretch in any

part of the spring is proportional to the length ℓ of that part; in particular the mid-point of the spring has only been stretched $x/2$ beyond its unstretched length $L/2$. The force of extension is transmitted undiminished through every part of the spring, thus the force constant, k_ℓ, of a part of length ℓ is $k_\ell = F/x_\ell$ but $x_\ell = x\ell/L$ so that $k_\ell = (F/x)\,(L/\ell) = k\,\frac{L}{\ell}$ which is the assertion made above, that the spring constant, k_ℓ, is inversely proportional to the length, ℓ.

EXERCISE 5: A 1.0-kg mass moving with a speed of 10 m/sec strikes a spring, compressing it a distance of 0.2 m. The mass rebounds with a speed of 7 m/sec.

(a) What is the loss in kinetic energy of the mass?

(b) What happens to this lost energy?

(c) Which of the force-compression curves in Fig. 18 do you think is most likely correct for this spring?

This is a good problem to test the student's grasp of the fundamental idea of potential energy. The part of the problem requiring thought does not require any calculations.

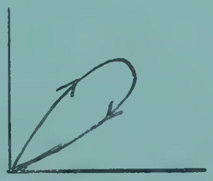
The initial kinetic energy is $\frac{1}{2}mv^2_{\text{initial}} = \frac{1}{2}(1)\,10^2\ \text{kgm}^2/\text{sec}^2 = 50$ joules. The final kinetic energy is $\frac{1}{2}mv^2_{\text{final}} = \frac{1}{2}\cdot 1\cdot 7^2 = 24.5$ joules.

a. The energy loss is $50 - 24.5 = 25.5$ joules.

b. It has gone as the work to deform the spring permanently. Alternatively, in a real spring whose mass is not negligible it is quite possible that as the mass leaves, the spring will vibrate. Since this energy of oscillation of necessity comes from the initial kinetic energy, the mass will leave with less energy than it had initially.

c. Of the three graphs in Fig. 18 we can immediately exclude that in the right part of the page, because even though the path is not straight nevertheless the ingoing and outgoing paths are the same which means that no energy is lost.

The correct graph must be the first because of the fact that the compressive path (ingoing) lies above the extensive (or outgoing) path, thus less energy is given up by the spring than was put into it.



Note that the direction around the loop in the middle or bottom graph is just the opposite to the correct graph. The implication of such a path is that somehow the ingoing compression triggered a mechanism that released energy so that more energy was available after the interaction than before, contrary to the hypothesis of the problem. Note that the above argument makes no mention of the numerical values attached to the graphs; it is only the direction around the loop that was used in deciding which was the correct graph.

Again, as in part b, the bottom graph would be possible, if the spring were vibrating before the mass struck it, and the timing was just right so that the energy was absorbed from the spring.

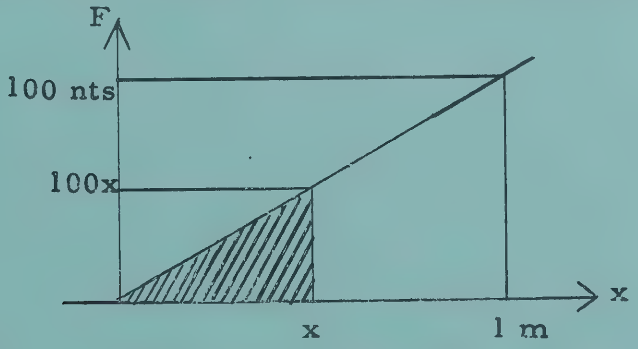
EXERCISE 6: A linear elastic spring is compressed 0.2 m by a force of 20 newtons.

(a) What is the force constant r (or force-compression ratio) of the spring?

(b) What is the equation for the potential energy stored by the spring as a function of its compression?

We believe that the force constant was meant to be k instead of r. This is a simple problem. Refer to figure 5. We are told the spring is linear and elastic. Therefore, $F = kx$ is the equation relating the force to the amount of compression. Rewriting we have $k = F/x$, in particular $k = \frac{20\text{ nt}}{0.2\text{ m}} = 100\text{ nts/m}$

If we plot this relation we see from $F = 100x$ that the area of the triangle of base x is $\frac{1}{2} \cdot x \cdot 100x = 50x^2$ joules and this is the potential energy. That is, the potential energy is $50x^2$ joules, where x is the compression measured in meters.

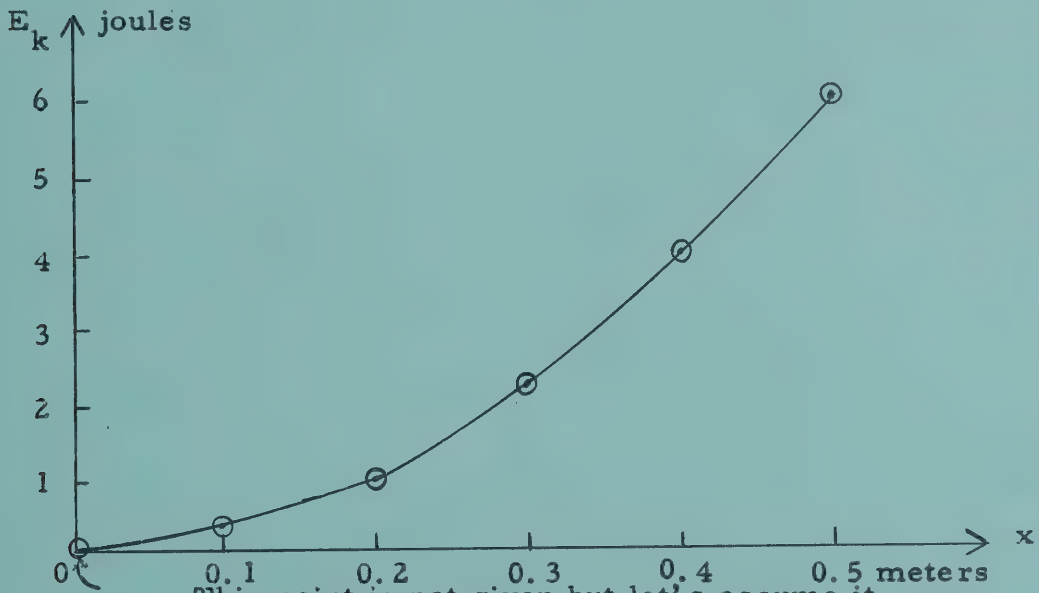


EXERCISE 7: The following table gives the kinetic energy E_k of a 2.0-kg mass when it leaves a spring compressed different distances x.

E_k Joules	x Meters
0.25	0.1
1.00	0.2
2.25	0.3
4.00	0.4
6.00	0.5

(a) Plot the potential-energy curve of the spring.
(b) Does this spring exert a linear restoring force?

This is a straightforward problem.



This point is not given but let's assume it.

Here we plot E_k, the final kinetic energy of a mass as it leaves the spring which was initially compressed different distances x, as a function of x. If we choose the zero of potential energy to be the undisturbed spring, then the initial potential energy of the spring is equal to E_k, the final kinetic energy of the mass. Therefore, the above plot is also the plot of the potential energy that was asked for.

b. As we have just seen in problem 6 (preceding) the potential energy of a spring varies as the square of the compression, x. Since the above curve resembles a squared (parabolic) relationship let us note that if $F = kx$, the potential energy is $\frac{1}{2}kx^2$. Let us compute $\frac{2E_k}{x^2}$. We expect that if the spring is exerting a linear restoring force this quantity will be constant.

x	x^2	E_k	$2E_k/x^2$
0.1	0.01	0.25	50
0.2	0.04	1.00	50
0.3	0.09	2.25	50
0.4	0.16	4.00	50
0.5	0.25	6.00	48

And we see that the spring is linear for the greater part of its range but the last point deviates slightly. This frequently indicates that the material of the spring has been permanently deformed, that is, the shape has been changed.

EXERCISE 8: Assume an archer's bow acts like a linear spring.

(a) How does the initial velocity of an arrow shot with the bow string pulled back 10 cm compare with the initial velocity of the arrow when the string is pulled back 20 cm?

(b) How many times higher can the archer shoot in the second case?

This is a straightforward problem.

a. If the bow behaves like a linear spring, then the potential energy varies as the square of the extension. Thus the potential energy for a 20 cm pull is four times larger than the potential energy for a 10 cm pull. The kinetic energy varies as the square of the speed and by conservation of energy the potential energy of the bow is converted into (equals) kinetic energy. If the zero of potential energy of the bow is the unstretched state, and if x_o is the initial pull of the bow, and if the arrow is initially at rest, and if v is the final speed of the arrow as it leaves the bow, then we have x_o^2 varies as the loss of potential energy which equals the gain of kinetic energy which varies as v^2. If v^2 varies as x_o^2, then the final speed v is proportional

to the initial pull x_o. The speed for a 20 cm pull is twice the speed for a 10 cm pull.

b. The potential energy in the earth's gravitational field for ordinary positions varies directly with the vertical displacement, h. At the top of its path the arrow has lost all of its kinetic energy but it has gained the same amount of potential energy. Applying conservation of energy to the vertical motion, the potential energy at the maximum height h is equal to the kinetic energy at $h = 0$. Thus h varies as v^2, where v is the velocity at $h = 0$. But v^2 is proportional to x_o^2, so that h varies as x_o^2 Doubling the distance of pull sends the arrow to 4 times the height.

EXERCISE 9: Fig. 19 is a picture of a roller skate with a wig-wag on it. The mass M, mounted on top of the hack-saw blades, is equal to the mass of the roller skate plus the base of the wig-wag. If you hold the roller skate and pull the mass M out to the left, and then let the whole system go, the mass moves to the right and the skate to the left. Describe the further motion, indicating the form of the energy at various stages.

This is a difficult problem involving a thorough understanding of most of the elements of mechanics. It is completely qualitative, however.

The system of the mass and the roller skate is completely analogous to that of two masses connected by a spring and sliding without friction on a horizontal plane. If the two masses are released from rest they move in opposite directions and oscillate in such a way that the center of mass remains fixed.

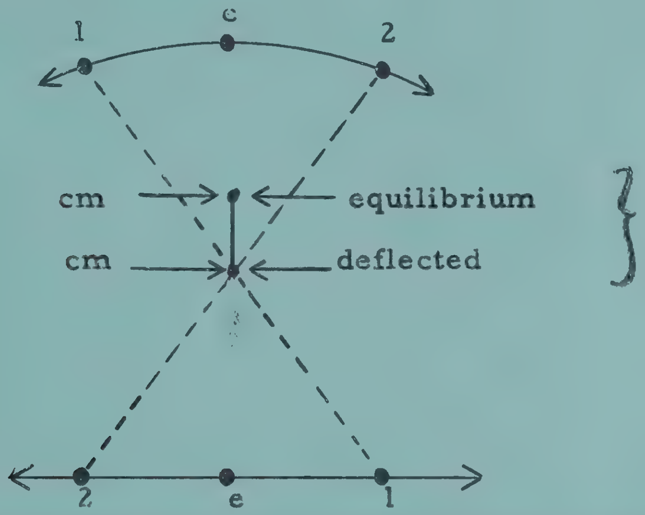
The roller skate is free to roll and this implies that we should think of the skate, blade, and mass as an isolated system, at least in the direction of possible motion. We have learned that an isolated system is one on which no external forces act. In this case there are no external forces acting on the skate in the horizontal direction. The force that the floor exerts on the skate is vertical, and therefore we ignore it in this problem.

Now we have also seen that the center of mass represents a convenient way to describe the position of the body, and, moreover, for many problems the motion of the center of mass is all that we need to consider. We also learned that the external forces, if any, can be thought of as acting on the center of mass. In this problem the center of mass will move with no horizontal acceleration because there are no horizontal external forces acting once the system has been released. The implication of the wording of the problem is that the skate is released at rest. This means that the center of mass speed is zero. Furthermore, it will remain zero because there are no external (horizontal) forces acting on the skate.

To describe the motion, one notes that the hacksaw blades always tend to return to the normal unbent position and that the restoring force is zero when the blades are straight. This means that when the mass M is released it will move toward the equilibrium position but that when it reaches the equilibrium position there is no force exerted on the mass, and, since it is moving, it will continue to move (by Newton's 1st Law) until the force of the bending blade brings the speed to zero. At this instant the mass is in the symmetrically displaced position, at rest, and is acted on by a force tending to restore the equilibrium configuration. So the mass accelerates (now in opposite direction) (the acceleration is not constant) and moves closer to the equilibrium position with increasing speed until at the equilibrium position it has maximum speed and is not further accelerated by the force. As above, it moves through the equilibrium position and starts to decelerate under the ever increasing force urging it back toward the equilibrium position. Under the action of this force the mass continues to slow down and it finally comes to rest at exactly (if there were no friction) the same position that it started. This is one cycle of the motion. Thus the motion of the wig-wag may be summarized by this figure where the dots represent the center of mass of the block, the skate and the combination of the two. The skate can only execute horizontal motion. The center of mass of the two can only move up and down (since no horizontal external force act) and the block swings back and forth in an arc going to the left when the skate is to the right (1-1) and then to position (2-2) and back again.

When the system is passing through the equilibrium position (e-e), the potential energy has disappeared and the kinetic energy is a maximum. When the system is in the starting position 1-1 or 2-2 it is instantaneously at rest and therefore the kinetic energy is a minimum (zero, in fact) and the energy has been stored as potential energy in the bent or deflected hacksaw blade.

This periodic, unending, transformation of energy back and forth from the kinetic to potential form is a characteristic of all oscillatory motion, whether or not the system is mechanical as in this case or in the case of pendulum or a swing, or electrical as in the case of various parts of radio transmitters and receivers.



This motion is exaggerated

EXERCISE 10: What will happen if the 3-kg mass in Problem 2 hooks itself onto the spring and remains fastened to the spring?

This problem is of moderate difficulty, because the question "what will happen" implies that in addition to a qualitative description of the oscillatory motion some numerical values of the motion should be calculated.

It is reasonable to assume that the bumper can be stretched as well as compressed. If the mass remains fastened to the spring it will oscillate to and fro. It executes simple harmonic motion as in Chapter 2, page 10.

After the mass has compressed the spring and come to rest under the action of the restoring force of the spring, the kinetic energy is zero and the energy has been converted to potential energy stored in the spring. The restoring force continues to urge the mass toward the equilibrium position. Now the potential energy is converted into kinetic energy until the potential energy is zero and the kinetic energy is a maximum. At this instant the mass is in the equilibrium position, and because no force acts on the mass, the state of motion remains unchanged by Newton's First Law. This means that the mass continues to move through the equilibrium position and away from the spring bumper, but since we are told that the two remain connected, we know that the mass now stretches the bumper, and, being displaced from its equilibrium position, the bumper exerts a force on the mass directed toward the equilibrium position which it just passed through. This force ultimately brings the mass to rest and starts it moving back toward the equilibrium position. At the instant that the mass stopped, the kinetic energy had all been transformed into potential energy of the spring. The transformation now proceeds in reverse.

This back and forth motion is called an oscillation, and in principle if there were no friction force no energy would be lost (dissipated) as heat and the oscillation would continue forever.

If we refer to problem 2 we see that the initial kinetic energy of the mass at the instant that it strikes the spring is 6 joules. The work required to stop the mass is 6 joules. The work required to compress the spring a distance x from equilibrium is $\frac{1}{2}kx^2$ where k is 100 nts/m. If we set $\frac{1}{2}kx_o^2 = 6$ the value of x_o so obtained is the distance that the mass has compressed the spring when it has come to rest. We obtain $x_o = \sqrt{\frac{12}{100}} = .2\sqrt{3}$ or $x_o = 0.346$ meters.

The mass will extend the spring the same distance from equilibrium.

The period, the time required for the mass to execute one complete oscillation, is found by using the formula of Chapter 2, page 12.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3}{100}} = \frac{2\pi\ 1.732}{10} = 1.09 \text{ seconds.}$$

EXERCISE 11: A 0.50-kg mass free to slide on a table is attached to a linear spring with a force-extension ratio k of 50 newton/m. The mass is pulled back and released so that it vibrates with an amplitude of 0.10 m.

(a) What is its maximum velocity?

(b) What is its velocity when it is 0.06 m away from its equilibrium position?

This problem is to make quantitative the ideas expressed above in problems 9 and 10. It is of moderate difficulty.

The potential energy of a spring compressed a distance x is $\frac{1}{2}kx^2$, where k is the force to extension ratio. When the mass stops and reverses its motion the kinetic energy is zero and the potential energy is a maximum. At this instant the mass has reached its maximum displacement, x_o, from the equilibrium position. At any other position, since the total energy, E, is a constant, we may write

$$\text{kinetic e. and potential e.} = \frac{1}{2}Mv^2 + \frac{1}{2}kx^2 = E$$

When $x = +x_o$ or when $x = -x_o$, $v = 0$ that is, the mass is at rest for an instant before it reverses its direction. Therefore E is given by

$$\frac{1}{2}M \cdot 0^2 + \frac{1}{2}x_o^2 = E$$

Therefore, at any point of the motion we have generally,

$$\frac{1}{2}Mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_o^2$$

This enables us to calculate the speed, v, at any position, x. In particular the maximum speed is when x = 0. So that for a) we have

$$\frac{1}{2}Mv_o^2 + o = \frac{1}{2}kx_o^2$$

Solving, $v_o = x_o\sqrt{\frac{k}{m}}$

Substituting $x_o = 0.10$ m

$k = 50$ nt/m

$M = 0.50$ kg

$$v_o = 0.10\sqrt{\frac{50}{.5}} = 0.10\sqrt{100} = \underline{1 \text{ m/sec}}$$

Since the motion is in a straight line, the maximum velocity is a vector of this magnitude pointing in either direction along the line.

Solving the general equation for v^2

b)

$$v^2 = \frac{kx_o^2 - kx^2}{M}$$

$$v = \sqrt{k(x_o^2 - x^2)/M}$$

and substituting the same numbers as above and putting x = 0.06 m

$$v = \sqrt{k(x_o^2 - x^2)/M} = \sqrt{k(x_o - x)\ (x_o + x)/M} = \sqrt{50(.1 - .06)(.1 + .06)/0.5}$$

$$= \sqrt{50\ (0.04)(0.16)/0.5} = (10)\ (0.2)\ (0.4) = \underline{0.8 \text{ m/sec}}$$

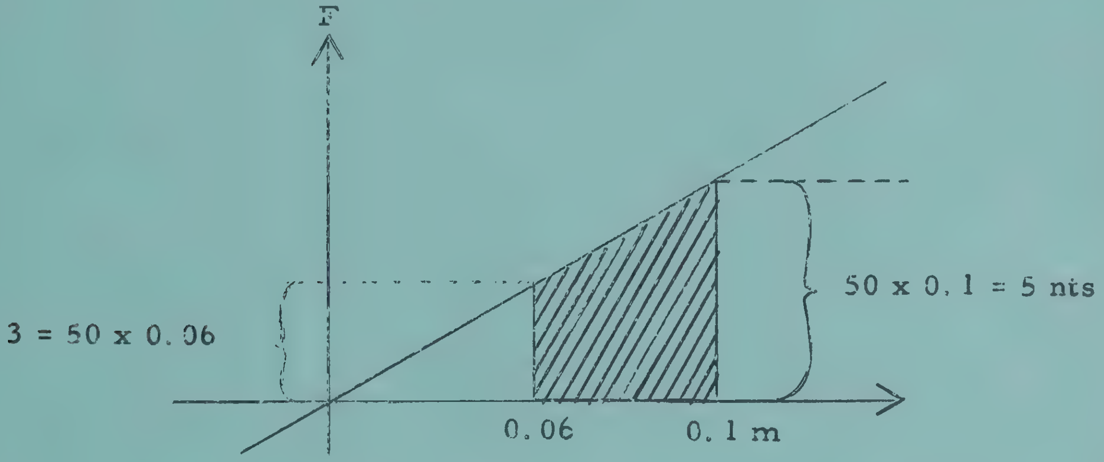
Let us do part b) of the problem again in terms of Figure 5. Since the amplitude of the motion is given to us as 0.1 m, this means that the mass (0.5 kg) comes to rest when $x = \pm 0.1$ m. We know that the loss of potential energy is equal to the gain in kinetic energy and in this particular case, since the mass starts at rest the gain in kinetic energy is the kinetic energy. Thus we have

$$\text{Loss in p. e.} = \text{area of shaded portion of graph} = \frac{1}{2}(5+3)(0.1-0.06)\text{joules}$$

$$= \text{gain in k. e.} = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2}(0.5)\,v^2$$

$$\therefore v^2 = \frac{8}{0.5}(0.04) = \frac{16 \times 4}{100} \quad \therefore v = 0.8 \text{ m/sec, as before.}$$



EXERCISE 12: A spring bumper with restoring force F = 200x where F is in newtons and x in meters is compressed 0.10 meters. A 0.5-kg mass is placed next to the end of the spring and the whole thing let go.

(a) With what momentum will the mass leave the spring?

(b) We do the same thing with masses 0.125 kg, 2.00 kg, and 8.00 kg. What is the momentum of each as it leaves the spring?

(c) What is the energy of each as it leaves the spring?

This problem is straightforward. The student must compute the momentum given the kinetic energy.

We let v_o be the velocity of the mass as it leaves the spring. Then $E_{k_o} = \frac{1}{2}mv_o^2$ is the kinetic energy as it leaves the spring.

Then $2mE_{k_o} = m^2v_o^2$, or $mv_o = \sqrt{2mE_{k_o}}$ is the corresponding momentum.

Now since the kinetic energy gained is equal to the potential energy lost,

$$E_{k_o} = \frac{1}{2}kx^2 .$$

We are given k = 200 nt/m and x = 0.1 m

so that

$$E_{k_o} = \frac{1}{2} 200 \times 0.01 = 1 \text{ joule}$$

Therefore

(a) $mv = \sqrt{2 \times \frac{1}{2} \times 1} = 1$ kg m/sec $(\frac{1}{2}$ kg)

(b) $mv = \sqrt{2 \times \frac{1}{8} \times 1} = \frac{1}{2}$ kg m/sec $(\frac{1}{8}$ kg)

$mv = \sqrt{2 \times 2 \times 1} = 2$ kg m/sec (2 kg)

$mv = \sqrt{2 \times 8 \times 1} = 4$ kg m/sec (8 kg)

(c) $E_{k_o} = 1$ joule for all of the above masses since the original potential energy was the same in each case.

EXERCISE 13: Fig. 20 is the graph of the displacements as a function of time of two interacting masses $m_1 = 3$ kg and $m_2 = 1$ kg.

(a) What is the initial kinetic energy of m_1?

(b) What is the final kinetic energy of m_1?

(c) What is the final kinetic energy of m_2?

(d) What is the minimum total kinetic energy?

(e) What is the maximum potential energy?

This is a time consuming problem of moderate difficulty. It is a good review problem for the student because it recalls material of previous chapters.

This problem concerns the one-dimensional collision (''encounter'' is the better word) between two masses. The figure appears to be the same as those of Chapter 5 (Figs 8 and 9) and Chapter 6 (Fig 7) except for the time scale. We believe that, the intent of the author of this problem was to have the encounter be elastic. It will be evident that if this was the intent then there is a drafting error because the lower curve in Figure 20 does not join smoothly with the straight line at the right-hand side of the figure.

Since kinetic energies are asked for in this problem, it is necessary to determine the speeds from the distance versus time graphs following the procedure of Vol 1, Chapter 5, Section 5. Since initial and final kinetic energies are asked for, one must draw the tangent lines to the curves before and after the encounter, and then choose convenient points to measure d_1, t_1, d_2 and t_2 for each of the four lines. One can achieve greater accuracy if the tangent lines are drawn completely across the figure. A ruler with 1/8″ division is very convenient for reading the time scale and a ruler with 1/10″ division for reading the distance scale. We have taken the 1 kg mass (the upper curve) to be initially at rest (zero slope).

In part (d) the slope must be determined at an intermediate point. One locates this point in time by the criterion that the distance between the two curves must be a minimum. The reasoning behind this choice is the same as that of Section 2 of this chapter. The total kinetic energy is a minimum when the mutual potential energy is a maximum. This latter condition occurs when the masses are closest together, and this is when the speeds are the same. By sliding a piece of paper along the time axis and noting the separation between the curves with pencil marks along the vertical edge of the paper we determine the time of closest approach to be at 0.076 seconds. Here we draw the tangent lines to either curve. If we draw both, they should be parallel as a check that this is the correct point.

In passing we note that it is possible to measure the slopes by finding the tangents of the angles of inclination of the tangent lines. These angles can be measured with a protractor. One must remember that the angles depend on the choice of scales used for the time and distance axes. To take account of this properly all tangent of angles in Figure 20 should be multiplied by f (1.25 m/sec) which is the speed corresponding to a line sloped at 45°. In the following table we show the speed computation by the above method as well as by forming $\Delta d/\Delta t$.

		d_1 (m)	d_2 (m)	t_1 (sec)	t_2 (sec)	Δd (m)	Δt (sec)	α (o)	$\tan\alpha$	$f\tan\alpha$ (m/sec)	$\frac{\Delta d}{\Delta t}$	$\frac{1}{2}mv^2$ (joules)
Initial	m_1	0	.241	0	.240	.241	.240	38.5	.796	.999	1.00	1.50
	m_2	.100	.100	0	.240	0	.240	0	0	0	0	0
Final	m_1	.034	.166	0	.240	.132	.240	23.8	.441	.552	.550	.45*
	m_2	0	.300	.013	.218	.300	.205	49.5	1.17	1.47	1.46	1.06
Intermediate		.068	.243	0	.240	.175	.240	30	.577	.723	.730	.80 (m_1) .27 (m_2) 1.07

In the last column the kinetic energies are calculated. The initial kinetic energy is 1.50 joules; the final, is 1.51 joules. Let us assume that this difference is due solely to an error in measurement. (Unless one uses extreme care it is quite easy to have an error of 1° in the inclination α; this corresponds to an error of about 3% in any one speed or 6% in any one kinetic energy. Our error is about 1 part in 150 or 0.7%. In other words, the assumption that the energy is constant is consistent with our measurements).

At the instant of minimum separation the total kinetic energy of the two masses is a minimum and the mutual potential energy of the two masses has reached a maximum. The potential energy is the total energy minus the kinetic energy and its maximum value is 1.50 - 1.07 or 0.43 joules [answer to (e)].

*The slope of the curved portion rather than the straight line was used. The slope of the line is 0.396 m/sec and the kinetic energy is 0.235 joules (quite an energy loss).

It may be of interest to determine the law of the force of interaction, that is, the shape of the graph of the force versus the separation-distance. When we are given the distance versus time curves for each interacting mass, we can determine the velocity versus time curves using the methods of Chapter 5, Volume 1. From these velocity curves we can determine the acceleration versus time curves. By Newton's Second Law we then have the force of interaction versus the time. By subtracting the two distance curves we can also determine the separation-distance versus time curve. Using this curve we can then plot the force versus separation-distance curve. In principle we can construct this curve for any interaction. We know that the force of interaction is zero in Figure 20 when the separation-distance is greater than d because the graph of distance versus time becomes a straight line for each of the two masses. [The slope is constant; this means the speed is constant; this means the acceleration is zero, and therefore no force acts.]

The resemblance of Figure 20 to the earlier figure suggests that possibly the force F might be constant when the separation is less than d. If this is the case then the graph of d_1 versus t for the first mass should be given by

$$d_1 = v_{o1} t - \frac{1}{2} a_1 t^2$$

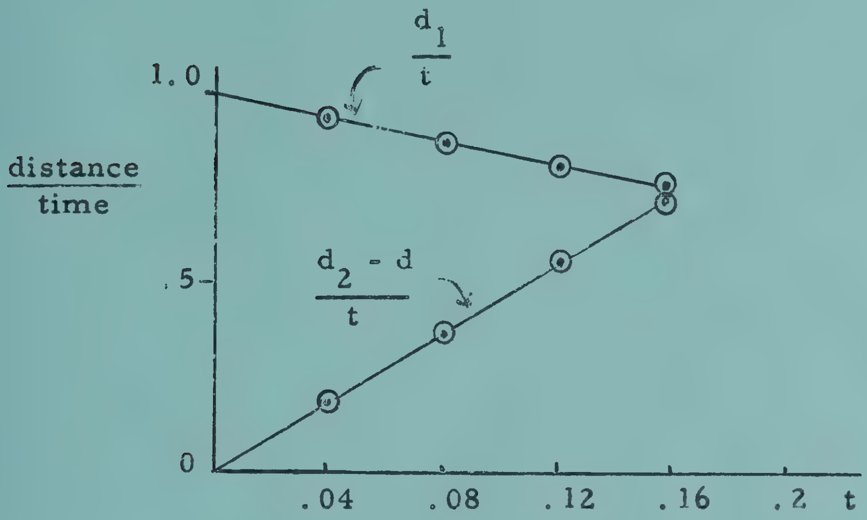
and that of d_2 by,

$$d_2 = d + v_{o2} t + \frac{1}{2} a_2 t^2$$

We can test this hypothesis by plotting d_1/t and $(d_2 - d)/t$ versus t using values taken from the graph, where d = 0.1 m; the plots should be straight lines if our hunch is correct.

We collect the data from Fig. 20 in the following table.

t	d_1	d_2	$\frac{d_1}{t}$	$\frac{d_2 - d}{t}$
0	0	.1	--	--
.04	.037	.107	.925	.175
.08	.070	.129	.875	.363
.12	.098	.165	.816	.541
.16	.123	.215	.770	.72



One sees that the plots are straight lines thus verifying our guess that F is constant and that the intercepts, v_{o1} and v_{o2}, are consistent with the slopes 1.00 and 0 m/sec as measured previously. The slopes of the straight lines are $-a_1/2$ and $a_2/2$, respectively. Therefore,

$$a_1 = 2\,\frac{.975-.770}{.16} = 2.56\ \text{m/sec}^2$$

$$a_2 = 2\,\frac{.720-0}{.16} = 9.00\ \text{m/sec}^2$$

We can use these accelerations to determine the force of the interaction. Actually there are several ways in which the force can be measured from the graphs of Figure 20 and we will give a few of them. Since they are all based on measurements we are not surprised to find different values for F.

We expect $F = m_1 a_1 = m_2 a_2$. From the above separate measurements $m_1 a_1 = 7.7$ nts, $m_2 a_2 = 9.0$ nts, which shows that there are errors both in constructing and measuring Figure 20.

Another value for F can be found from the measurement of the maximum potential energy, and the minimum distance of separation (it is .058 m). The potential energy (taking the potential energy as zero when the masses are further apart than d) is the force times the distance that the masses have moved toward each other since the beginning of the interaction it is d-.058 or .042 m. (This is because the force is constant). Using the value 0.43 joules found above we have

$$F = \frac{0.43}{.042} \text{ joules/m} = 10.1 \text{ nts}$$

Yet another independent value of F can be found from the time of interaction. Using the fact that F is constant and the curves are parabolas on Fig 20, we have for the distance between the curves

$$d_2 - d_1 = d + (v_{o2} - v_{o1})t + \frac{a_1 + a_2}{2} t^2$$

and so the distance between the curves in d when t = 0 and when $t = \frac{2(v_{o1} - v_{o2})}{a_1 + a_2}$ and this latter value (the time of the interaction) is 0.167

sec by inspection. Using $v_{o1} = 1$ m/sec we have $a_1 + a_2 = \frac{2.1}{.167} = 12$ m/sec^2 but $a_1 + a_2 = \frac{F}{m_1} + \frac{F}{m_2}$ so that

$$F = \frac{m_1 m_2}{m_1 + m_2}(a_1 + a_2) = \frac{3}{4} \cdot 12 = 9 \text{ nts}$$

Another independent value of F comes from the final velocity of m_2 and the time of interaction (.167 sec) using $v_2 = a_2 t$. This gives $F = \frac{1.46 \times 1}{.167} = 8.8$ nts. This last value is one of the best single determinations since it uses the best determined slope of all those measured.

The poorest determination of F is the one based on the potential energy since it uses squares of speeds and differences of squares and the various errors tend to be compounded. We will assume that Figure 20 was constructed using F = 9 nts.

One last comment and that concerns the sudden break in the slope of the lower curve at the end of the interaction. The break is not to be expected because the interaction force suddenly drops to zero. A sudden change in force can only cause a sudden change in curvature. The only way that the slope can suddenly change is to have an impulse, that is, a large force act for a short time. This can cause a change in momentum and this implies a sudden change in speed. An impulsive force acts every time the total energy of the mass is suddenly changed. It is as though the mass suddenly rolls down a very steep embankment along its path. It was because of the implication of non-constant energy that we ascribed the break in slope to a drafting error, and used the slope of the curve for the speed determination.

EXERCISE 14: Two masses $m_1 = 1$ kg and $m_2 = 2$ kg are pushed together despite an elastic interaction force until their potential energy is 27 joules greater than it is when they are beyond the distance at which they interact. If they are let go, what will be the final kinetic energy of each?

A hard problem combining the conservation of energy and of momentum, illustrating the same principles used to compute the energy of the earth in Section 3.

In this problem we know that the interaction force (we can think of it as a spring that is compressed between the two masses--but the actual law of force versus distance is not important) will push the two masses apart so that each is moving after the interaction is over. We know that the mutual potential energy will be converted completely into kinetic energy of motion, because the force is given as elastic. But how do the two masses share the energy? Is the division arbitrary, hit-or-miss, helter-skelter?

In most of the examples discussed so far: the spring bumper, the motion of a mass in the gravitational field of the earth, etc. the fact that the potential energy is a mutual quantity is frequently overlooked because the mass of the earth and the mass of whatever the spring bumper is fastened to are so large that the kinetic energy is delivered almost entirely to the smaller mass.

The sharing is, of course, not at random and is completely determined by the ratio of the interacting masses. The principle that we must use is the conservation of momentum. At every instant during the interaction the changes of momenta of the two masses are equal and opposite whatever the law of force and this fact permits the ratio of kinetic energies to be determined. It is implied that the masses are at rest before they are released. The total momentum changes are then equal in magnitude, and the analysis of section 3 is applicable. The result is

$$\frac{E_{k1}}{E_{k2}} = \frac{m_2}{m_1} = \frac{2}{1}$$

$$E_{k1} + E_{k2} = 27$$

Using either algebra or the rules of proportion we have

$$\frac{E_{k1}}{E_{k1} + E_{k2}} = \frac{2}{3}$$

$$\text{or} \quad E_{k1} = \frac{2}{3}\,(E_{k1} + E_{k2}) = \frac{2}{3}\,27 = 18 \text{ joules,}$$

$$\text{and} \quad E_{k2} = 9 \text{ joules.}$$

EXERCISE 15: (a) A ball of mass 0.25 kg is thrown to the right at a speed of 2 meters per second. It starts along the frictionless surface at the left of Fig. 21. How high up the slope at the right will it go before coming momentarily to rest? What kind of a motion will it perform?

(b) If the ball is released from rest at point P, what kind of a motion will it perform? How high up the slope on the right will it rise? What is the binding energy (the extra energy needed to make the ball escape from the well in the center of the figure)?

(c) What is the binding energy in part (a)?

A straightforward exercise on energy conservation stressing an understanding of binding energy.

(a) We will assume that the ball is moving along the horizontal surface with the given speed. It has a kinetic energy of $\frac{1}{2}$ joule. It will gain kinetic energy (and lose potential) as it drops into the valley but then lose it as it rises up the other side until at the instant it reaches the original elevation it will again have $\frac{1}{2}$ joule k.e. It continues to rise to a vertical height h such that

$$mgh = \frac{1}{2} \text{ joule.}$$

Therefore

$$h = \frac{1}{2}\ \frac{1}{0.25} \cdot \frac{1}{9.8}$$

$$h = \frac{2}{9.8} = 0.204 \text{ m}$$

The slant height = $\frac{h}{\sin 45^\circ} = 0.288$ m

The ball can be thought of as rebounding; it comes in from the left, rolls up the hill, and then rolls off to the left again with the same speed as it had initially.

(b) The motion will be oscillatory. It will rise as high on the right as the left. If it had an additional amount of energy equal to mgh where h is the height of the level section above p = 0.25 x 9.8 x 0.5 = 1.23 joule it could just escape. This is the binding energy, if it starts from rest at P.

If the ball is at rest at the bottom of the well, then it needs mgh_o = 0.25 x 9.8 x 1 = 2.45 joules to escape where h_o is the full depth of the well. This is the binding energy in this case.

(c) The binding energy is negative, if the particle has kinetic energy left over in the separated state. One must take away $\frac{1}{2}$ joule of energy to leave the ball just free of the well (no kinetic energy on the horizontal surface.)

EXERCISE 16: A linear spring whose force-extension ratio k is 40 newton/m hangs vertically, supporting a 0.80-kg mass at rest. The mass is then pulled down a distance of 0.15 m.

(a) How high will it rise?

(b) What will be its maximum velocity?

(c) How would the answers to (a) and (b) differ if the experiment were done on the moon?

A difficult problem combining two forms of potential energy (gravitational and elastic). This problem requires a strict and careful application of the principles of energy conservation and computation of potential energy. Students who are unwilling to be systematic will experience difficulties. We give two solutions. The first involves direct application of energy principles. The second is based on considering the forces, and serves to give added insight.

In considering the potential energy we must compute both the gravitational and elastic terms. The former is given simply enough as mgh where h is the height above any convenient reference we choose to take. The elastic potential energy is given as $\frac{1}{2}kx^2$ where x is the distance the spring is stretched (or compressed) from its unstretched length. Thus if we stretch a spring .20 meters, and then stretch it an additional .05 meters, the extra elastic energy we must supply is not $\frac{1}{2}k(.05)^2$ joules but is rather $\frac{1}{2}k(.25)^2 - \frac{1}{2}k(.20)^2$ joules.

In our problem, the .80 kgm mass when hanging in equilibrium stretches the spring. The additional stretch of .15 m given before release must be treated as we have just remarked.

We shall therefore locate the mass by a distance x measured down from the position of the end of the unstretched spring. The .80 kgm mass then hangs in equilibrium at the point x_o at which the weight downward, mg, is just balanced by the upwards pull of the spring, kx_o.

$$\text{Thus } kx_o = mg \text{ or } x_o = \frac{mg}{k} = \frac{(.80) \times 9.8}{40} = .196 \text{ meters}$$

At any point x the spring has potential energy $\frac{1}{2}kx^2$, and the gravitational potential energy is $-mgx$, (the minus sign arising from our choice of x as positive downwards).

The total potential energy, U when the mass is at point x is therefore

$$U = \frac{1}{2}kx^2 - mgx$$

The potential energy at the equilibrium point is

$$U_{\text{equilibrium}} = \frac{1}{2}kx_o^2 - mgx_o$$

$$= \frac{1}{2}k\left(\frac{mg}{k}\right)^2 - \frac{(mg)^2}{k} = -\frac{1}{2}\frac{(mg)^2}{k}$$

(a) In this part we must recognize that at both the highest and the lowest points, the mass is instantaneously at rest. Therefore at these points the total energy is all potential. Thus, the energy at the lowest point, .15 m below x_o, is

$$\frac{1}{2}k(x_o + .15)^2 - mg(x_o + .15)$$

Equating this to the energy at the top (position x)

$$\frac{1}{2}kx^2 - mgx = \frac{1}{2}k(x_o + .15)^2 - mg(x_o + .15)$$

We substitute $x_o = \frac{mg}{k}$, collect terms, and multiply through both sides by 2/k to get

$$x^2 = 2\left(\frac{mg}{k}\right)x + \left(\frac{mg}{k}\right)^2 = (.15)^2$$

or $x = \frac{mg}{k} \pm .15$ for the two roots.

$= x_o \pm .15$

The plus sign corresponds to the starting (lowest) point, and the minus sign to the top point, both of which have the same potential energy.

We note that the mass oscillates between extremes of ± .15 m about its equilibrium at x_o, just as a mass and spring on a horizontal surface oscillates equal distances on each side of their equilibrium. The effect of gravity seems to be simply to shift the zero.

It is interesting to express the potential energy in terms of the displacement, y, from the equilibrium x_o. The total displacement is then $x = x_o + y$ (y is therefore positive downwards). Substituting in our expression for potential energy we get

$$U = \frac{1}{2}k(x_o + y)^2 - mg(x_o + y)$$

$$= \frac{1}{2}kx_o^2 + kx_o y + \frac{1}{2}ky^2 - mgx_o - mgy$$

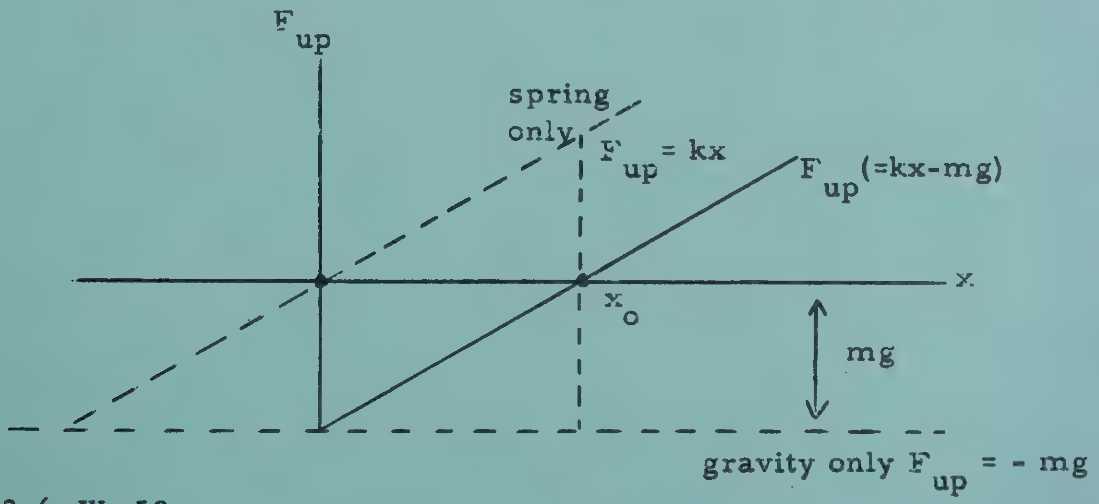
Using $x_o = \frac{mg}{k}$

$$U = \frac{1}{2}ky^2 - \frac{(mg)^2}{2k}$$

When $y = 0$ $\qquad U = -\frac{(mg)^2}{2k}$

The difference between the potential energy at displacement y and that at equilibrium (y = o) is then $\frac{1}{2}ky^2$. This is just the expression we would have for a spring which is unstretched at y = 0 and not acted upon by the gravitational force.

We can also see the result by considering the total force on the mass. It consists of mg down and kx up. Thus $F_{up} = kx - mg$



We see that the total force is a straight line parallel to that of the spring alone. The equilibrium point x_o occurs where $F_{up} = 0$ (i.e. where the line crosses the x axis). The total force is just as though we had the spring only, but with the spring unstretched when the mass is at x_o.

(b) In view of our remarks above, the maximum velocity occurs when $x = x_o$, where the potential energy is a minimum. Even without our remarks above we know that the equilibrium point is the point of lowest total potential energy, just as it is for a mass hanging from a string. Therefore

$$\frac{1}{2}mv^2 + U(x_o) = U(x_o + .15)$$

or

$$\frac{1}{2}mv^2 = \frac{1}{2}k(x_o + .15)^2 - mg(x_o + .15) - \frac{1}{2}k x_o^2 + mgx_o$$

Or, after some algebraic manipulation employing $x_o = \frac{mg}{k}$

$$\frac{1}{2}mv^2 = \frac{1}{2}k(.15)^2$$

$$v = .15\sqrt{\frac{k}{m}} = .15\sqrt{\frac{40}{.8}} = 1.06 \text{ m/sec}$$

(c) On the moon g would be smaller. Looking at our resulting formulas above we see that the equilibrium distance x_o would be different. However, the highest point would still be .30 m above the lowest point, because the displacement from equilibrium is the same by hypothesis. The maximum velocity would still be 1.06 m/sec.

EXERCISE 17: A 0.2-kg stone is thrown upward from a point 20 meters above the earth's surface at an angle of 60 degrees with the horizontal and with a speed of 20 m/sec.

(a) What is its total energy?

(b) What will be its total energy when it is 15 m above the earth's surface?

(c) What will be its speed 15 m above the earth?

A straightforward exercise involving the zero of gravitational energy and conversion of kinetic to potential. Although the direction of the initial velocity is given, we do not need it. The problem does not require solution of the path of the projectile since we are only asked for its speed.

To compute the total energy of the stone we must first select a zero of potential energy. For convenience we choose the surface of the earth.

(a) The initial kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}(.2)(20)^2 = 40$ joules. The initial potential energy, mgh, is $mgh = .2 \times 9.8 \times 20 = 39.2$ joules. Since h is 20 m above the surface of the earth, our zero

The total energy is the sum of the kinetic and potential energy at any instant:

$$39.2 + 40 = 79.2 \text{ joules total energy.}$$

(b) Since the total energy in this system is conserved (we neglect air resistance) the total energy is 79.2 joules at all points on the path.

(c) At 15 m above the earth's surface the potential energy $mgh = .2 \times 9.8 \times 15 = 29.4$ joules. Therefore using the conservation of energy the kinetic energy, $\frac{1}{2}mv^2$, is given by

$$\frac{1}{2}mv^2 + 29.4 = 79.2$$

$$\text{or} \quad \frac{1}{2}mv^2 = 49.8 \text{ joules} \qquad v = \frac{2 \times 49.8}{m} = \frac{2}{.2} \times 49.8$$

$$v = 22.4 \text{ m/sec.}$$

Note that the velocity is larger than the initial velocity as it must be since the stone is below the initial point.

Had we chosen the zero of potential energy to be the launching point (20 m above the surface of the earth) the total energy would have been initially entirely kinetic (40 joules). The answers to parts (a) and (b) would then be 40 joules, but of course the answer to (c) would still be 22.4 m/sec since the velocity of the stone is not affected by the choice of a zero for potential energy.

For a discussion of the path of the projectile, see Teacher's Guide, Vol 3, Chapter 2, Sections 3 and 4.

EXERCISE 18: Use the equations $v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$ and $v_2' = \frac{2m_1}{m_1 + m_2} v_1$ for the final speeds of two masses m_1 and m_2 interacting elastically to estimate:

(a) the ratio of the initial to the final kinetic energy of m_1 if $m_2 = 10^{24} m_1$.

(b) the ratio of the final kinetic energy of m_2 to the initial kinetic energy of m_1.

(c) Do the answers justify the statement that a mass colliding elastically with a spring bumper solidly fastened to the earth loses practically no kinetic energy as a result of the interaction?

A fairly straightforward problem involving substitution into known formulas. The major source of difficulties is likely to be handling two numbers of greatly different sizes ($m_2 = 10^{24} m_1$). It is appropriate to use approximation methods. The proper steps may be hard for students to grasp. Incidentally, the formulas given come from analysis of a collision in which m_2 is initially at rest. v_1 is the initial speed of m_1.

(a) The initial kinetic energy of m_1 is $\frac{1}{2} m_1 v_1^2$

The final kinetic energy of m_1 is $\frac{1}{2} m_1 v_1'^2$

Therefore the required ratio is

$$\frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1v_1'^2} = \frac{v_1^2}{v_1'^2}$$

But $v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$, so that

$$\frac{v_1^2}{v_1'^2} = \left(\frac{m_1 + m_2}{m_1 - m_2}\right)^2 = \left(\frac{m_1 + m_2}{m_2 - m_1}\right)^2 = \frac{(10^{24} + 1)^2}{(10^{24} - 1)^2}$$

Since 10^{24} is much bigger than 1, it is an excellent approximation to replace $10^{24} + 1$ by 10^{24} and $\frac{(10^{24})^2}{(10^{24})^2} = 1$

A correction term to the approximation may interest the class and is relevant to part (b). The calculation of the correction illustrates a very useful technique of approximation. Let us write

$$m_1 + m_2 = m_2\left(1 + \frac{m_1}{m_2}\right)$$

where $\left(\frac{m_1}{m_2}\right) = 10^{-24}$ is much less than 1. Let us introduce the symbol a for $\frac{m_1}{m_2}$; a is therefore 10^{-24}. We have, then,

$$\frac{v_1^2}{v_1'^2} = \frac{m_2^2}{m_2^2}\frac{\left(1 + \frac{m_1}{m_2}\right)^2}{\left(1 - \frac{m_1}{m_2}\right)^2} = \frac{(1 + a)^2}{(1 - a)^2}$$

Now $(1 + a)^2 = 1 + 2a + a^2$

Since a is much less than 1, we can say that the three terms on the right contribute in various approximations to the term on the left. That is, our first approximation is

$$(1 + a)^2 = 1$$

A more accurate result is

$$(1 + a)^2 = 1 + 2a = 1 + 2 \times 10^{-24}$$

while the exact answer is

$$(1 + a)^2 = 1 + 2a + a^2 = 1 + 2 \times 10^{-24} + 10^{-48}$$

We note that the three terms (1, 2×10^{-24}, 10^{-48}) are progressively smaller, the ratio of successive terms being only about 1 part in 10^{24}.

Likewise $(1 - \frac{m_1}{m_2})^2 = (1-a)^2 \cong 1-2a = 1-2 \times 10^{-24}$

We now approximate $\frac{1}{1-2a}$ by $1 + 2a$

The validity of this approximation is seen by simply performing long division:

$$\begin{array}{r l}
 & 1 + 2a + 4a^2 + \ldots \\
1-2a \,\big) & 1 \\
 & \underline{1 - 2a} \\
 & \quad 2a \\
 & \quad \underline{2a - 4a^2} \\
 & \qquad 4a^2 \\
 & \qquad \underline{4a^2 - 8a^2} \\
 & \qquad\qquad 8a^2 \\
 & \qquad\qquad + \ldots
\end{array}$$

The remaining terms involve higher and higher powers of a. In the spirit of the approximation of neglecting a^2 we have, then

$$\frac{1}{1-2a} = 1 + 2a$$

Thus,

$$\frac{v_1^2}{v_1'^2} = \frac{(1+a)^2}{(1-a)^2} \cong \frac{1+2a}{1-2a} \approx (1+2a)^2 \approx 1 + 4a$$

$$\frac{v_1^2}{v_1'^2} \approx 1 + 4 \times 10^{-24}$$

(b′ We compute the ratio of the final kinetic energy of m_2 to the initial kinetic energy of m_1 in a similar way: Using the formula for v_2' the ratio is

$$\frac{\frac{1}{2}m_2 v_2'^2}{\frac{1}{2}m_1 v_1^2} = \frac{m_2}{m_1} \frac{4m_1^2}{(m_1+m_2)^2}$$

$$= \frac{4\frac{m_1}{m_2}}{\left(1 + \frac{m_1}{m_2}\right)^2} \cong \frac{4\,m_1}{m_2} \text{ since } \frac{m_1}{m_2} \ll 1$$

$$= 4 \times 10^{-24}$$

Notice that from part (a) we have

$$\frac{v_1'^2}{v_1^2} = \frac{1}{1 + 4 \times 10^{-24}} = 1-4 \times 10^{-24}$$

using the approximations of (a). Note this implies that m_1 loses a fraction 4×10^{-24} of its initial kinetic energy. The fact that

$$\frac{\frac{1}{2}mv_2'^2}{\frac{1}{2}mv_1^2} = 4 \times 10^{-24}$$

means that the kinetic energy lost by m_1 is given to m_2.

(c) Yes, since the mass of the earth is 6×10^{24} kgm, much larger than a typical mass used in the laboratory.

EXERCISE 19: Fig. 22 illustrates a very common toy. The rotating wheel and shaft stick on the supports magnetically. The toy is held upright and the rotor allowed to move down one side starting from rest. It will go around the curve at the bottom and climb up the other side. If you can get hold of such a toy, find out how much energy is lost from mechanical energy (the sum of gravitational potential plus kinetic energy) in one period of the motion. What fraction is this of the maximum kinetic energy during the motion? Into what form of energy does the lost energy probably go?

A problem for class demonstration if one can get the apparatus. Excellent for class discussion even if the toy is not available.

A minor point of definition: By "the period" of the motion is meant the time to go completely around and back--that is, so the rotor has returned to the same place and is going in the same direction.

To compute the fraction of the maximum kinetic energy represented by the energy lost, we consider Fig. (a) Starting at rest from the top the

spool would acquire its maximum kinetic energy at the bottom and, if there were no losses the kinetic energy at the bottom would be equal to the initial potential energy, mgh. The maximum kinetic energy with losses will be a little less. As a rough approximation we shall simply use mgh.

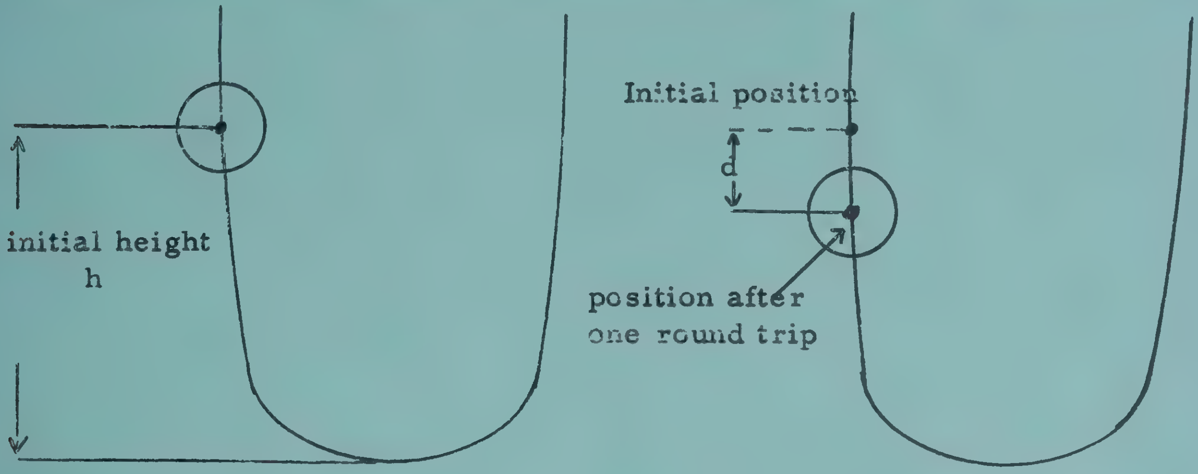


Fig. (a)

If on completing one round trip the rotor's highest point is a distance, d, below its starting point it has lost energy mgd. Therefore the ratio of the energy loss to the maximum kinetic energy is

$$\frac{mgd}{mgh} = \frac{d}{h}$$

A better approximation may be found by dividing the energy loss equally between the two sides. Then by the time the rotor first reaches the bottom its kinetic energy is mgh minus the loss on dropping. But the total loss in the motion down on the left, up on the right, down on the right, up on the left is mgd. Therefore mgd/4 is the loss going down. Then the maximum kinetic energy is

$$mgh - \frac{1}{4}mgd \text{ and the ratio is } \left(\frac{d}{h - d/4}\right)$$

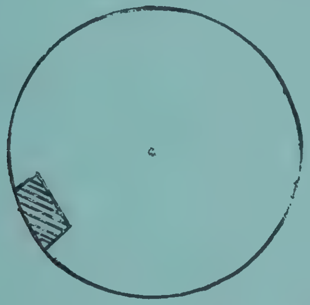
If the toy is not available, you can ask the class to describe the motion, first of all if there are no energy losses, then if there are losses. You should be able to lead them to recognize that loss will manifest itself by the failure of the rotor to climb up all the way on both left and right hand sides.

In this example the energy loss will produce a temperature rise since it is presumably due to friction and to electric currents set in motion by the rolling magnet, the currents in turn producing a temperature rise. If a large fraction of energy is lost, the latter source is undoubtedly the cause.

EXERCISE 20: Can you design a simple come-back toy? The outside is a cylindrical container. When the container is rolled away from you on a table, it will come to rest in a short distance, reverse its direction of motion, and come back to you.

The toy, to come back, must have a potential energy which varies as the cylinder rolls such that the toy can not, so to speak, surmount the potential energy barrier.

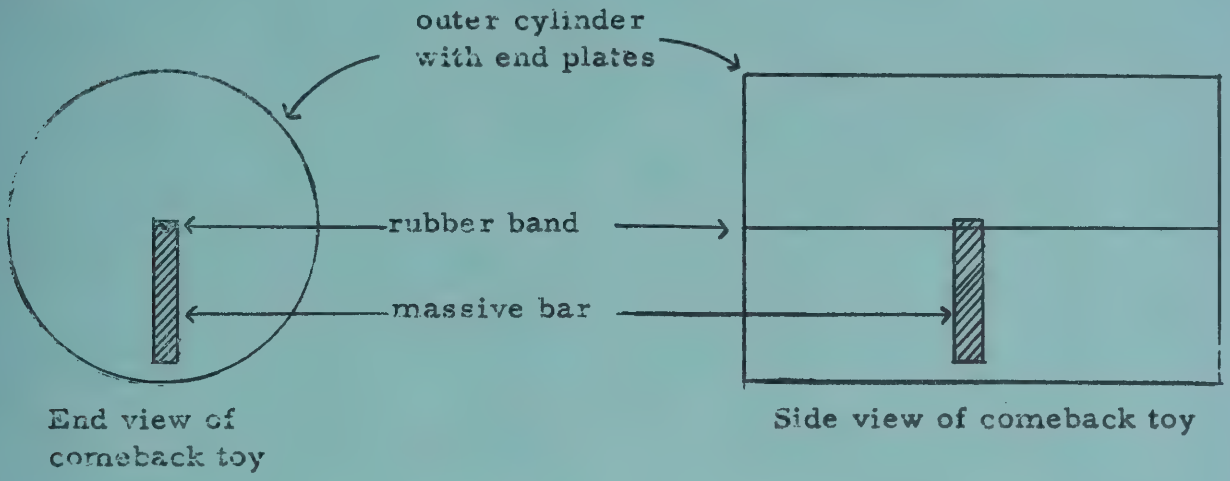
If a mass is attached to the inside of the cylinder, (see figure), the potential energy has a maximum when the mass is at the top and a minimum when it is at the bottom. The toy is much like a pendulum which can swing through the full 360°.



mass fastened to the cylinder.

If the cylinder is rolled with a small initial kinetic energy, it will roll until the kinetic energy has gone entirely into gravitational potential energy from raising the mass, then roll back. If the initial kinetic energy is too large, the cylinder will never come to rest, but its motion will be somewhat jerky. Since it is the change in potential energy which counts, we will expect different behavior depending on whether or not the mass is at its lowest point when we first start the cylinder rolling.

The cylinder will not come back if it rolls very far. To make a device that will return after many revolutions, we must make the potential energy change more slowly with angle of rotation. One way to do this is shown in the figure:



The bar is suspended from a rubber band in such a way that the rubber band twists when the cylinder is turned relative to the bar. As the toy rolls, the bar resists turning, causing the rubber band to twist. It takes many turns of the cylinder for the bar to turn through much of an angle. Eventually the original kinetic energy is converted entirely to potential energy (both gravitational, from raising the bar, and elastic from twisting the rubber band), and the toy comes to rest. It then rolls back.

EXERCISE 21: When an object of mass m moves from r to r′ in the earth's gravitational field the potential energy changes by $\Delta U = -GmM\left(\frac{1}{r'} - \frac{1}{r}\right)$, where M is the mass of the earth. Show that, if the object moves away from the earth a distance Δr which is small compared to its distance from the earth's center, the above expression reduces to $\Delta U = mg\,\Delta r$.

A moderately hard problem involving the use of mathematical approximations. This problem fills in the steps at the bottom of the first column of section 4 of the text. See also Teacher's Guide, Appendix to section 4, Chapter 6.

We write $r' = r + \Delta r$. Substituting we find

$$\Delta U = -GmM\left(\frac{1}{r'} - \frac{1}{r}\right) = -GmM\left(\frac{1}{r + \Delta r} - \frac{1}{r}\right)$$

$$= -GmM\left[\frac{r - (r + \Delta r)}{r\,(r + \Delta r)}\right] = +\frac{GmM\,\Delta r}{r\,(r + \Delta r)}$$

Since $\Delta r \ll r$, we may approximate the denominator $r\,(r + \Delta r)$ by r^2. Therefore $\Delta U \cong \frac{GmM}{r^2}\Delta r$. If we define $\frac{GM}{r^2}$ as g we have $\Delta U \cong mg\Delta r$

EXERCISE 22: Find the binding energy, to two significant figures, of:

(a) You to the earth.

(b) The moon to the earth.

An easy numerical problem primarily involving substitution into formulas focusing attention on the important concept of binding energy.

The basic relation needed is found in Section 4: The binding energy, BE, is

$$BE = \frac{GmM}{r} - \frac{1}{2}mv^2$$

We recall that the first term represents the potential energy we must supply to move m from r to infinity. However, if m has velocity v when at the initial distance r and we supplied a total energy equal to $\frac{GmM}{r}$, m would still have the same kinetic energy when a long distance away from M. By bringing m to rest at the great distance we can now "get back" $\frac{1}{2}mv^2$ of energy.

(a) We shall take a 70 kgm man (a 160 lb man would be 72.6 kgm), and use $G = 6.670 \times 10^{-11} \frac{\text{nt} \cdot \text{m}^2}{\text{kgm}^2}$ (Text, P. 3-14)

$r = 6.38 \times 10^6$ m, the radius of the earth (Text, P. 3-9)

$M = 5.98 \times 10^{24}$ kgm, the mass of the earth. (Text, P. 3-9)

$v = 0$ for a man at rest. (See TG 3-6, Sec. 4, App. I)

$$\text{Then BE} = \frac{GmM}{r} = \frac{(6.67 \times 10^{-11}) \times 70 \times 5.98 \times 10^{24}}{6.38 \times 10^6}$$

Binding energy of 70 kgm man = 4.3×10^9 joules

(b) Part (b) is closely related to Problem 23.

To compute the binding energy of the moon we must compute both $\frac{GmM}{r}$ (where m is now the mass of the moon, and r the radius of the moon's orbit about the earth) and also the moon's kinetic energy, $\frac{1}{2}mv^2$.

The moon goes a path of length $2\pi r$ in a time T, its period. Therefore

$$v = \frac{2\pi r}{T} \text{ and } KE = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{4\pi^2 r^2}{T^2} = 2\pi^2 m\frac{r^2}{T^2}$$

From P. 3-9 in the text we get

$m = 7.34 \times 10^{22}$ kgm

$r = 3.8 \times 10^8$ meters

$T = 2.36 \times 10^6$ seconds,

$$\text{So KE} = 2\pi^2 m\frac{r^2}{T^2} = 2\pi^2 \times 7.34 \times 10^{22} \times \left(\frac{3.8 \times 10^8}{2.36 \times 10^6}\right)$$

$= 3.8 \times 10^{28}$ joules.

The potential energy, $\frac{GmM}{r}$, is found using

$$G = 6.67 \times 10^{-11} \ \frac{nt \cdot m^2}{kgm^2}$$

$$M = 5.98 \times 10^{24} \text{ kgm}$$

$$r = 3.8 \times 10^{8} \text{ meters}$$

$$PE = \frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22} \times 5.98 \times 10^{24}}{3.8 \times 10^{8}}$$

$$= 7.7 \times 10^{28} \text{ joules}$$

The binding energy is therefore

$$BE = \frac{GmM}{r} - \frac{1}{2} mv^2$$

$$= (7.7 - 3.8) \times 10^{28} \text{ joules}$$

$$= 3.9 \times 10^{28} \text{ joules}$$

Alternatively we may make use of our knowledge of circular motion to express $\frac{1}{2} mv^2$ in terms of G, m, M, and r. Then Newton's second law tells us

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

Thus $mv^2 = \frac{GmM}{r}$ or $\frac{1}{2}mv^2 = \frac{1}{2}\frac{GmM}{r}$

Therefore

$$BE_{moon} = \frac{GmM}{r} - \frac{1}{2}mv^2 = \frac{GmM}{r} - \frac{1}{2}\frac{GmM}{r}$$

$$= \frac{1}{2}\frac{GmM}{r}$$

so $BE_{moon} = \frac{1}{2} \times \frac{(6.67 \times 10^{-11}) \times (7.37 \times 10^{22}) \times (5.98 \times 10^{24})}{3.8 \times 10^8}$

binding energy of moon = 3.9×10^{28} joules

(Notice that $BE = \frac{1}{2}\frac{GmM}{r} = \frac{1}{2}mv^2$ but due to the fact that our figure for the radius of the moon's orbit has only two significant figures, we find $\frac{1}{2}mv^2 = 3.8 \times 10^{28}$ joules whereas $\frac{1}{2}\frac{GmM}{r} = 3.9 \times 10^{28}$ joules).

EXERCISE 23: A satellite is in a circular orbit about the earth.

(a) Express its kinetic energy in terms of F_c, the centripetal force on the satellite, and r, the satellite's distance from the center of the earth.

(b) Using the fact that F_c is the gravitational force of attraction on the satellite, find the ratio of the satellite's kinetic energy to its potential energy.

(c) How much additional energy is needed to make the satellite escape from the earth? Express your answer in terms of its present kinetic energy.

A straightforward but non-trivial problem related to the concepts of circular motion and binding energy, involving algebraic rather than numerical manipulation. The results are useful in problem 22b.

(a) The centripetal or center-seeking force (or force along the line towards the "center"), F_c, acting on the satellite, is related to the acceleration of the satellite towards the center, v^2/r, by Newton's second law:

$F_c = m\frac{v^2}{r}$ where m is the mass of the satellite.

Therefore $mv^2 = rF_c$. Since the kinetic energy, E_k is $\frac{1}{2}mv^2$ we have

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}r\,F_c$$

(b) Since F_c is the gravitational force of attraction of the earth (mass M) on the satellite

$$F_c = \frac{GmM}{r^2}$$

Therefore from (a) $E_k = \frac{1}{2}rF_c = \frac{1}{2}r\frac{GmM}{r^2} = \frac{1}{2}\frac{GmM}{r}$

But the potential energy, PE, is $\frac{GmM}{r}$ in magnitude

$$\frac{E_k}{PE} = \frac{\frac{1}{2}\frac{GmM}{r}}{\frac{GmM}{r}} = \frac{1}{2}$$

(c) The additional energy to enable the satellite to escape is what we mean by the binding energy. Using the results of Ch. 6, Sect. 4, we have binding energy $= \frac{GmM}{r} - \frac{1}{2}mv^2$. But $\frac{GmM}{r} = 2 \times E_k$. Therefore binding energy $= 2\,(E_k) - (E_k)$

$$= E_k$$

We must supply an extra energy exactly equal to the present kinetic energy barely to free the satellite.

EXERCISE 24: The force field between a pair of protons is repulsive. Does the potential energy increase or decrease as a pair of protons are brought together?

A non-mathematical exercise which stresses a point that is often confusing.

Since the force between the protons is repulsive, they resist being pushed together or fly apart if released from rest. Since particles naturally move "down" a "potential hill", their potential energy must decrease the farther they are separated, and conversely, their potential energy increases as they are brought closer.

Another way to phrase the answer is to remark that we must do positive work to push the protons together. Consequently we increase their potential energy when we bring them closer.

EXERCISE 25: The force of repulsion in newtons between two electrons is is given by $F = \frac{2.3 \times 10^{-28}}{x^2}$ where x is the separation between them in meters. If each of the electrons has a mass of 9×10^{-31} kg and they are held 10^{-10} m apart and then released, what will be:

(a) The kinetic energy of each when they are 2×10^{-10} m apart?

(b) The velocity of each?

A difficult exercise requiring a generalization of the expression for gravitational potential energy. Exercise 24 should precede.

(a) The force, F, between two electrons is given by

$$F = \frac{2.3 \times 10^{-28}}{x^2}$$

This is an inverse square law and therefore depends on the separation in just the same way as does the gravitation force. The fact that this is a force of repulsion means however, that in any relative displacement the change in potential energy is the negative of what it would be if the force on each

particle pointed in the opposite direction--that is, the force were one of attraction.

Were the force one of attraction we see, by analogy to the expressions for gravitational attraction that the potential energy of the particles at a separation x would be: PE at x = $-\frac{2.3 \times 10^{-28}}{x}$ where we take the potential energy to be zero when the particles are a long distance apart. Since the force is one of repulsion we need the negative of this result to get the actual potential energy at separation x:

$$\text{PE at separation } x = +\frac{2.3 \times 10^{-28}}{x}$$

Notice that as x increases, the potential energy decreases, as discussed in problem 24.

(a) Since we release the electrons from rest they move in opposite directions, their velocities being equal and opposite to conserve the total momentum as zero. Their kinetic energies are equal. Denoting the mass of either particle by m and magnitude of the velocity of either by v, we have $\left(2\,\frac{1}{2}\right)mv^2$ + potential energy = constant (the total energy) = $\frac{2.3 \times 10^{-28}}{10^{-10}}$.

Since when x = 10^{-10}, v = 0.

At separation 2 x 10^{-10} the potential energy, PE, is given by

$$\text{PE} = \frac{2.3 \times 10^{-28}}{2 \times 10^{-10}}$$

$$2 \times (\frac{1}{2}\, mv^2) = \frac{2.3 \times 10^{-28}}{10^{-10}} - \frac{2.3 \times 10^{-28}}{2 \times 10^{-10}}$$

$$\text{or} \quad \frac{1}{2}\, mv^2 = .625 \times 10^{-18} \text{ joules}$$

(b) Since $m = 9 \times 10^{-31} = .9 \times 10^{-30}$ kgm

$$v = \sqrt{\frac{2}{m} \times .625 \times 10^{-18}} = \sqrt{\frac{2 \times .625 \times 10^{-18}}{.9 \times 10^{-30}}} = 1.18 \times 10^{6} \text{ cm/sec}$$

EXERCISE 26: What is the binding energy of the proton and electron in a hydrogen atom if the proton and electron are 0.5×10^{-10} m apart and the force of attraction between them is given by $F = \frac{2.3 \times 10^{-28}}{r^2}$ newtons, when r is in meters. Remember, the electron will not stand still long with a force on it. Assume it is moving in a circle around the proton.

A problem the solution of which follows readily from Problem 23.

Since the force of attraction is $\frac{2.3 \times 10^{-28}}{r^2}$, it is an inverse square attractive force like the gravitational force, and we can take the potential energy to be given in analogy as $-\frac{2.3 \times 10^{-28}}{r}$.

(That is, it is as though $GmM = 2.3 \times 10^{-28}$)

Using the result of Problem 23, the work to remove the electron, which is its binding energy, is

$$\frac{2.3 \quad 10^{-28}}{r} - \frac{1}{2}mv^2$$

but $\frac{mv^2}{r} = \frac{2.3 \times 10^{-28}}{r^2}$ so that $mv^2 = \frac{2.3 \times 10^{-28}}{r}$

and binding energy $= \dfrac{2.3 \times 10^{-28}}{r} - \dfrac{1}{2}\,\dfrac{2.3 \times 10^{-28}}{r} = \dfrac{1}{2}\,\dfrac{2.3 \times 10^{-28}}{r}$

since $r = .5 \times 10^{-10}$ meters

binding energy $= \dfrac{1}{2}\,\dfrac{2.3 \times 10^{-28}}{.5 \times 10^{-10}} = 2.3 \times 10^{-18}$ joules

EXERCISE 27: A drain pipe pointing slightly downhill sticks out from a retaining wall at the side of the road. A child throws a ball up the pipe and it comes back with a greater speed than that with which it was thrown.

(a) If this happened to you, would you be surprised?

(b) If this occurred a few times over, what would you suspect?

A non-mathematical problem good for class discussion and a few laughs.

a. Whether or not you would be surprised no doubt depends on whether you realized that the ball when returning to its starting point has gained in total energy (the potential energy is the same as when it was thrown since it is at the same place, but the kinetic energy is larger). Let's hope the students would be surprised!

b. Whether or not the ball returns at a greater speed more than once is immaterial. If it ever gains energy we suspect someone caught it and threw it back down!

EXERCISE 28: With about what fraction of the original mass can a rocket using chemical fuel "escape" from the earth?

A problem of medium difficulty based on the discussion of rockets in Ch 4 and on the concept of binding energy.

Since the binding energy of a body which is at rest is $\frac{GmM}{r}$, the rocket must acquire this much energy from its fuel. The fuel gives the rocket kinetic energy $\frac{1}{2}mv^2$ while the rocket is still close to the surface of the earth. This kinetic energy is gradually converted to potential energy as the rocket moves away from the earth. Therefore the kinetic energy just after the fuel is burned must equal $\frac{GmM}{r}$, where r is approximately the radius of the earth, m the mass of the rocket, and M the mass of the earth.

Therefore $\frac{1}{2}mv^2 = \frac{GmM}{r}$

$$\text{or } v = \sqrt{\frac{2\,GM}{r}}$$

From P. 3-9 and 3-14 we have

$$G = .667 \times 10^{-10} \frac{m^3}{\text{kgm. sec}^2}$$

$$M = 5.98 \times 10^{24} \text{ kgm}$$

$$r = 6.38 \times 10^6 \text{ meters}$$

$$\text{So } v = \sqrt{\frac{2 \times .667 \times 10^{-10} \times 5.98 \times 10^{24}}{6.38 \times 10^6}} = \sqrt{1.25} \times 10^4 \text{ meters/sec}$$

$$= 1.1 \times 10^4 \text{ meters/sec}$$

(see text, P. 6-12)

From the text, P. 4-14 we take the exhaust speed of the gases, v_e, to be 2×10^3 m/sec

Therefore $\frac{V}{V_e} = \frac{1.1 \times 10^4}{2 \times 10^3} = 5.5$

From the graph of P. 4-13 we find $\frac{M_a}{M_o}$, the ratio of the mass remaining, M_o, to the original mass, M_o.

Answer: the rocket has mass of only .004 of the original mass.

Chapter 7 - Heat, Molecular Motion and the Conservation of Energy.

PURPOSE (1) To extend the law of energy conservation to include thermal energy, for the particularly simple case of an ideal gas, regarded as a collection of "billiard-ball" molecules. To generalize this result to more complicated systems.

(2) To show how the kinetic theory of an ideal gas leads to the concept of temperature, and to quantitatively relate the average kinetic energy of translation of the molecules to temperature as defined customarily.

CONTENT a) The pressure exerted by a gas or liquid stream is directly proportional to the kinetic energy per unit volume associated with the motion of the centers of mass of the particles which comprise the fluid.

b) For an ideal gas, we may associate the absolute temperature with the kinetic energy of the motion of the centers of mass of the particles by the relation $\frac{1}{2} mv^2 \doteq \frac{3}{2} k T$, where $k \doteq 1.37 \cdot 10^{-23}$ joules/° k.

c) Thermal energy may be stored in matter in ways other than by changing the kinetic energy of motion of the centers of mass of molecules, e.g. by causing rotational motion of molecules or changes in configuration; thus the heat content (the measure of the total thermal energy) of a body is not necessarily reflected by the temperature of the body (the kinetic energy of motion of the centers of mass of the molecules). In particular, a body may absorb or evolve heat during a change of state (the "latent heat"), without a change in temperature.

d) Experiments show that whenever a given amount of mechanical energy is "lost" in an interaction, an identical amount of heat appears; thus heat and mechanical energy are completely equivalent-the temperature of 1 gm of water at 15° C will be raised 1° C by a dissipation of exactly 4.185 joules of energy.

e) The relation between heat and mechanical energy is reversible.

f) If we include all forms of energy, thermal energy, mechanical energy, radiant energy, etc., then the total energy within any closed system is conserved.

EMPHASIS The concept of the generality of energy conservation is one of the great achievements in all of physics, and should be fully appreciated by all students. This can be best accomplished by a full, quantitative treatment of this material.

However, your time schedule may not permit this "full-treatment" for the present chapter. In this eventuality, it would be best to skim rapidly over the first three sections on the kinetic theory of gases (which contain most of the mathematics), and concentrate on a qualitative coverage of energy conservation), referring to Joule's experiment as proof of the equivalence of heat and energy. Even an abbreviated treatment should be properly spiced with some appropriate exercises. Problems 10, 11, 16, 17, 22, and 24 are all concerned with energy transformation and the conservation of energy, and are particularly appropriate for highlighting the chapter.

COMMENT To most professional physicists, there is no other concept in physics with a grandeur equal to that of the law of conservation of energy. All interactions in the universe are inter-related through this one quantitative framework. Yet even this law, like all physical laws, rests ultimately on our limited experimental observations. It is entirely possible that over the whole universe, of which we have accurately observed only a tiny fraction, energy is not conserved. While such a possibility cannot be excluded, it should be understood that a valid observation of non-conservation of energy would be more than a ripple on the placid pond of physics. Our fundamental ideas about space and time are intimately connected with our belief in energy conservation, and a change in this "law" would cause an enormous revolation in physics, comparable to that which was triggered by Galileo and Newton when they first broke away from the Aristotelian concepts.

DEVELOPMENT The detailed treatment of the gas laws may well escape your slower students. While it is not essential that they "learn" this development (to the point of reproducing the calculation on an exam!), they should appreciate, at a minimum, the identification of temperature with the kinetic energy of motion of the centers of mass of molecules.

Although the present chapter provides ample material to lead one into a full-fledged treatment of the usual phenomenological material of "heat" (conduction, convection, radiation, mixtures, etc.), this is an indulgence we simply cannot afford in the present course. Interested students should, of course, be encouraged to do outside reading.

Section 1 - Pressure and Energy Density: Simple Gas Theory

PURPOSE Using a simplified model, to show that the pressure exerted by a fluid is proportional to the kinetic energy per unit volume associated with the motion of the centers of mass of the fluid particles.

CONTENT a) A small mass of water, Δm, which strikes a wall and stops imparts an impulse $F \Delta t = (\Delta m)v$ to the wall.

b) The force per unit area exerted by a steady stream of water of density ρ, is $P = \frac{F}{A} = \rho v^2 =$ 2x kinetic energy per unit volume.

c) The pressure exerted by an ideal gas is equal to $\frac{2}{3}$ x kinetic energy per unit volume associated with motion of the centers of mass of the gas molecules.

EMPHASIS To do this section properly will require a good deal of time. If your schedule is tight, skip it. Slower students may encounter some difficulty with the mathematics. Therefore thorough coverage may require several days of class time. You may be able to afford only a superficial job. If so, make certain that all students at least understand that it is possible to show the equivalence of the pressure and the kinetic energy density. Stress the fact that it is only the kinetic energy associated with motion of the centers of mass of molecules which determines the pressure.

COMMENT Students may be disturbed about the analogy between the pressure of a gas and that exerted by a stream of water which stops after striking the wall. The impact of the water stream is obviously inelastic (since the kinetic energy is essentially zero after collision), while the impact of perfect gas molecules is elastic (they rebound with the same speed as they hit, hence their kinetic energy, $\frac{1}{2}mv^2$, is unchanged by the collision). However, the argument presented is only

concerned with the change in momentum of the fluid particles, and momentum is conserved in both elastic and inelastic collisions. Indeed, the statement that the water "has no momentum" after striking the box is true only if the wall has infinite mass (or is rigidly attached to the earth).

Some alert students may see that the derivation given in the text is highly simplified. They may note that it implicitly assumes that all particles or molecules travel with the same speed.

Even if all molecules started off with the same speed, they would eventually collide with one another, and even if these collisions be elastic they produce shifts in speed which would eventually lead to a broad spectrum of speeds. When two such objects collide elastically while their center of mass is at rest, no change in speeds results. When their center of mass is in motion (e.g. collision at right angles) changes in the speeds result. A molecule moving with speed 3 v would have nine times the kinetic energy of one moving with speed v, but would transmit only six times the impulse in an elastic collision $(2m \cdot 3v)$ as the slower moving molecule. Thus the relation between pressure and kinetic energy must somehow depend on the actual distribution of speeds of the various molecules. However, students may be reassured by the statement that a rigorous proof is possible, (one which takes into account the actual distribution of velocities), and it still gives the same result which has been so simply derived in this section.

Section 2 - Temperature and Molecular Kinetic Energy

PURPOSE To show the quantitative relation between the kinetic energy of motion of the centers of mass of molecules of an ideal gas and the absolute temperature of the gas.

CONTENT a) The absolute temperature is related to the kinetic energy of motion of the centers of mass of the molecules by

$$kT = \frac{2}{3}\left(\frac{mv^2}{2}\right),$$

where K is a universal constant equal to $1.37 \cdot 10^{-23}$ joules/°K. This value corresponds to 12.6 joules/°K per mole for any monatomic gas.

b) Polyatomic gases, as well as liquids and solids, may store energy by exciting vibrational and rotational oscillations, and by changes in the mutual potential energy between interacting molecules, as well as by increase in the kinetic energy of motion of the centers of mass of the molecules. Such substances will require more energy than would be required by a monatomic gas to effect a certain increase in temperature.

c) The total energy stored in a substance by virtue of changes in the kinetic energy of center of mass motion, rotational and vibrational oscillations and changes in potential energy of interaction is defined as the thermal energy of the substance; only that fraction associated with the kinetic energy of the motion of the centers of mass of the molecules can be identified with the temperature of the substance.

EMPHASIS Should be treated fully. Even if time does not permit a thorough treatment, you should be certain that students understand the basic difference between thermal energy and temperature.

COMMENT In this section, we are at the very heart of the Kinetic Theory of Gases, one of the great achievements in physics. We have shown (or at least presented plausibility arguments in lieu of more complicated but exact proofs) that the bulk thermal properties of matter can be described exactly in terms of the dynamics of the motion of the individual molecules comprising a substance. Yet some students may feel that, after all, we have defined the equivalence of temperature and kinetic energy by our theory--so why should it be surprising that they turn out to be equivalent! The answer

is that there is nothing at all in dynamics which corresponds to the temperature. The only parameters we had to consider were things like mass, force, speed, acceleration, momentum, energy, etc. The remarkable feat of the theory is that we have been able to identify such a complicated macroscopic property as temperature with so simple a microscopic dynamical property as the kinetic energy of motion of the centers of mass of the molecules.

Of course, the whole of kinetic theory would be purely formalistic if it were not possible to confirm the model by direct observation. There are several easily verifiable predictions which have been found to be true. Some of these have been discussed earlier in Volume 1, Chapter 9, but might well be reviewed in the present context. For example, from the simple relation $kT = \frac{2}{3} \frac{(mv^2)}{2}$, we can conclude:

1) That the average speed of motion of molecules of mass m at absolute temperature T is $\bar{v} = (\frac{3\,kT}{m})^{\frac{1}{2}}$. Typically, for hydrogen

(H_2, $m \approx 3 \cdot 10^{-27}$ Kg) at room temperature ($T \approx 300^\circ K$),

$v \approx (\frac{3 \cdot 1.37 \cdot 10^{-23} \cdot 3 \cdot 10^{2}}{3 \cdot 10^{-27}})^{\frac{1}{2}} \; 2 \cdot 10^{3}$ m/sec, a bullet-like speed which can easily be measured by letting molecules escape one by one into a vacuum system;

2) For a collection of molecules of different masses, all at the same temperature, the relative average speeds will be inversely proportional to the square root of the mass--a fact which is verifiable by measuring the relative rates of diffusion through a porous membrane (this is the basis for the separation of uranium isotopes by the gaseous diffusion process used at the great plants at Oak Ridge).

We can also find in the kinetic theory the answers to seemingly trivial questions, such as "If gas molecules have mass, why don't they all fall down to the floor in a gravitational field?" To answer such a question, we may ask ourselves "if a molecule strikes the floor with a speed v and rebounds elastically, how high will it bounce?" From previous considerations, we know that at a speed v, the molecule has a kinetic energy $\frac{1}{2}mv^2$. If it is moving upward with the initial speed v after hitting the floor, it will move to a height L such that the gravitational potential energy, mgh, is equal to the initial kinetic energy. Thus:

$$\frac{3}{2}kT = \frac{1}{2}mv^2 = mgh$$

$$h = \frac{v^2}{2g} = \frac{\frac{3}{2}kT}{mg}$$

Using the first expression, assuming g to be constant, and employing our earlier result for $\bar{v}$ for hydrogen ($2 \cdot 10^3$ m/sec.), we see that the molecule would rebound to a height of about

$$\frac{(2 \cdot 10^3)^2}{2 \cdot 10} \approx 2 \cdot 10^5 \text{m},$$

which is some bounce! If we consider the variation of h, it would be even larger. Thus, our molecules would indeed fall to the "floor", but only if we are considering a "room" about 1000 km high!

Our model has also taken us far beyond our original premises, at least in principle. Not only can we completely explain the thermal properties of simple gases, but we can also understand why "un-simple" substances like polyatomic gases and liquids and solids exhibit different thermal behavior. We can actually account for the differences quantitatively for the polyatomic gases. However, for condensed systems, we are limited by the extent of our knowledge of the actual forces of interaction between the molecules which comprise a "many-body" system.

Actually, the extension of the theory to polyatomic gases is reasonably simple. We note from the equation

$$\frac{3}{2} kT = \frac{1}{2} mv^2$$

that we have a thermal energy $\frac{1}{2}$ kT associated with each of the three degrees of freedom of motion of the center of mass of the molecule. If we consider a simple diatomic molecule, like H_2, there are two more degrees of freedom which the molecule can have associated with two possible independent rotations of the two-actions about their center of mass. (There are more degrees of freedom associated with vibration of the two atoms along the line joining their centers←○→←○→, but this motion is not usually excited at room temperatures.) Into each of the awakened degrees of freedom we must put an additional amount of kinetic energy $\frac{1}{2}$ K(ΔT) in raising the temperature through ΔT degrees. Thus the specific heat of the simple diatomic gas must be

$$\frac{\frac{5}{2} K \Delta T}{\frac{3}{2} K \Delta T} = 5/3 \text{ times that of the monatimic gas,}$$

or $\frac{5}{3} \cdot 12.4 = 20.6$ joules per mole per deg. K.

For more complicated gases we proceed in an identical manner, by determining the total number of degrees of freedom associated with all possible separable motions of the molecule and associating a thermal energy $\frac{1}{2}$ kT with each degree of freedom. The "specific heat" of the ideal gas is always simple the ratio of an incremental change in thermal energy to the resulting incremental change in temperature. Thus, for a system with a total of n degrees of freedom, the specific heat per mole (Avogadro's number, N) is simple $\frac{1}{2}$ n Nk. The quantity Nk is frequently designated by the symbol R, and is called the "gas constant". Since both Avogadro's number, N, and Boltzmann's constant, k, are universal constants, R is also a universal constant equal to 8.36 joules per mole per deg. K.E. (1.98 calories per mole per deg. K).

The words "ideal gas" recur frequently in discussions of the kinetic theory. An "ideal" gas is one in which the molecules are approximated to be geometrical points which interact only through very short range forces, by colliding with one another in the manner of perfectly elastic spheres. This is our basic model for the kinetic theory, and is approximated by simple gases at low pressure.

Of course, if molecules actually did not interact through long-range forces, we would never see matter in the condensed liquid and solid states. Thus we know that the "ideal" gas model is obviously only an approximation. One of the most striking evidences of the presence of long-range forces between molecules is in the drop in temperature of a gas which is suddenly allowed to expand freely by passing through a nozzle into a large chamber. This phenomenon is usually called the Joule-Thompson effect, and is the basis for most refrigeration machines. If there were no long-range interactions between the molecules, their average kinetic energy should not be affected by the distance between them, so there should be no temperature changes associated with a free expansion.

Section 3 - Mechanical Energy of Bulk Motion and Internal Energy

PURPOSE Prologue to the law of conservation of energy by consideration of the storage of energy in "invisible" internal degrees of freedom in simple mechanical systems.

CONTENT a) While the total linear momentum of an extended object can always be determined exactly from the motion of the center of mass, the total energy may depend, in addition, on "invisible" motion of parts of the system with respect to the center of mass.

b) Loss of mechanical energy in an interaction is frequently accompanied by evolution of heat.

c) It is reasonable to suppose that thermal and mechanical energy are, in some way, equivalent.

EMPHASIS This section should be treated briefly, in conjunction with sections 4, 5 and 6.

COMMENT This material should go over quite smoothly. There is one point which seems obvious when it is once understood, but which may be puzzling at first encounter. This regards the question of changes of internal potential energy. As the text states, changes of this type are associated with changes of molecular configurations. If, on the microscopic scale, the average distance between molecules is increased, then on a macroscopic scale the volume occupied by a given amount of a substance will increase. Thus when water boils, at atmospheric pressure, energy is added to the water at 100° C. and the temperature does not change. The energy, instead, does work in separating the molecules against the forces of intermolecular attraction. The change of state which occurs gives, as its product, water vapor. The volume of a given mass of water vapor is much greater than the volume of an identical mass of water.

Section 4 - The Equivalence of Mechanical and Thermal Energy

PURPOSE To describe Joule's critical experiment proving the equivalence of mechanical energy and heat.

EMPHASIS Students should understand the nature of Joule's apparatus, (Fig. 3, p. 7-7), but no special emphasis need be devoted to this section. Treat as part of a longer assignment.

Section 5 - Heat Flow

PURPOSE Extension of the previous section to include cases involving heat flow, to show the equivalence of direct and indirect heating.

EMPHASIS Qualitative material. Treat briefly. Avoid extending the present simple discussion to the general phenomenology of heat flow, mixtures, conduction convection, radiation *et al*. All of these are interesting subjects, but a sufficiently detailed presentation would be diversionary and time-consuming.

Section 6 - Quantitative Relation of Energy Dissipation and Temperature Rise

PURPOSE To introduce the mechanical equivalent of heat (4.185 joules to raise 1 gm of water from 14.5° C to 15.5° C).

EMPHASIS Important material, but a brief treatment should suffice.

COMMENT In order to emphasize the magnitude of the mechanical equivalent of heat, ask students to work out a few simple energy balance equations. For example, "How far can a 50 kg man climb on one slice of bread?" Assume we have about 100 Calories or about $4.2 \cdot 10^5$ joules in the piece of bread. Then the man can increase his potential energy by an amount $mgh = 4.2 \cdot 10^5$ joules "on one slice of bread". Solving, $h = \frac{4.2 \cdot 10^5}{50.9 \cdot 8} \approx$ 850 m, quite a fair climb! (Overweight students may see why it is better not to have eaten that "one slice of bread" than to try to exercise it off!) See problem 24.

Students (at least those who do not exercise violently) may wonder where all the food energy in the 2000-odd Calorie daily intake goes to. Fortunately (for the lazy ones), most of it is consumed in just keeping us alive and warm, keeping our heart and lungs going and our brains clicking away. For an average person, this basic amount comes to about 100 watts (10^2 joules/sec or about $8.6 \cdot 10^6$ joules per day). Since 2000 Calories is about the same amount, we can hold our own without climbing ten mountains a day.

Section 7 - Conservation of Energy

PURPOSE To discuss the reversibility of mechanical-thermal energy conversion, and the generality of the law of conservation of energy.

EMPHASIS Important material. Best handled by numerical exercises.

COMMENT Avoid becoming involved in a quantitative treatment of the second law of thermodynamics, Carnot engines, refrigerators, etc. These are all fascinating subjects, but a detailed discussion would be too time-consuming.

Many large factories maintain a complete energy balance to know how they are spending their money. A power plant using river cooling usually knows the input and output temperatures of the cooling water, the temperatures of stack gases, the heating of generators and the energy rating of their fuels. Of course they also keep a strict record of the electrical energy delivered to their customers. Some students might be interested in visiting a local power plant to see how such an "energy accounting" is carried out.

Appendix - p. 7-11

PURPOSE An extension of section 1 for mathematically-inclined students. Not to be treated as general class work.

EMPHASIS Reading assignment only.

HISTORICAL BACKGROUND The story of man's quest for an understanding of the nature of heat is a fascinating one. Of course, people knew about heat long before concepts about energy were formulated. Early speculations concerning the nature of heat rested on two rival hypotheses: first, that heat was somehow connected with motion (fire by friction, etc), and second, that heat is a material substance.

In 1620 Francis Bacon stated in his Novum Organum that "Heat itself . . . . is motion and nothing else." Later in the Seventeenth Century both Robert Boyle and Robert Hooke expressed similar ideas, but the theory held little weight with the majority of the scientists of the following century mainly because no one was able to explain why, if heat were motion, it would be conserved in experiments where two bodies, initially at different temperatures, reach thermal equilibrium when placed in contact.

In general, during the Eighteenth Century, a theory of heat came into use which assumed heat to be a subtle, elastic fluid present within any "hot" body. The particles of the "fluid" supposedly repelled each other, but were attracted to the particles of ordinary matter. This "fluid" heat came to be known as "caloric", and the concept of heat as a material substance was accordingly called "the caloric theory".

The foundation of the caloric theory was the notion that heat is conserved. The majority of the observations and experiments of the calorists were performed under limited conditions which allow the total quantity of heat to remain invariable, and which suggest that heat is a conserved quantity. It was thus convenient to think of heat as a substance which could neither be created nor destroyed, but which could flow from one material to another.

It is of interest to examine explanations for heat phenomena offered by the calorists. They assumed, as was customary at that time, that the basic units of matter were impenetrable. Despite the mutual attractions of the particles of the substance, they argued that these particles could not be in actual contact; otherwise contractions could not take place when a body was compressed. Therefore, there must be some counterbalancing repulsive force, and this force was due to the caloric present between the particles. Because of the self repulsion of the caloric substance, heat would flow from a hot material to a cold one. Whether a substance existed in the solid, liquid, or gaseous state depended upon the quantity of caloric which entered into its composition. When it contained large amounts of caloric, the substance would assume the form of a gas; solids and liquids would contain less caloric and thus occupy less volume. As a substance was cooled it was thought to be drained of caloric; the removal of caloric which supplied the repulsive force explained the fact that most substances contract upon cooling.

Although the caloric theory of heat is no longer used, some of its language has been carried over into the modern description of heat. This is particularly convenient when one discusses heat "flow" and "transfer" of heat. We still speak of an object "soaking up heat". And, of course, our unit of heat, the calorie, was originally used as a measure of caloric substance.

Count Rumford's (1753-1814) experiments, begun about 1787, were purposefully designed to attack the caloric theory at its very foundations. Rumford set about, at first, to show that heating a body did not increase its mass. One might argue that if caloric were matter, then it must possess the fundamental property of all matter: it must have mass. By a series of extremely careful experiments, he showed that the weight of a body did not depend on its temperature. However, this offered no serious challenge to the calorists. They merely answered that caloric was not ordinary matter, and was therefore not necessarily subject to gravitational forces.

Rumford then turned toward experiments with the heat produced by friction. While supervising the cannon boring operation at the Munich arsenal, he noticed the very high temperatures attained by a cannon while it was being bored.

On the basis of the caloric theory, the attractive force that was supposed to exist between the molecules of the metal and caloric must have been diminished when the metal was broken into chips releasing caloric which appeared as heat. However, the heat released in these experiments seems to be inexhaustible. From this, Rumford concluded: "It appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner in which heat was excited and communicated in these experiments, except it be MOTION."

The physical basis upon which the caloric theory was founded was the notion that heat was a conserved quantity. It was just at this point that critics of the theory attacked. They showed that in friction experiments, such as Rumford's cannon, heat could be produced in seemingly endless quantities.

Apparently the first person to realize how important an identification of heat as energy could be was Julius Robert Mayer, who lived from 1814 to 1878. Mayer had the idea that energy could not be destroyed and that one form of energy was in some sense equivalent to another. He pointed out:

> "In numberless cases we see motion cease without having caused another motion or the lifting of a weight; but an energy once in existence cannot be annihilated, it can only change its form; and the question therefore arises, what other forms is energy . . . . . capable of assuming?"

He argued that since work can be converted to heat, heat must be a form of energy.

> "If potential energy and kinetic energy are equivalent to heat, heat must also naturally be equivalent to kinetic energy and potential energy."

Here, Mayer made his most penetrating observation. If heat was just a converted form of kinetic or potential energy, and if energy as a whole is conserved, then a given quantity of heat must be the result of a proportional amount of mechanical energy. Work done on an object must produce a proportional amount of heat in the object; if its kinetic and potential

energy is unchanged. From experiments already done on gases, Mayer was able to derive a quantitative relation between mechanical work and heat which is in fairly good agreement with the best values that have been obtained since. The definitive work of James Prescott Joule (1818-1898) came a bit later, and spelled the end of the caloric theory.

For Home, Desk, and Lab

GENERAL A number of the exercises deal in a qualitative manner with energy transformations involved in systems of interest to the students, like gasoline engines, a machine gun. If you are pressed for time, a discussion of some of these problems will be the most effective way to bring home the concept of heat as kinetic energy of molecular motion. The exercises most suitable for this purpose are 3, 5, 11, 21, and 22, and they are underlined in the table. Exercises 8 and 9, involving the behavior of an ideal gas, and exercises 17 and 24, concerned with the mechanical equivalent of heat, are very interesting, if you have time for them.

Section	Easy	Medium	Hard	Class Discussion	Home Experiment
1		1,2	3	3	
2	5	4*, 6, 7	8, 9	5	
3	10				
4					13, 14, 15**
5-6	12, 18, 19	20,24			
7	17	11, 22, 26	21, 23, 25	11, 21, 22	16, 27, 28

* Contains a misprint. See discussion.
** Very difficult to obtain the desired result with home equipment. See discussion.

The first three exercises are intended to drive home the concept that pressure is due to the collision of molecules with the walls of the container. However, all three exercises deal with inelastic rather than elastic collisions, so the formulas obtained will differ from those derived in Section 2 by factors of two in some cases. In any case, do not stress these formulas, but rather the physical results that the pressure depends on the momentum and the flux of the incident stream of particles, hence on the kinetic energy density in the stream.

EXERCISE 1: Water issues from a fire hose in a horizontal jet of diameter 10 cm. The water emerges with speed 30 m/sec. A fireman plays this stream on a vertical wall. Calculate the pressure on the 10-cm-diameter circular patch of wall that the stream hits.

The first problem illustrates the fact that the pressure is proportional not only to the momentum of each molecule, but to the flux of molecules. It does this by using the analogy with a jet of water.

The water has a velocity 30 m/sec., and if we assume that on collision with the wall it comes instantaneously to rest, then 1 kg of it loses momentum

$$mv = 30 \text{ kg m/sec.}$$

In each interval of one second the mass of water which actually hits the wall is the mass contained in a cylinder of diameter 10cm = 0.1m, and length 30 m. (This is the volume of water which passes any point in one second.) This volume is

$$\pi \frac{0.1}{2}^{2} \times 30 \text{ m}^3 = 0.24 \text{ m}^3.$$

The density of water is 10^3 kg/m^3, so the mass of this volume of water is 0.24×10^3 kg. Thus in one second the momentum imparted to the stream by this area of the wall is

$$m\Delta v = 0.24 \times 10^3 \times 30 \text{ kg.m/sec.} = 0.72 \times 10^4 \text{ kg.m/sec.}$$

The force required to produce this much change in momentum in one second is given by $F\Delta t = m\Delta v$ where $\Delta t = 1$ sec, so

$$F = 0.72 \times 10^4 \text{ kg.m/sec}^2 = 0.72 \times 10^4 \text{ nt.}$$

The pressure is the force exerted by the wall per unit area. The area is $\pi \frac{0.1}{2}^{2}$ m^2, so

$$P = \frac{2.8 \times 10^4}{\pi (0.1)^2} \text{ nt/m}^2$$

$$= 9.0 \times 10^5 \text{ nt/m}^2.$$

This pressure is fairly large (about 10 atmospheres). That is why fire hoses are used not only for putting out fires, but also to disperse riotous mobs of students. One way to mine gold which lies near the surface of the earth is to direct a high-pressure hose at a hillside. The surface layers of earth and (hopefully) gold nuggets are rapidly eroded away. The resulting sludge can then be separated.

Improved Solution

Notice that the area of the jet of water appeared twice in the calculation: in calculating the flux of water and in obtaining the pressure from the force. Consider a jet of area A. The flux of water per second is

$$A \times 30 \times 10^3 = 3 \times 10^4 \, A \text{ kg}.$$

The change of momentum in one second is

$$3 \times 10^4 \, A \times 30 = 9 \times 10^5 \, A \text{ kgm/sec},$$

the force is therefore 9×10^5 A nt, and the pressure is 9×10^5 nt/m^2. Thus the area is immaterial: the pressure on the patch of wall that the stream hits depends only on the velocity of the jet, and its density. ($P = \rho v^2$ section 1 of Chapter 7.)

EXERCISE 2: A machine gun fires a stream of 10-gram bullets at the rate of 400 rounds per minute. The bullets, moving at velocity 300 m/sec, hit a wall of solid rock and stop dead. Calculate:

(a) The force on the wall.

(b) The kinetic energy of the bullets arriving at the wall in one minute.

(c) The kinetic energy of the bullets in 1 meter length of the stream as they approach the wall. Compare twice this answer with your answer to (a).

Very similar to (1) from the point of view of physics(!). We have merely traded water for bullets. This exercise illustrates the fact that we get the same average result when the mass comes in small bundles.
(a) The flux of bullets is 400 per minute, or 6.67 per second. Each bullet loses an amount of momentum $mv = 10^{-2}$ kg x 300 m/sec = 3 kg.m/sec

The change of momentum of the stream of bullets in one second (400 bullets/min = $6\frac{2}{3}$ bullets/second) is

$$\text{flux} \times mv = 6.67 \times 3 \text{ kg.m/sec} = 20 \text{ kg.m/sec}.$$

The average force required to produce this change in momentum is given by

$$F \Delta t = \text{change of momentum}$$

where Δt is one second, so

$$F = 20 \text{ kg.m/sec}^2 = 20 \text{ nt.}$$

(b) The kinetic energy of each bullet is

$$\frac{1}{2} mv^2 = \frac{1}{2} \times 10^{-2} \times (300)^2 \text{ joules} = 450 \text{ joules}$$

In one minute 400 bullets hit the wall. Their total kinetic energy was

$$450 \times 400 \text{ joules} = 1.8 \times 10^5 \text{ joules.}$$

(c) The bullets are fired at the rate of $6\frac{2}{3}$ every second, i.e. one every $\frac{3}{20}$ second. The distance that one bullet will have travelled by the time the next is fired is therefore

$$\frac{3}{20} \times 300 \text{ m} = 45 \text{ m.}$$

The bullets in the stream are 45 m apart! On the average, one meter of the stream of bullets will contain $\frac{1}{45}$ of a bullet, so its kinetic energy will be $\frac{1}{45}$ of the kinetic energy of a bullet i.e. $\frac{1}{45} \times 450 = 10$ joules. Twice this is 20 joules. The force on the wall, obtained in (a), is 20 nt. This result corresponds to the fundamental relation of section (1):

$$P = 2 \frac{(\text{Kinetic energy})}{\text{Volume}}.$$

The correspondence between these numerical results can be multiplying both sides of the above equation by an area, A, of the wall on which the pressure, P, is being exerted

$$PA = F = 2 \frac{(\text{Kinetic energy})}{\text{Volume}} A = 2 \frac{(\text{Kinetic energy})}{\text{length}}$$

Thus the average force on the area A is equal to twice the kinetic energy per unit length in the molecules of gas (or bullets) which are set to strike that area, A.

EXERCISE 3: Suppose there is a tiny sticky area on the wall of a container of gas. Molecules hitting this area stick there permanently.

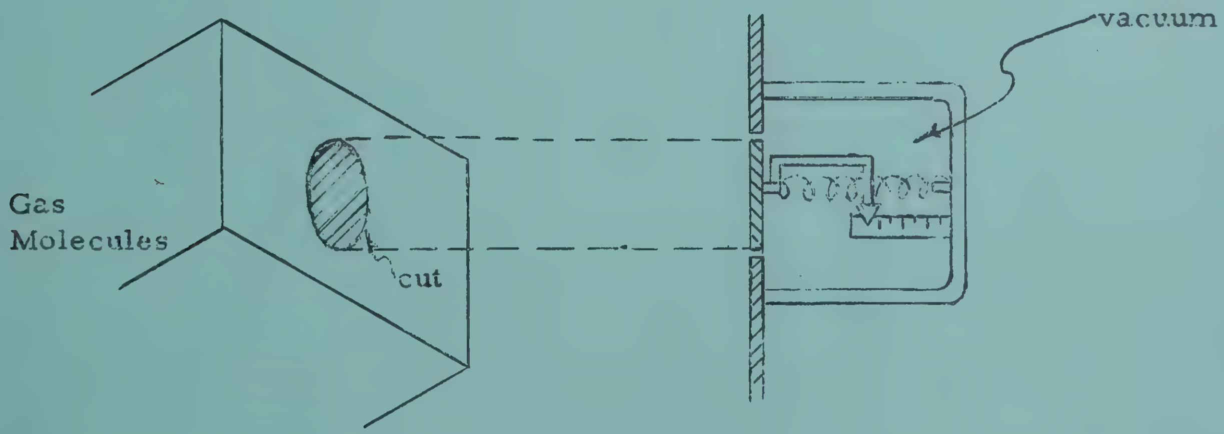
(a) Is the pressure there greater or less than on the ordinary areas of the wall?

(b) If the sticky region is cut out, leaving a hole, what is the pressure on the hole?

To stimulate thinking of gases in terms of motion of molecules, two hypothetical situations are presented which can be understood most easily in those terms.

The exercise is intentionally ambiguous. The answers depend on what you adopt as the definition of pressure, i.e. the way in which you would actually measure it. Two alternative definitions are given, of which you may wish to discuss only the first.

It is implicit in this chapter that the way the pressure on a wall is measured is to cut around a small area of the wall, so that is is free to move. (The cut is so narrow that no molecules escape.) This piece of wall is then connected to a spring balance, so that the force on it can be measured. The pressure is then calculated as Force/Area. (The space around the spring balance is evacuated, so that there is no counter-acting pressure.) On this assumption the answers are as follows:



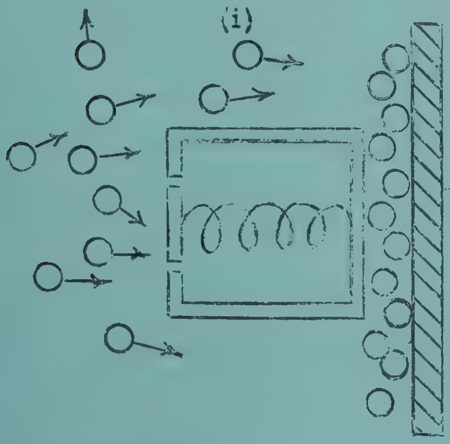
(a) If the excised piece of wall is just like the rest of the wall, the force it must exert on the stream of molecules approaching it must be sufficient to reverse their component of momentum perpendicular to the wall. If it is sticky, so that molecules that strike it stay on its surface, the change in momentum of the molecules in halved, and the force, and consequently the pressure, experienced by the sticky piece of wall will be halved.

(b) Our definition of pressure does not encompass this situation. The molecules approaching the hole suffer no change in momentum, but there is now no measuring device for us to obtain the value of the pressure. (We have defined the "pressure on the wall", and here there is no wall.)

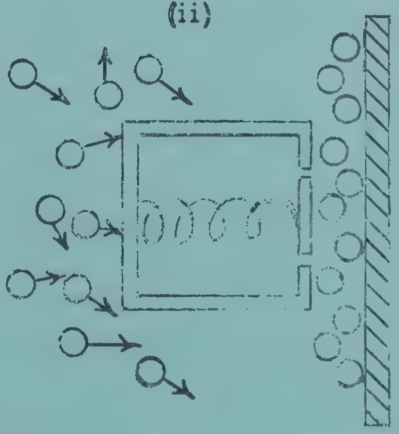
Alternative Definition

The pressure on the walls of the container depends on the mass and average velocity of the gas molecules, and these quantities are the same throughout the volume of the gas. It is useful to extend our definition of pressure so as to include the concept of the pressure of the gas, a property possessed by gas throughout its volume. We now imagine our pressure-measuring device to be a small, evacuated box with a movable piece of wall connected to a spring balance, very similar to our earlier apparatus. This box is placed at any point in the volume of the gas, and the pressure on the piston measured in some way. Our answers to the exercise are now a little different:

(a) We place our pressure-measuring box close to the sticky part of the wall. Molecules are approaching the wall, but do not leave it. The pressure measured depends on the orientation of the box!



(i)



(ii)

If the piston faces away from the wall, the molecules which hit its surface have the same average velocity as the molecules everywhere else in the container, and the pressure measured is the same as that obtained throughout the volume. On the other hand, if the piston faces towards the sticky part of the wall, no molecules ever hit it, since they do not rebound from the wall. The pressure measured is zero.

If a gas is in equilibrium, the pressure must be the same for all directions of the box, since exchange of momentum by collision between molecules will ensure that the average momentum is the same in all directions. The situation we have here is clearly not one of equilibrium, since eventually all of the molecules would be stuck to this part of the wall.

(b) As in part (a), the force on the piston will depend on the orientation of the box. If the piston faces into the container the pressure measured will be the same as in the interior of the container. If it faces out of the hole, no molecules will hit it, and the pressure measured will be zero. This is not surprising, since as in (a) the system is not in equilibrium.

Do not go into detail about the construction of the pressure-measuring box, or about how to read the pressure when it is inside the container and you are outside. The object of the exercise is to start the class thinking of a gas as a collection of molecules, to which the laws of motion of particles can be applied.

EXERCISE 4: In a certain gas 2/5 of the energy of the molecules is tied up in motion of the atoms around each other and 3/5 in motion of the centers of mass.

(a) On the average, what is the kinetic energy of the center of mass motion of one such molecule when the temperature is $300^{\circ}K$?

(b) If the temperature is raised 1° C, what energy must be supplied to a mole (0.6025×10^{19} molecules) of the gas?

A numerical problem on energy content of an ideal gas.

Note: This problem contains a misprint. A mole is 0.6025×10^{24} molecules, not 0.6025×10^{19}.

(a) The temperature measures only the average kinetic energy of the center of mass of the molecules. Numerically,

$$kT = \frac{2}{3}\left(\frac{1}{2}mv^2\right)$$

where $k = 1.37 \times 10^{-23}$ joules/°K. Thus at 300° K (approximately room temperature) the average kinetic energy of the center of mass of a molecule is

$$\frac{1}{2} mv^2 = \frac{3}{2} kT = \frac{3}{2} \times 1 \cdot 37 \times 10^{-23} \times 300 \text{ joules}$$

$$= 6.2 \times 10^{-21} \text{ joules.}$$

This is the value quoted in the text (section 2).

(b) We are told that this is only 3/5 of the actual kinetic energy. The other 2/5 exists in the form of rotational kinetic energy. When we raise the temperature of the gas by 1°K, the center of mass K.E. of one molecule will increase by $\frac{3}{2}$ k.

The total kinetic energy of one molecule will therefore be increased by

$$\frac{5}{3} \times \frac{3}{2} k = \frac{5}{2} k.$$

Since there are $.6025 \times 10^{24}$ molecules in a mole of gas, an amount of energy equal to

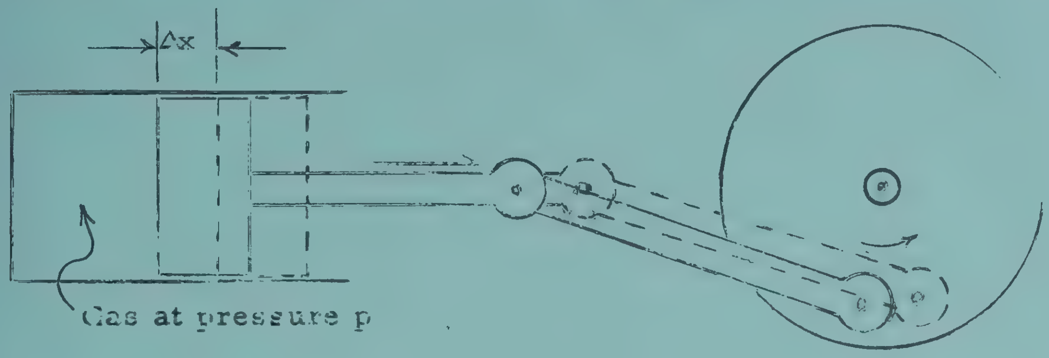
$$.6025 \times 10^{24} \times \frac{5}{2} k \text{ joules} = 20.6 \text{ joules}$$

must be supplied to raise its temperature by 1°K.

In the text (section 2) the value 12.4 joules is quoted for the energy to be supplied to the center of mass motion of the molecules in a mole of gas to raise its temperature 1°K. Because of the extra rotational kinetic energy required in our case, this is only $\frac{3}{5}$ of the result we get.

EXERCISE 5: A gas in a cylinder pushes a piston out, increasing its volume by ΔV. The face of the piston has an area A and the gas exerts a pressure P on it. The piston transfers energy $F \Delta x$ from the gas to some outside machinery, where F is the force exerted by the gas on the piston and Δx is the distance it moves out. Show that the work $F\Delta x$, is equal to $P\Delta V$ (This is a very useful expression for work whenever we deal with a gas or liquid pushing a piston. Work = pressure times change in volume.)

A qualitative exercise on molecular behavior of a gas. It can be used to discuss, from the molecular point of view, exactly how work is done on or by a gas.



This problem can be answered at various levels of insight. On the surface, making no inquiry about the constitution of the gas, it is almost trivial: the force F on the piston is just PA where A is the area of the piston. When the piston moves outwards through a distance Δx, the work done is FΔx (the force experienced by the machine times the distance through which the force moves). Now FΔx = PAΔx, and from the geometry of the piston, AΔx is just the volume swept out by the end of the piston: it is the increase in volume of the gas. Thus FΔx = PΔV.

Where did the energy come from? This depends on what else happened to the gas while the piston moved. If the cylinder was completely insulated from other sources of heat (i.e. energy) then the energy must have come from the gas itself, i.e. from the kinetic energy of the individual molecules. What was the exact mechanism of the energy transfer? When molecules collide with the walls of the cylinder, or with the piston when it is stationary, their perpendicular component of momentum is reversed, the other components are unchanged, and consequently their kinetic energy is unchanged. But when molecules collide with the piston moving outward, their perpendicular component of momentum is reduced in magnitude, and their kinetic energy is reduced. (It is analogous to the case of a rubber ball which is bounced off a wall which is free to recoil. The ball's speed will be reduced by the collision.) By collision with other molecules, this loss in energy is soon shared with all of the molecules in the gas. The average kinetic energy is reduced, which means that the gas temperature is lowered.

Another possibility is that the cylinder is in contact with some other source of heat energy at the same temperature. In this case the tendency of the gas temperature to reduce is counteracted by a flow of heat from the other source (due to the temporary temperature difference). Thus the final temperature of the gas in the cylinder will be the same as it was initially-consequently the total kinetic energy of the molecules is unchanged-but energy has been supplied by the heat source. We have changed heat into work. [This fact does not conflict with the second law of thermodynamics, because we have only considered a small part of the cycle process necessary to do this continuously. When the rest of the cycle is considered, we find that not all of the heat can be turned into work. Some of it must be passed on to a heat source at a lower temperature.]

EXERCISE 6: One mole of an ideal monatomic gas (in practice, helium or argon) is placed in a cylinder at temperature $273^{\circ}K$. The gas is at atmospheric pressure, 1.02×10^{5} newtons/m^2. At this pressure and temperature the gas occupies 2.24×10^{-2} m^3. A piston in the cylinder is then pushed in to decrease the volume by 2.45×10^{-4} m^3.

(a) How much mechanical work must be done to push the piston in? Neglect the change in pressure. (Why can you?)

(b) What is the final temperature of the gas if the container is completely insulating? (Remember 12.4 joules of energy raise its temperature one degree.)

Exercises 6, 7, 8 and 9 provide practice with the ideal gas "equation of state".

(a) The amount of work done by some machine in pushing the piston in a small distance is $P\Delta V$ where ΔV is the decrease in volume. The argument is the same as that of problem (5), except that the work must be done by some external agency.

$$P\Delta V = 1.02 \times 10^{5} \times 2.45 \times 10^{-4}\ (\text{nt/m}^2)\ (\text{m}^3)$$

$$= 25.0 \text{ joules.}$$

We have neglected the change in pressure, i.e. we have assumed that the pressure of gas stayed at its initial value of 1.02×10^{5} nt/m^3. In fact it will change. The order of magnitude of the change can be obtained by looking at our "equation of state" for the gas, $PV = NkT$, and assuming that T does not change appreciably. (In part (b) we shall verify this assumption.)

Then in some approximation, PV is constant during the compression, and the relative increase in P must be equal to the relative decrease in V. Since $\Delta V/V$ is 10^{-2}, the pressure therefore changes by about one percent during the compression. Our error in neglecting the change in P is of this same order of magnitude, i.e. one percent.

(b) If the container is completely isolated, then all of the work done on the gas in compressing it (25.0 joules) must be transformed into kinetic energy of the gas molecules. The molecules are in fact single atoms, with no kinetic energy of rotation, so the energy required to raise the temperature of one mole of the gas 1°C is just 12.4 joules. Thus our 25 joules will raise its temperature by $\frac{25.0}{12.4}$ °C, i.e. a fraction over 2°C.

The temperature change is less than one percent, and the approximation made in calculating $\Delta P/P$ in (a) is justified. (Actually, we have oversimplified this problem somewhat, by calculating the changes in pressure and temperature as though they were unrelated. If no heat flows in or out of the gas when work is being done, the reaction is termed adiabatic, and is governed by the "equation of state" PV^{γ} = const, where γ is a constant about equal to 1.7 thus $\frac{\Delta P}{P} = \gamma \frac{\Delta V}{V}$, not $\frac{\Delta V}{V}$. However, the term is still small and can be neglected in the present exercise. We consider this effect in the next change?

EXERCISE 7: In Problem 6, by what fraction of its original value does the pressure change?

We justified our neglect of the change in pressure, in problem (6), by assuming that there was no temperature change. Since the effect would in any case affect the calculated work done on the gas by about one percent, this was sufficiently accurate. Now, however, we know that the temperature did change: it increased by $\Delta T = 2°C$. To obtain the change in pressure, ΔP, we must make a more careful calculation, allowing for the simultaneous changes in T and V. We use the equation of state for the gas, $PV = NkT$.

When V is decreased by ΔV, and T is increased by ΔT, P is increased by ΔP. The new state of the gas still satisfies the equation of state:

$$(P + \Delta P)\,(V - \Delta V) = Nk\,(T + \Delta T)$$

$$\text{i.e. } PV + V\Delta P - P\Delta V - \Delta P\Delta V = Nkt + Nk\Delta T.$$

We neglect the $\Delta P\Delta V$ term. Whatever ΔP turns out to be, this term will be small compared with those we are retaining, and will involve corrections to ΔP of order one percent, which we can neglect. We divide the left side of the above equation by PV and the right side by NkT, these two factors being equal because of the equation of state. Then

$$1 + \frac{\Delta P}{P} - \frac{\Delta V}{V} \simeq 1 + \frac{\Delta T}{T}\ ,$$

$$\text{i.e. } \frac{\Delta P}{P} = \frac{\Delta V}{V} + \frac{\Delta T}{T}$$

Since $\Delta V/V = 10^{-2}$, and $\Delta T/T = 2^\circ/273^\circ = 0.7 \times 10^{-2}$,

$$\frac{\Delta P}{P} = 10^{-2} + 0.7 \times 10^{-2} = 1.7 \times 10^{-2}$$

The relative change in pressure is 1.7%. We estimated it in problem (6a) to be about 1% neglecting ΔT, this was clearly adequate for our purpose.

EXERCISE 8: Suppose we take 18 grams of water (one mole) at boiling point and turn it into vapor in a cylinder, closed by a frictionless piston, which is so light that the gas remains at atmospheric pressure at all times.

(a) What volume will the vapor occupy if it behaves as an ideal gas? (Water vapor is not an ideal gas, but the error due to the assumption will be less than 10 percent.)

(b) What work must be done in pushing the piston out against atmospheric pressure as the whole of the water vaporizes?

(c) Given that each gram of water takes 540 calories to tear its molecules apart into vapor, how much heat is needed to convert the water to vapor and push the piston out?

(d) What fraction of the total heat needed is converted to work in pushing the piston out?

(a) We assume that water vapor behaves like an ideal gas, i.e. that it satisfies the equation of state $PV = NkT$. In this equation, P is now atmosphere pressure, 1.02×10^5 nt/m^2, T is 373°K, the temperature of boiling water, N is the number of molecules in a mole, $.6025 \times 10^{24}$, and k is 1.37×10^{-23} joules/°K,

Hence

$$V = \frac{NkT}{P}$$

$$= \frac{.6025 \times 10^{24} \times 1.37 \times 10^{-23} \times 373 \text{ m}^3}{1.02 \times 10^5}$$

$$= 3.0 \times 10^{-2} \text{ m}^3 .$$

(b) The work done in pushing the piston out a small volume ΔV was shown in problem (5) to be $P\Delta V$. Here we arrange to have P constant throughout the expansion, so the total work done is just Px (total change in volume). Since the initial volume is approximately 18 cm^3 (there are 18 gm of water, and its density is 1 gm/cm^3 or $1{\cdot}8 \times 10^{-5}$ m^3, we can neglect it in comparison with the final volume. Thus the total work done is

$$PV = 1.02 \times 10^5 \times 3.0 \times 10^{-2} \text{ joules}$$

$$= 3.1 \times 10^3 \text{ joules}$$

(c) Each gram of water requires 540 calories or $540 \times 4.2 = 2.27 \times 10^3$ joules to become vapor. The total energy needed for the 18 gm of water is

$$18 \times 2.27 \times 10^3 \text{ joules} = 4.1 \times 10^4 \text{ joules.}$$

The total energy required, including the work done when the vapor expands, is thus

$$(4.1 \times 10^4 + 3.1 \times 10^3) \text{ joules} = 4.4 \times 10^4 \text{ joules.}$$

(d) Only $\frac{3.1 \times 10^3}{4.4 \times 10^4} = \frac{1}{14}$ of the total energy required goes into pushing out the piston. The rest is required to tear the molecules of water apart i.e. to overcome the very strong attraction water molecules exert on each other in the liquid state.

EXERCISE 9: (a) Estimate the speed of oxygen molecules at room temperature from the following data: 32 grams of oxygen at room temperature (20°C) at one atmosphere pressure, 1.02×10^5 newtons/m^2, occupy $2.4 \times 10^{-2} m^3$.
(b) The same volume of hydrogen at the same temperature and pressure weighs only 2 grams. Estimate the average velocity of hydrogen molecules at room temperature.
(c) From your answer to (a), estimate the average velocity of nitrogen molecules at room temperature, within 10 percent.
(d) What is the average velocity of air molecules at room temperature correct within 10 percent?
(e) What is the velocity of oxygen molecules at room temperature and two atmospheres pressure?

(a) In Section 1, the following relation among P, V and the molecular kinetic energy was obtained:

$$P = \frac{N}{V} \left(\frac{2}{3} \cdot \frac{1}{2} mv^2\right)$$

This can be rearranged to give

$$V = \left[\frac{3PV}{NM}\right]^{\frac{1}{2}} .$$

We are given that P = 1 atmosphere = 1.02×10^5 nt/m^2, and V = $2.24 \times 10^{-2} m^3$. The other quantities in the expression can be obtained when we realize that Nm, the number of molecules in a mole of the gas, times the mass of each molecule, is just the mass of a mole of the gas. This is given to be 32 g = 3.2×10^{-2} kg. The average velocity is then given by

$$v = \left[\frac{3 \times 1.02 \times 10^5 \times 2.24 \times 10^{-2}}{3.2 \times 10^{-2}}\right]^{\frac{1}{2}} \text{ m/sec}$$

$$= \left[2.14 \times 10^5\right]^{\frac{1}{2}} \text{ m/sec.}$$

$$= 4.62 \times 10^2 \text{ m/sec.}$$

(b) For one mole of hydrogen under the same conditions, we can go through the same procedure. The only difference is that we must now use in our expression the mass of one mole of hydrogen, which is 2 gm = 0.2×10^{-2} kg. This is one sixteenth of the value for oxygen, and since our expression involves it to the (-1/2) power, the average velocity is four times that for oxygen, i.e.

$$v_H = 1.85 \times 10^3 \text{ m/sec.}$$

It is a little simpler to use the following slightly different argument: at the same temperature, the average kinetic energy of translation of hydrogen is the same as that of oxygen:

$$\frac{1}{2} m_H v_H^2 = \frac{1}{2} m_o v_o^2$$

$$v_H / v_o = \sqrt{m_o / m_H}$$

Since the masses are in the ratio one to sixteen, the velocities must be in the ratio $(16)^{1/2}$ to one, i.e. four to one.

(c) The mass of a mole of nitrogen is 28 g. Using the argument of section (b), the average velocity of the nitrogen molecules differs from that of oxygen by $(32/28)^{1/2} = 1.07$. Thus to within ten percent they are the same.

(d) Air is composed prodominantly of oxygen and nitrogen (in the ratio 20% : 80%). Since the velocities of their molecules are the same to 10%, the average velocity for air molecules is to this approximation the same as that obtained in section (a), i.e. $v_{av} = 4.6 \times 10^2$ m/sec.

(e) The kinetic energy of the air molecules depends only on T, so the average velocity will be the same even though the pressure is doubled, provided T is the same (which it is). In the expression used in Section (a), P is doubled, so that (because of the equation of state) V is halved, and PV stays constant.

Exercises (10), (11), (22), and (23) are qualitative discussions of the various forms that energy takes in actual physical processes. They can be thought-provoking and very instructive.

EXERCISE 10: A large bag of sand is hung from a tree by a long rope. A boy shoots a bullet into the sand bag and the bullet stays in the bag.

(a) Describe the energy changes.

(b) Suppose a 10-gram bullet is moving at 300 m/sec when it hits the bag, and the bag, is a 1990-gram bag of sand. Calculate:

(i) how much kinetic energy the bullet had originally.

(ii) how much the bullet and bag have after collision.

(iii) how much disappears.

(c) What fraction of the original kinetic energy of the bullet goes into heat?

(a) The complications of what happens to the energy in the gun, etc. are dealt with in more detail in problem (11). The main parts of the energy chain are

(i) Kinetic energy of bullet; this is changed to

(iia) Heat of bullet and sand surrounding it after it comes to rest, and

(iib) kinetic energy of the bag plus bullet. The bag swings, and we get the cyclic process of kinetic energy becoming potential energy, and vice versa. Very slowly, because of air friction, this will cease, and the energy will become

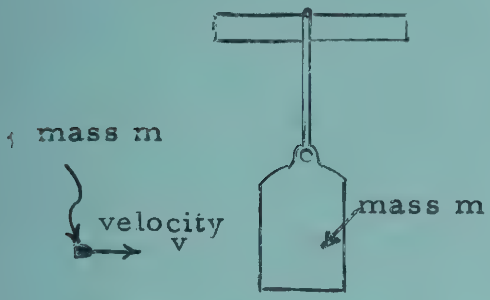
(iii) heat energy of the surrounding air.

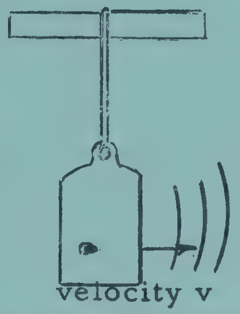
(b) (i) Originally the bullet has kinetic energy

$$\frac{1}{2} mv^2 = \frac{1}{2} 10^{-2} (3 \times 10^2)^2 \text{ joules}$$

$$= 450 \text{ joules.}$$

(ii) During the collision of the bullet with the bag (which is completed before the bag moves appreciably) momentum is conserved. (So is energy, so long as we include heat energy, but mechanical energy of bulk motion by itself, which is what mechanics considers, is not.)





If, after the collision, bullet and bag move with velocity V, then

$$mv = (m + M)V$$

$$10^{-2} \times 3 \times 10^{2} = 2V$$

$$V = 1.5 \text{ m/sec.}$$

The kinetic energy of the bag plus bullet is

$$\frac{1}{2}(M_b + M)V^2 = \frac{1}{2} \times 2 \times (1.5)^2$$

$$= 2.25 \text{ joules!}$$

(iii) Before collision, the bullet had a kinetic energy of translation of 450 joules. After collision, only 2.25 joules exists in the form of k.e. of bulk motion. 447.75 joules has disappeared from view. It has gone mainly into heat, of the bullet and the sand immediately surrounding it. A little goes into potential energy of separation of the sand particles disturbed by the passage of the bullet.

(c) If we assume that all of the "lost" energy goes into heat, this is a fraction of the initial kinetic energy.

$$\frac{447.75}{450} = 99.5\%$$

EXERCISE 11: A machine gun fires lead bullets at a wall and they stop dead. Describe the changes of energy from the stage when it is in the explosive to the stage when the bullets have been lying on the ground at the bottom of the wall for several hours.

The stages in the "chain" of energy transmutations are:

(i) Chemical energy of the gunpowder;
(ii) kinetic energy of thermal motion in the hot exhaust gases;
(iii) kinetic energy of bulk motion of the bullet, plus that of recoil of the gun;
(iiia) heat and sound in the air due to the passage of the bullet, and sound when it strikes the wall;
(iv) heat, and potential energy of deformation of the spent bullet, and the region of the wall hit;
(v) heating of the air in the vicinity as the bullet and wall cool.

We neglect bulk potential energy, assuming that the bullet's path is short, and fairly flat. In that case, and assuming that the bullet falls only a few feet after hitting the wall, it is negligible.

The transmutation (i)→(ii) involves the chemical combination of elements in the gunpowder into more complicated and more tightly bound molecules. The potential energy of the whole system is reduced, and the liberated energy appears as thermal motion of these molecules.

In the step (ii)→(iii) the expanding hot gases do work $\int PdV$ on the bullet to give it kinetic energy. Not all of their thermal energy is transmitted, however. When the bullet leaves the end of the barrel, the gases there just rush out, and mingle with the surrounding air, heating it up as they cool.

The intermediate step (iiia) reduces the kinetic energy of the bullet as it travels. The bullet travelling through the air generates a "shock wave", a very intense pressure pulse. When this passes through the air it leaves

it at a slightly higher temperature. As the shock travels further and further from the bullet's path its intensity decreases. The sound of a higher velocity bullet passing close by is a sudden sharp "crack", while that of a spent bullet (where the shock wave has lost much of its energy) is a more continuous, less intense whine.

The transmutation (iii)→ (iv) is the most sudden. The bullet may be brought to rest in a centimeter, whereas it was accelerated over a distance of almost a meter. The bullet is then completely deformed, and it is the violent relative motion of the lead which produces the heat.

All of this energy originated in the chemical process of explosion. Ask the class to consider where it really came from. You can generate a discussion of arbitrary length on the problems of extracting the elements from which gunpowder is made from the places where they are found naturally. And how did they come to be there? Everything can be traced to the sun's energy, which is far enough back for us to go. Exercises 12), 17 and (18) give quantitative practice with problems of energy transformations, and the mechanical equivalent of heat.

EXERCISE 12: In one of his most famous experiments, Joule churned water with a paddle wheel driven by two loads, each of mass 14 kg, each falling vertically about 2 meters. He had about 7 kg of water to be heated. After each churning he hauled his loads up and let them fall again. What temperature rise would you expect him to find after twenty falls? (Of course, this is a cruel reversal of the logic of Joule's great experiment. You know that 4.2 joules will always raise the temperature of a gram of water 1° C; Joule was trying to find out.)

In each fall, the two loads lose an amount of potential energy.

$$2\ mgh = 2 \times 14 \times 9.8 \times 2 \text{ joules}$$
$$= 550 \text{ joules.}$$

The process is repeated twenty times, so the total loss in potential energy is

$$20 \times 550 = 1.1 \times 10^4 \text{ joules.}$$

Assume that this is all transformed into thermal energy of the water. The amount of energy required to raise the temperature of 7 kg of water by 1° is $7 \times 10^3 \times 4.2 = 2.9 \times 10^4$ joules. Hence 1.1×10^4 joules supplied by the falling loads will raise the temperature by

$$\frac{1.1 \times 10^4}{2.9 \times 10^4} = 0.38^\circ \text{ C.}$$

This is only a very small rise in temperature. The experiment is in fact a difficult one to do accurately.

Where did the energy come from? It came from the poor laboratory attendant who had to keep lifting the loads after each fall!

EXERCISE 13: Make or buy a small spring of steel wire. Stretch it and let it go many times. Hold it to your cheek to feel whether it grows warmer. Make a similar spring of soft copper wire and repeat the trial. Copper wire is easily softened by heating with a gas flame. (If you overstretch the spring, just push it back into close coiling.)

A simple experiment, with results that are easy to observe. No appreciable heating of the steel spring will be detected, but the copper "spring" will become hot to the touch after ten or twenty rapid flexings. To avoid buckling of the copper spring, you can wind it around a smooth glass rod, and then flex the spring, keeping it on the rod. By observing that the glass rod does not heat up very much, you can verify that the heating of the spring is not caused by the friction of its rubbing on the glass. An even simpler demonstration of the effect is to take a piece of copper wire in your hands and try to break it in half by repeatedly bending it. Be careful when testing the temperature at the break: the copper can get hot enough to burn you!

An explanation of this phenomenon in terms of work done on the springs is as follows: the steel spring is (so far as we are concerned) perfectly elastic. The force you must exert on it depends only on the amount it is stretched or compressed, and not on whether you are stretching it or letting it go back to its normal unstretched position. Thus all of the work which you do on the spring in stretching it reappears as work done by it on you when it is allowed to unstretch. There is no net work done on it, and no energy is lost. Therefore no heat can be developed. With the copper spring, on the other hand, you stretch it, and it stays there. You have to exert a force on it to compress it again. Thus you do perform a net amount of work on it in one cycle, and this must appear as heat. The copper heats up.

EXERCISE 14: Swing a hammer fast and give several violent blows to a small block of lead on a firm anvil of stone or iron. Feel the lead before and after. Make very rough estimates of the kinetic energy lost by the hammer and of the thermal energy gained by the lead. (Assume that 1 kg of lead needs about 130 joules to raise its temperature 1° C.)

Another home experiment to illustrate the conversion of other forms of energy into heat. Ten or so sharp blows with the hammer will produce a distinct (of order 10°) rise in temperature. The explanation is very similar in principle to that given in exercise (13). All of the kinetic energy of the hammer (which comes from the student's muscles) is absorbed by the lead, which is deformed. This energy appears as heat.

In making a rough estimate of the kinetic energy lost by the hammer, it is easier (and more accurate) to estimate the force exerted on the hammer, thus the work done, and finally its kinetic energy at the end of the swing, rather than estimating the velocity of the hammer directly, and computing $\frac{1}{2}mv^2$. The energy input, when obtained in this way, should agree within a factor of two with the energy inherent in the measured temperature rise.

EXERCISE 15: Experiment. (a) Obtain an electric egg beater and run it at full rate for 5 minutes in a measured mass of water. (If you put the water in an aluminum pan, you may allow for the heating of the pan also. 1000 grams of aluminum take as much heat as 200 grams of water. An ingenious way to avoid having a container that takes heat is to hold the water in a thin plastic bag like the ones used in a deep-freeze--but this makes a good deal of mess if the egg beater touches the bag.)

(b) Repeat the experiment for twice the time; and again with half or twice the mass of water. In each case, estimate the heat delivered by the egg beater per minute.

As the result of exercise (12) shows, experiments on the mechanical equivalent of heat are very difficult to do accurately. The expected rise in temperature due to the work done on the water is very small, and may easily be masked by the other ways in which heat can be exchanged with the surroundings. With this particular experimental arrangement, with water at room temperature (to avoid ordinary heating or cooling due to contact with the air), it is difficult to avoid a drop in temperature. This is due probably to the increased evaporation caused by the agitation of the water. If the container is reasonably well sealed off from circulating air, this effect will be minimized.

EXERCISE 16: Estimate the energy delivered in 5 minutes by a small electric light of known wattage. Do this (i) experimentally by running the lamp in ink (why ink?) (ii) by calculation from other knowledge: note that one kilowatt hour, which is likely to cost you 3 or 4 cents, is worth 3.6×10^6 joules.

By using an automobile headlight bulb connected to a 6-volt battery, and immersed in a small bottle of ink, an easily measurable temperature rise can be obtained (about 2°C in five minutes). The students should be warned not to use light-bulbs of higher wattage powered by 110 volts, because of the danger of electric shocks.

(i) The lamp is immersed in ink so that the radiation emitted by it will almost all be absorbed.

(ii) The calculated rise in temperature, with the above arrangement, agrees with the measured one to better than a factor two. For a 15 watt lamp, the total energy delivered in 5 minutes is: $5 \times 60 \times 15 = 4500$ joules.

If we use about 100 cm^3 of ink, the temperature rise should be about
$\frac{4500}{100 \cdot 4.2} \approx 11$ centigrade degrees.

EXERCISE 17: A high-speed swimmer uses up in his muscles food fuel that releases 120,000 joules of energy in a half-minute race. Three-quarters of that energy is released as waste heat, however he swims. The rest is dissipated by his hands and legs by mechanical work.

(a) In that time, 30 seconds, he swims 100 meters. Estimate the average force opposing his motion.

(b) Describe the changes of form of energy in the swimming.

(c) Where and in what form is the energy that has been released when he has finished the race?

(a) If the average force opposing the swimmer's motion through the water is F nt., then in swimming 100 meters against this force he does work equal to $Fx = 100F$ joules. He uses up 120,000 joules, of which only a quarter i.e. 30,000 joules goes into useful mechanical work. Hence

$$100\ F = 30{,}000$$

$$F = 300 \text{ nt.}$$

This is the average force opposing his motion. (This speed for 100 meters is almost twice as fast as the world's record.)

(b) Chemical energy of the food he has eaten is turned into mechanical energy by his muscles. The swimmer's kinetic and potential energies remain constant, of course, since his velocity is roughly constant, and he swims on the surface of the water! Hence all of the useful work he does is taken up in overcoming the resistance of the water. It becomes kinetic energy of turbulent motion of the water through which he has passed, and it gradually dies down, leaving the water at a slightly higher temperature.

One could ask why swimmers do not try to create less turbulence by making smoother motions. Firstly, a racer's aim is to get through the water as fast as possible, not as easily as possible. Long-distance swimmers do use a much smoother stroke, partly to reduce this energy loss. Secondly, it is impossible not to create some turbulence. You can show from hydrodynamics that the only way to get "thrust" from the fluid is to create eddies--otherwise you just don't move.

(c) We have already answered this in (b): the energy expended finishes up as heat; the water in the pool is slightly warmer. With a pool of usual dimensions (30m x 15m x 3m) containing about 10^6 kg of water, the temperature rise due to the addition of 120,000 joules would be

$$\Delta T = \frac{120{,}000}{4.2 \times 10^3 \times 10^6}\ {}^\circ K = 3 \times 10^{-5}\ {}^\circ K.$$

We have included all of the chemical energy that the swimmer expended, since that part of it which goes to increase the swimmer's body temperature will also contribute to heating up the water. In any case we see that cooling the pool will not be a major problem!

EXERCISE 18: With what velocity must a 2-kg block of ice at 0° C be thrown against a stone wall so that the entire mass of ice melts upon contact? 1 kg of ice takes 3.36×10^5 joules to melt. What effect does the mass have on the velocity?

To melt a 2 kg block of ice at 0°C requires

$$2 \times 3.36 \times 10^5 \text{ joules} = 6.72 \times 10^5 \text{ joules}$$

In order for the block of ice to melt on contact with the wall (we assume that all of its kinetic energy is transformed into heat of melting of the ice) then

$$\frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times v^2 = 6.72 \times 10^5 \text{ joules}$$

$$v = 6.72 \times 10^5 \text{ m/sec.}$$

$$= 820 \text{ m/sec.}$$

The mass enters the calculation twice, in such a way as to cancel out. The heat required to melt the ice is proportional to the mass, but so is the kinetic energy for a given velocity.

The problem is somewhat unrealistic. When jet planes achieve speeds such as we have obtained above, the surface of the plane becomes red hot, due to friction with the air. If our block of ice travels very far before reaching the wall, it will all have melted before it gets there.

EXERCISE 19: One mole of helium at temperature 24°C is placed in contact with water at 26° C and insulated from the rest of the world. The final temperature of both is measured to be 25° C. How much water was there?

A simple numerical exercise concerning the specific heat of an ideal monatomic gas.

It takes 12.4 joules of energy to raise the temperature of one mole of a monatomic gas like helium one degree. Hence the helium must have absorbed 12.4 joules of energy from the water. The water cooled one degree. Since it takes 4.2 joules to raise the temperature of 1 gram of water one degree, the amount of water which will yield 12.4 joules when

cooled one degree is

$$\frac{12.4}{4.2} = 2.96 \text{ gm.}$$

The mass of water was about 3/4 the mass of helium. (One mole of helium is 4 gms.)

Exercises (20), (21) and (25) involve detailed qualitative descriptions of the processes by which energy transformation occur.

EXERCISE 20: Suppose you have two large boxes of negligible mass. Box A contains one mole of hot helium at 60°C, the other box B, contains one mole of cool argon (also a noble gas) at 10° C.

(a) The two boxes are placed side by side in contact with all their outer surfaces insulated. After some time, both gases are at the same temperature. What is the temperature and why?

(b) Instead of being placed side by side, the two boxes are joined together to make one large box without change of volume, so that the gases mix. What will be the final temperature of the mixture?

(c) Now suppose that box B contains one mole of cold nitrogen at 10° C instead of the argon. The hot helium and cold nitrogen are allowed to mix as in (b) above. Will the final temperature be the same as in (b) or higher or lower?

(d) Give a clear reason for your answer to (c).

Both helium and argon are monatomic gases, and therefore require 12.4 joules of energy to raise one mole of either of them one degree. (Note that it is assumed that the two sides which are in contact are not insulated.)

(a) From the point of view of their temperature response when heat is transferred to them, the two gases are identical. Thus the one will cool by as many degrees as the other is heated. Since they have the same final temperature, this temperature must be just half-way between their initial temperatures, i.e. 35° C.

In the form of an equation, our solution is as follows: with a final temperature T°C, the helium will have lost an amount of heat energy 12.4 (60°C-T). The argon will have gained an amount of heat energy 12.4 (T-10°C). Since no energy is lost to the outside world, these energy changes must be equal:

$$12.4\,(60^\circ - T) = 12.4\,(T - 10^\circ C)$$

$$T = \frac{1}{2}\,(60^\circ C - 10^\circ C)$$

$$= 35^\circ C.$$

The energy is transferred through the common wall separating the gases. The rapidly moving helium atoms strike this wall and give some of their K. E. to the atoms in it, leaving with slightly reduced energy. This increase in kinetic energy of the atoms on the helium side of the wall is shared, by mutual interactions in the solid, with all of the atoms in the wall. (The heat is "conducted" through the wall.) The slowly moving argon atoms strike the rapidly vibrating atoms of the wall, and on the average go off with slightly increased kinetic energy, the wall atoms losing a corresponding amount of energy. Thus heat energy is transferred from the helium to the argon, and the process stops only when the temperatures are the same, i.e. when the average kinetic energy of the helium, argon, and wall atoms are all the same. There is then (on the average) no tendency for them to change any more, and the system is in equilibrium.

(b) In equilibrium, the double box will contain a mixture of the two gases (they are "noble" gases, and do not combine chemically with anything), all at a uniform temperature. That means that all of the molecules (helium and argon) have on the average the same translational kinetic energy. No energy is gained or lost from the system, so the kinetic energy gained by the argon in heating up to the final temperature T must be exactly the same as the kinetic energy lost by the helium in cooling down to T. The situation, so far as temperature changes are concerned, is identical with that in (a). The final temperature will therefore be also 35°C.

The energy exchange has now been effected by direct collision of the more energetic helium atoms with the less energetic argon atoms, without the intervening wall. This does not affect the final temperature. There is however, a physical difference between the two ways of achieving equilibrium: in the second case, we have a mixture of the two gases, and one would have to go to some trouble (using some rather complicated techniques of physics) to separate them again. We have lost something. This is the sort of physical process which the second law of thermodynamics describes. It says that the entropy of systems always tends to increase; in simpler terms, the randomness i.e. the inability to make definite statements about the properties, always tends to increase. After (b) we can no longer say which box the argon or helium is in, whereas before, we could.

(c) The difference between this and (b) is that nitrogen is a diatomic gas. When it is heated, some energy must be transferred to the rotational degrees of freedom. In fact, as we calculated in problem (4), it takes 20.5 joules of energy to raise the temperature of one mole of any ideal diatomic gas by one degree.

Not all of the energy supplied by the cooling helium will appear as increased translational kinetic energy of the nitrogen. Consequently its rise in temperature will be less than that of the argon. Its final temperature will therefore be lower.

(d) Our reason has already been given in (c). We can obtain the actual final temperature T by modifying the equation used in part (a). The energy supplied by the cooling helium is still $12.4(60^\circ - T)$ joules. For the nitrogen, the increase in energy is now $20.5(T-10^\circ)$. Since the energy lost by the helium must be the same as that gained by the nitrogen,

$$12.4\,(60^\circ - T) = 20.5\,(T-10^\circ),$$

$$T = \frac{12.4 \times 60 + 20.5 \times 10}{20.5 + 12.4}$$

$$= 28.8^\circ C$$

EXERCISE 21: A rocket is equipped with insulated containers full of very hot gas instead of combustible fuel. The hot gas rushing out of a nozzle drives the rocket forward.

(a) Where does the momentum that the rocket acquires come from?

(b) Where does the rocket's kinetic energy come from?

(c) If the gas is ejected into a box standing on the ground as the rocket starts out, and the temperature of the gas collected in the box is measured after the rocket has left, do you expect that temperature to be higher, lower, or the same as the original temperature of the store of gas in the rocket?

This exercise, while very topical, involves rather difficult and subtle reasoning. It is probably not worth while to spend much time on the detailed mechanism, although a complete description is given here.

(a) The total momentum of the whole system of rocket, fuel and expelled gas must be conserved. A rocket acquires a forward momentum by ejecting mass with considerable momentum in the backward direction. This is true for any type of rocket. In the present gas-filled rocket, the "fuel" chamber contains many molecules, travelling with high momentum in all directions, and the nozzle allows only those travelling in the backward direction to leave the rocket. The momentum of an individual molecule inside the rocket is not constant--it is continually exchanging momentum with other molecules and with the rocket itself--so that although molecules travelling in just the right direction to escape through the nozzle will do so, other molecules will very quickly also acquire this correct momentum. This explains how there will be a steady stream of gas out of the nozzle. It also explains how, from an initial state in which the rocket is at rest and the momenta of the molecules are distributed randomly, it is possible to get to the final state in which the rocket is travelling with a high speed in the forward direction, and the gas molecules are travelling, on the average, in the opposite direction.

Where, actually, does the push come from? If we were looking at the gas as a continuous fluid exerting pressure on the container walls, the forces produced by the pressure would not balance because of the hole

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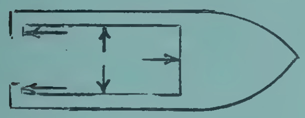

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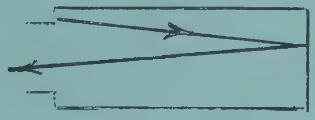

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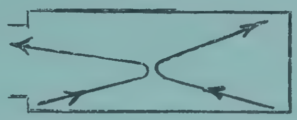


at the back of the rocket. The net force on the rocket would be PA, where A is the area of the hole. From the point of view of molecules, when does the push occur? For a molecule which travelled in the path shown in Fig. (ii), the exchange of momentum between it and the rocket came when the molecule hit the front wall of the fuel chamber. For the one in (Fig. (iii), it will occur when the other molecule which provided just the correct momentum reaches the front wall. In more complicated paths, the momentum might take longer to reach the body of the rocket, but eventually we know that the system of rocket plus the rest of the fuel molecules will acquire a forward momentum just equal and opposite to that of the escaping molecule.

(b) There has been no change in the total energy of the system. (We assume that the rocket is in outer space, where there will be no air friction or gravitational attraction.) The kinetic energy of the rocket plus the kinetic energy of the backwards-travelling molecules must be exactly equal to the initial thermal energy of the gas molecules.

[Is this compatible with the Second Law of thermodynamics? This law says that in a heat engine, some of the thermal energy at high temperatures is converted to work, but some is rejected as waste heat at a lower temperature. The stream of escaped molecules move predominately in the backward direction; so their kinetic energy is partly kinetic energy of bulk motion. They are not all moving with the same velocity, however, so the stream also has thermal energy. This is the heat rejected at the lower temperature, as required by the second law. The actual temperature of the stream is difficult to calculate. It depends very much on the collisions which a molecule makes while escaping through the nozzle. We are, moreover, considering systems not in equilibrium, so simple thermodynamics cannot be applied safely.]

(c) This we have answered in parts (a) and (b), essentially. Because of energy conservation, since the rocket has acquired kinetic energy, the gas must lose some, i.e. it will be at a lower temperature. We saw from Figs. (ii) and (iii)

that on the average an emerging molecule will have slightly less momentum, and hence the gas will have slightly lower temperature, because some momentum (and energy) has been given to the rocket in the last collision made by the molecule inside the rocket.

There might seem to be a difficulty in reconciling this result with our answer to problem (25c). That problem relates to a "free expansion", which is essentially what is happening to the gas in the rocket. Its property is usually taken to be that for an ideal gas there is no temperature change. This result is obtained, however, by assuming that there was no change in energy of the container; either the gas expands uniformly in all directions, or the container is rigidly attached to the earth, effectively an infinite mass, so that although the requisite momentum is transferred to the earth, the energy associated with this small momentum transfer to a large mass is negligible.

EXERCISE 22: A new small airplane is to be tested. To maintain secrecy, the test is conducted in a large, closed hangar whose walls are completely insulated from the outside world. The airplane is filled up with 10 kg of gasoline, flies around and around the hangar for half an hour, and then lands with its gas tanks empty again.

(a) In what form was the energy that the airplane uses in flying supplied to it originally?

(b) Discuss the changes of energy during the flight.

(c) Where is the energy and in what form, half an hour after the flight?

Qualitative discussions of the energy transformations involved in running an internal combustion engine, and in the mechanism of flight, are required. Very instructive for the mechanically minded students.

(a) Clearly the gasoline provides the energy. The energy is in the form of chemical potential energy of the gasoline molecules.

(b) The energy transformation resembles those occurring during the firing of a bullet [see exercises (10) and (11)]. An internal-combustion engine is, as we know, a device for transforming the chemical energy of the gasoline into work (and also waste heat energy) as a continuous process. The various forms that the energy takes in the engine are as follows:

(I) chemical energy of the gasoline-air mixture;
(II) thermal energy of the hot, high-pressure gas produced when this mixture burns;

(III) work in pushing back the pistons, which eventually (with a little frictional loss) becomes work in turning the propeller at a constant speed.

(IV) The next stage involves the mechanism of flight, which is complicated. The essential point is that in order to progress at a constant speed through still air, an airplane must do work at a constant rate. This work becomes kinetic energy of bulk motion of the air the airplane has passed through.

(V) The turbulent motion of the air gradually dies down as the relatively ordered motion of eddies changes into completely random thermal energy of the air molecules, due to air viscosity.

(Va) There is also the exhaust heat from the engine, an unavoidable consequence of trying to change heat energy into work.

The transformation (I)→(II) is a chemical reaction in which, under the stimulation of the intense point of heat provided by the spark-plug, the gasoline-vapor and air mixture changes into exhaust gases whose molecules are tighter atomic structures. The potential energy which the molecules have given up appears as kinetic energy of thermal motion of these very hot gases.

(c) Half an hour after the flight, all of the bulk kinetic energy has been changed to thermal energy of the air in the hangar. The end result is then that the air temperature has been increased a little.

Let us estimate the rise in temperature. The energy supplied when 1 kg of gasoline burns is approximately 4×10^7 joules. The total energy supplied is therefore 4×10^8 joules. Suppose that the hangar is 100m long, 100m wide and 50m high. Its volume is thus 5×10^5 m^3. Since 1 mole of air at roughly room temperature and at atmospheric pressure occupies 2×10^{-2} m^3, there are

$$\frac{5 \times 10^5}{2 \times 10^{-2}} \approx 2 \times 10^7 \text{ moles}$$

of air in the hangar. The molecules of oxygen and nitrogen that constitute air are diatomic, so 20.5 joules are needed to raise the temperature of one mole one degree. The rise in temperature of all of the air in the hangar is therefore

$$\frac{4 \times 10^7}{20.5 \times 2 \times 10^7} = 0.1^\circ \text{ K.}$$

It is only a very small effect.

EXERCISE 23: Imagine you have a light bulb in a perfectly reflecting box which is otherwise empty--a perfect vacuum.

(a) By running the light bulb do you produce
  (i) kinetic energy of bulk motion?
  (ii) potential energy of the bulk matter of the box?
  (iii) thermal energy of internal atomic motion?
  (iv) thermal energy in the form of inter-molecular potential energy?
  (v) internal energy in some form?
  (vi) no additional energy in the box?

(b) By running a light bulb, do you produce
  (i) momentum of bulk motion?
  (iii) momentum of thermal motion of atoms?
  (vi) no momentum in the box?

An exercise involving the concept of radiation as a form of energy, subject to the same conservation laws as bulk mechanical energy and thermal energy. An understanding of radiation theory is far beyond the scope of this course, but an acceptance of these properties of radiation will enable the students to have a more complete picture of the flow of energy, and its transformation, in the whole solar system.

(a) The system under consideration consists of the heated filament, and whatever else is present in the evacuated space enclosed by the perfectly reflecting walls. Suppose that the system is initially at absolute zero: it contains no energy of any form. Suddenly the current in the filament is switched on. Electrical energy is being fed into the filament. Some of it is converted into heat, and the filament becomes hot. The filament then emits radiant energy. This radiant energy cannot escape through the walls of the box, but is completely reflected. Some of it will be reabsorbed by the filament. Briefly, the electrical energy fed into the system produces thermal of the heated filament, and radiant energy in the space enclosed by the box. Nothing moves. There are no other ways that energy can be stored in this system. We can now answer the first parts of the exercise.

(i) There is no kinetic energy of bulk motion.
(ii) There is no potential energy of bulk matter.
(iii) The only thermal energy of internal atomic motion in the bulb itself.
(iv) + (v) There is no place where intermolecular potential energy or any other kind of internal energy can be stored.
(vi) There is additional energy in the box: it is in the form of radiation.

(b) The filament and its enclosure is an isolated system, whose momentum stays constant. Assume that it is zero initially, and then it must always be zero. The radiation is continually being reflected from each of the walls of the enclosure. Thus parts of it are moving in all directions (like the molecules in a gas) and the total momentum averages to zero. Since the momentum of the whole system is also zero, the box and filament must have zero momentum. Thus

(i) There is no momentum of bulk motion.
(ii) The only momentum of thermal motion of atoms is in the filament, and it adds up to zero.
(iii) The radiation in the box is a form of energy, and has momentum associated with it, but the net momentum is zero.

Students may ask what happens to the system as more and more energy is fed into it by the electric current. The answer is inescapable: since no energy can leave the system by any means, the energy content must increase continuously, that is, the "temperature of the radiation" (and that of the filament) would increase. The bulb would quickly burn out. It is interesting to consider the system in equilibrium, i.e. with current now switched off. No energy is entering or leaving the system, so its temperature must stay constant. The filament is emitting radiation all of the time, and since the amount of radiant energy is constant, as much radiation must be absorbed as is emitted by the filament. Since the walls are perfectly reflecting, the only place where the radiation can be absorbed is by the filament. There is a continual emission and reabsorption of radiation by the filament, with the net amount of radiant energy staying constant.

You can contrast this situation with that of actually switching off an electric light. The temperature of the filament drops in a fraction of a second to a temperature so low that radiation in the visible wavelength region (i.e. light) is no longer emitted. This is because the glass envelope of a light bulb does not reflect very much radiation, so that most of it escapes.

As regards the momentum of the radiation, you can point out that if, in the original system of a perfectly reflecting enclosure, a hole is made at one side then radiation can escape. It will carry momentum with it, so that to conserve momentum, the filament and enclosure will start to move in the opposite direction. In principle, we have here a rocket propulsion device very similar to that discussed in exercise (21). It is probably not practicable because to obtain a useful amount of "thrust" requires temperatures much higher than can be obtained terrestrially.

EXERCISE 24: An average mountain climber can climb about 1,500 feet or 500 meters vertical rise per hour.

(a) How much energy does such a climber gain as gravitational potential energy in a five-hour climb up a mountain?

(b) The human body is an inefficient chemical-mechanical machine. At best its muscles deliver only 2.5 percent of the chemical energy used as useful mechanical energy. The other 75 percent or more is dissipated as waste heat. Assuming this efficiency, how much chemical energy does the climber use in 5 hours?

(c) Assume that, aside from mountain climbing, he needs 2.2×10^6 small calories per 24 hours. How much total energy should he take in in his daily diet if he makes that climb every morning?

(d) If the climber walks down the mountain every afternoon, he loses the potential energy he gained. Why does that not help him to reduce his diet?

(a) In five hours, the increase in height of the climber is 5×500 m = 2.5×10^3 m. If his mass is, say, 80 kg, his increase in gravitational potential energy is

$$mgh = 80 \times 9.8 \times 2.5 \times 10^3 \text{ joules}$$

$$= 2.0 \times 10^6 \text{ joules.}$$

(b) The above energy is only a quarter of that expended by the climber. The chemical energy used is therefore

$$4 \times 2.0 \times 10^6 = 8.0 \times 10^6 \text{ joules.}$$

(c) In calories, this chemical energy is

$$\frac{8.0 \times 10^6}{4.2} = 1.9 \times 10^6 \text{ small calories.}$$

This is just about the same as his daily calorie requirement, so he would in fact have to consume a total of $2.2 \times 10^6 + 1.9 \times 10^6 = 4.1 \times 10^6$ calories per day, i.e. double his requirements when not mountain climbing.

(d) The only way that the human engine accepts energy is in chemical form. It is not possible to increase its energy content by doing work on it (as you can to a spring-driven toy by winding it up). It is just not that kind of machine. When you awake in the morning, and feel that you need energy to go through the day's tasks, you don't connect yourself to an electric outlet (as you would if you stored energy like an electric cell). You don't connect yourself to some

mechanical device with moving parts that you can resist the motion of, in order to obtain energy from the work done on you by the machine. (You would if you stored energy by winding up a spring in your inside.) You have breakfast. The food contains chemical energy, which your body, by very complicated processes, can turn into work.

Discussion of this topic can be almost endless, and not too fruitful. The difference and similarities between the human engine and other kinds of engines can be brought out: even when "idling", it must be rigidly temperature-controlled, blood must be pumped, etc. An electronic computer uses almost as much energy when idle as it does when solving long problems, and a human being has some of the same attributes. When "working", energy must be expended in all movements, and even when not doing "work" in the mechanical sense, your body is expending energy internally. (Try holding your arm outstretched, unsupported, for a few minutes!)

EXERCISE 25: (a) A cylinder containing helium gas is closed with a movable piston which has negligible friction. A man pushes on the piston and drives it in quickly, compressing the helium. The helium heats up. Why does the helium heat up? Discuss the mechanism of the heating up in terms of molecular behavior.

(b) A cylinder with a movable piston contains compressed helium. The piston is released and the helium pushes it out and cools. Explain in terms of molecular behavior how the helium cools.

(c) A large box with a good vacuum in it contains a small bottle of compressed helium. A trigger is arranged to remove the stopper of the helium bottle. When the helium is let out of the bottle no change of temperature is observed after the release is all over. Explain, from the point of view of molecular behavior, why the helium does not change temperature when it expands into the big box.

(d) Although compressed helium shows no final change of temperature, some other gases show a noticeable cooling after they have expanded into vacuum from high compression. What does that tell you about these other compressed gases?

Another exercise involving a discussion of the detailed mechanism by which mechanical energy is changed into thermal energy of molecular motion. The complete treatment will probably require more time than you can afford, and the most important feature, occurring in part (a), has already been discussed in exercise (5).

(a) From the general discussion of the equivalence of energy of bulk motion and thermal energy, we can say that since work was done in pushing the piston into the cylinder against the pressure of the gas, the same amount of energy must appear somewhere. The only place it can appear is as in an increase in thermal energy of the gas, i.e. as a rise in temperature.

What actually happens? When the piston is at rest, the gas atoms continually collide with it and bounce off with the same kinetic energy as they had before they hit. (The component of momentum perpendicular to the piston is reversed, and the other components are unchanged. Since energy is

$$\frac{1}{2m}(P_x^2 + P_y^2 + P_z^2)$$

and does not depend on the sign of any component, it is unchanged.) When the piston is moving into the gas, the atoms hit a surface moving towards them. They recoil with increased kinetic energy. That is, on the average, we are increasing the kinetic energy of the molecules, i.e. the temperature rises.

We can check by an actual calculation that the energy of an atom, colliding with the advancing piston, is increased by the impact. Suppose the piston is pushed in with a speed u. An atom which approaches it with velocity v_x (this is just the component perpendicular to the piston) has a velocity relative to the piston of $v_x + u$. The mass of the piston is extremely large compared with the atomic mass, so the molecule recoils from the piston with this same relative velocity, i.e. $v_x + u$. Relative to the container, its velocity is therefore $(v_x + u) + u = v_x + 2u$. The increase in its kinetic energy is

$$\frac{1}{2}m\left[(v_x + 2u)^2 + v_y^2 + v_z^2\right] - \frac{1}{2}m\left[v_x^2 + v_y^2 + v_z^2\right]$$

$$= m(2v_x u + 2u^2).$$

It can be shown that the total increase in kinetic energy of the atoms is just equal to the mechanical work done in pushing the piston in.

(b) The argument is very similiar to that of part (a). Essentially, the atoms are colliding with a piston which is retreating from them. Since their momentum relative to the piston is reversed on impact, their momentum relative to the container is decreased. Thus, the thermal energy of the gas molecules decreases. On the other hand, work is done by the gas on the mechanism connected to the piston. It can be shown that the decrease in thermal energy is exactly the same as the work done by the expanding gas.

(c) This process is called "free expansion". The very important difference between it and the changes in volume produced by movement of the piston is that now the atoms never collide with a moving wall which could produce a change in their kinetic energy. The atoms will rush out of the opening in the bottle and will continue to travel until they meet the stationary wall of the box. Their kinetic energies will not change, and consequently the temperature will not change.

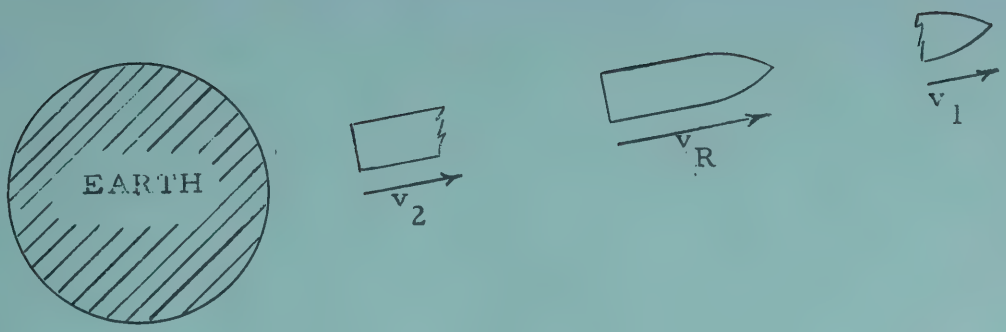
(d) In our previous arguments, we have neglected potential energy. For helium this is justified, since two helium atoms are very much like billiard balls: they only interact when they are almost touching. (They do not attract each other enough to form a molecule.) There are other gases, however, for which, although the molecules repel each other very strongly when very close together, there is a weak attraction between the molecules when they are relativel far apart. When the gas expands, the average distance between the molecules is increased. Their potential energy is therefore increased so there will be an equal drop in kinetic energy in addition to the changes produced by the means we discussed in part (b). Thus, even in a "free expansion" the potential energy of the molecules is increased, and their kinetic energy will decrease, i.e. they will cool.

EXERCISE 26: A rocket of mass 2×10^3 kg has been fired in an attempt to "escape" from the earth. However, when it is several thousand kilometers from the earth, its speed is only 30 m/sec, so it is evident that the rocket will fall back into the atmosphere. The control center on the surface therefore sends a radio impulse which fires off a small explosive charge (several kilograms) in the rocket. It explodes into two parts--one of them, with mass 0.5×10^3 kg, continues forward at 330 m/sec.

(a) Calculate the velocity of the other fragment.

(b) Calculate the kinetic energy of the original rocket, and of each piece after the separation.

(c) Why is there more kinetic energy afterwards than before?



(a) This is an isolated system, so momentum is conserved. We are given that one of the fragments continues in the same direction as that of the rocket, so the momentum of the other fragment must also lie on this line. As a vector equation,

$$P_R = P_1 + P_2$$

Since all momentum vectors lie on the same straight line, we may write the above vector equation as a simple algebraic equation, simply omitting the vector signs.

$$P_R = P_1 + P_2$$

The mass of the fragment whose velocity is unknown must be 1.5×10^3 kg, the difference between the rocket's mass and the mass of the first fragment. Substituting in our momentum equation,

$$M_2V_2 = M_RV_R - m_1v_1$$

$$= (2 \times 10^3 \times 30 - .5 \times 10^3 \times 330) \text{ kg.m/sec,}$$

$$=- 1.05 \times 10^5 \text{ kg.m/sec.}$$

Thus $$V_2 = \frac{-1.05 \times 10^5}{1.5 \times 10^3} = - 70 \text{ m/sec.}$$

The second fragment goes backwards with velocity 70 m/sec.

(b) The kinetic energy of the rocket is

$$\frac{1}{2}M_R V_R^2 = \frac{1}{2} x 2 x 10^3 x 30^2 \text{ joules}$$

$$= 9 x 10^5 \text{ joules.}$$

For the fragments, the total kinetic energy is

$$\frac{1}{2}M_1V_1^2 + \frac{1}{2} M_2V_2^2 = \frac{1}{2}\left[.5 x 10^3 x 330^2 + 1.5 x 10^3 x 49 x 10^2\right] \text{joules}$$

$$= 6.2 x 10^7 \text{ joules}$$

(c) The kinetic energy of bulk motion after the explosion is much greater than before. That there should be an increase in kinetic energy is to be expected--that is the only way to separate the two fragments. (The process is the reverse, in time, of one in which two bodies collide and stick together. In that case kinetic energy is always lost.) The increase in kinetic energy came from the detonation of the explosive charge, which transformed chemical energy partly into this bulk kinetic energy of translation, and partly into heating the rocket fragments.

EXERCISE 27: Experiment with the radiation from a glowing electric heating element. Place a sheet of asbestos tile (or a piece of damp cardboard will serve temporarily) between the heater and you. Make a small hole about 1 in. in diameter to let radiation through to you. Hold the back of your hand near the hole and feel the effect of radiation on your skin. Then try the following changes:

(a) Place a sheet of cardboard between the hole and your hand, and remove it quickly.

(b) Place a sheet of window glass between the hole and your hand and remove it quickly.

(c) Cover your hand with aluminum leaf (most easily done by clenching your fist, licking the back of your hand to make it wet with saliva, and placing a sheet of very thin aluminum leaf gently on it). Try the metal-coated skin near the hole.

(d) Keep the aluminum leaf on your hand but paint a little black paint or India ink on top of it. Again try that near the hole.

A home demonstration of energy transfer by radiation. The apparatus described works very well. What you observe is as follows:

(a) Quite noticeable heating of your hand. The reason for the sheet of cardboard which is suddenly removed is to enable you to make a reliable judgment of the heating effect. If you just put your hand near a heat source, or in warm water, very soon your hand becomes "used to the temperature", and you can no longer estimate the temperature. Your hand is only good for measuring sudden changes in temperature.

(b) The same effect as with cardboard is seen. Although glass is transparent to visible light it absorbes the long-wavelength radiation which produces the heating of your hand.

(c) No heating of the hand. The shiny aluminum surface is a poor absorber of heat, and the incident radiation is largely reflected.

(d) The hand can become uncomfortably warm! The dull black surface absorbs heat very well.

In connection with the discussion at the end of exercise (23a), where we obtained the result that good absorbers are good emitters to the same degree, one may ask why the blackened foil in part (d) does not radiate away all of the heat it absorbs, and leave the hand unaffected. The answer lies in the fact that the foil does not come to equilibrium with the radiation. In an experimental arrangement in which equilibrium would be reached, both shiny and blackened foils would reach the same temperature. In the arrangement of this problem we have a non-equilibrium situation. The rise of temperature of the blackened foil is then much faster than that of the shiny one. [As a matter of fact, when cooling occurs by convection and conduction as well as by radiation, the good absorber will always reach a higher temperature than the poor one.]

EXERCISE 28: Use the skin of the back of your hand or, better, your cheek to detect the radiation from a hot surface. For the source, use either a bright metal can of boiling water or, better, a bright sheet of copper which has just been made very hot with a gas flame. In each case, paint one side of the source black (a mixture of soot and alcohol serves well). Then make your estimate near each surface, bright and black, in turn.

A home demonstration of the fact that good reflectors of radiation are bad emitters and absorbers. It is not too easy to detect the radiation emitted by the shiny surface, but that emitted from the blackened one is very noticeable. (If you use a heated copper plate, be careful when testing the shiny side not to put your cheek too close!)

The shiny copper sheet will be dulled somewhat when heated, but this should only increase the radiation, and so the conclusion that shiny surfaces emit much less radiation than dull surfaces is not affected.

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